Confidence-Interval Based Multi-Objective Constrained Probabilistic Optimal Capacitor Placement in Distribution Systems

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Abstract

This paper presents a probabilistic confidence-interval based method for optimal placement of shunt capacitors in distribution networks by considering probabilistic characteristics of loads. The main objective function are reducing loss and improving the voltage profile. Backward forward sweep method has been employed to obtain the power flow results in distribution system. In addition, Integer Harmony Search Algorithm has been used to solve the optimization problem. The probabilistic aspects of problem have been solved using point estimation (PE) method. The novelty of this paper is introducing a confidence interval index using Gram-Charlier expansion. This index shows the risk of system to violate its security constraints when loads are considered as stochastic random variables. Using this index, planners can find the optimal sitting and sizing of capacitors in distribution systems based on their desired level of risk. Verification of the proposed method has been tested on 33-bus radial distribution system. Results demonstrate the effectiveness and merits of the proposed method.

KeyWords: Point Estimation method, Integer Harmony Search Algorithm, Optimal Capacitor Placement, Probabilistic Methods, Gram-Charlier expansion.

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Nomenclature

Z	function of *n* independent random variables
μ_Z	Expected value of random variable Z
M_{kZ}	k^{th} central moment of random variable Z
f(X)	Joint probability density function of X
n	Number of random variables
σ_j	standard deviation of the j^{th} variable
$x_{j,k}$	Estimating points in point estimate method $(k=1,2)$
p_j	Weights of estimating points in point estimate method
$\lambda_{j,3}$	Skewness of a random variable
$\lambda_{j,4}$	Kurtosis of a random variable
$Z_{m,k}$	output of the function for the point constituted by k^{th} point of m^{th}
$\varphi(x)$	PDF of normal distribution
C_i	constant coefficients obtained from cumulants of random variable
k_i	cumulants of a random variable
X	state vector
C	control vector
σ_j	Standard deviation of j^{th} variable
P_{g_j}	active powers injected to bus j
P_{g_j}	Reactive powers injected to bus j
P_{d_j}	active power demands at bus j
Q_{d_j}	reactive power demands at bus j
Q_{C_j}	total reactive power injected to bus j
$\alpha_{R\%}$	value that guarantee bus voltage to be in the admissible range
$\beta_{R\%}$	value that guarantee bus voltage to be in the admissible range
$\gamma_{R\%}$	value that guarantees line current amplitude to be less than the maximum

1. Introduction

Distribution systems are radial and too long. Because of such topology and the fact that current amount is high in distribution systems (i.e. due to their low voltage level), the ohmic loss in these networks is high and voltage at the end of these feeders has very poor regulation.

Optimal placement of capacitor banks in distribution networks results in reduction of power loss, improved voltage profile, and releases reactive capacity of power apparatus. Optimal Capacitor Placement (OCP) is a well-discussed subject of many other papers [1-14]. In [2], Ant Colony (AC), in [3], Tabu Search (TS), in [4 ،5]

Particle Swarm (PS), in [6-9] fuzzy logic theory, and in [10-14] Genetic Algorithm (GA) have been used to solve optimal capacitor placement problem. Recently, in many researches it has been found that Harmony Search Algorithm (HSA) has a better solution for such allocation problems because of its useful capabilities [15-17]. HSA has good robustness and does not need many mathematical requirements due to its evolutionary nature. The evolution operators make HSA effective at finding global minima, unlike the so-called trajectory methods, which only compare the nearby points in the search space. Due to its simple structure, HSA can find the optimal point in less time than many other evolutionary methods, which well suits high computation burden of probabilistic methods.

On the other hand, the deregulation of electric markets calls for consideration of financial costs and power quality issues with respect to real network conditions. Because of the probabilistic nature of loads, it is important to consider load forecasts to attain real system conditions. Load forecasts are probabilistic and may have different probability density functions and their variations may not be in sync, so the distribution system may encounter all combinations of loads. To address this issue, probabilistic methods should be used to identify real network conditions. Unfortunately, in many related researches, load variations have not been considered [1 ،2 ،18-21], and in some others, load variations have been considered at two or three different levels [6 ،7 ،12].

One of the most common methods to solve probabilistic problems is Monte-Carlo Simulation (MCS). This method solves problems by generating suitable random numbers with respect to input variables’ PDFs, solving the problem in a deterministic manner for each set of generated numbers, and determining the PDF of the outputs by analyzing the results. However, a huge computation burden makes this method time consuming [22].

Another probabilistic approach to find different moments of probabilistic output variables is Point Estimation (PE) method. PE has been widely used in literature to study different aspects of power systems including load flow [23-25]. PE considers just the first four moments of input variables’ PDFs to estimate the whole PDF. PE takes less time and has less computation burden than MCS while having a desirable accuracy.

When dealing with a probabilistic problem, a certain degree of reliability should be taken into account because of the probabilistic nature of both state and dependent random variables. In [26], the POCP problem has been solved using PE method, and all loads and state and dependent variables are considered to have a normal PDF. However, because of the nonlinear nature of the power system, neither the loads, nor the state and dependent variables, have normal PDF. Furthermore, the only degree of reliability used in this reference is the six-sigma index which does not represent the real system conditions. It has been shown in literature [27-29] that a PDF can be described by its first few moments using Gram-Charlier series expansion. Therefore, to meet the constraints of a probabilistic problem for a certain degree of reliability, Gram-Charlier series expansion can be used to approximate PDF and CDF of dependent variables.

To the best of our knowledge, reliability based point estimation method has not previously been applied to the problem of multi-objective constrained probabilistic optimal capacitor placement. In this paper, PE method has been used along with IHSA to solve the probabilistic optimal capacitor placement in radial distribution systems. Gram-Charlier series expansion is used to approximate the PDF and CDF of dependent variables and meet the probabilistic constraints for a certain degree of reliability.

The rest of this paper is organized as follows: Section 2 introduces different methods used in this paper. Problem formulation is illustrated in Section 3 and different probabilistic aspects of the minimization problem are also described. In Section 4, the method described in this paper is tested on a modified 33-bus radial distribution system and results are compared and discussed. Finally, the paper is ended with conclusions section.

2. Methodology

2.1. Point Estimation

In order to consider uncertainties, three points for each input variable are determined in this method and the problem is solved for each stochastic variable three times; once for the point below the mean, once for the point above it, and once for the mean point itself, while other variables are kept at their mean value. More details of this method are discussed below:

Let Z be a function of n independent random variables as follows:

$$Z = f(X) = f(x_1, x_2, \ldots, x_n) \quad (1)$$

Assume that the expected value and standard

deviation of the j^{th} variable are μ_j and σ_j, respectively.
The k^{th} central moment of the j^{th} variable with probability density function of $g_j(x_j)$ is calculated as follows:

$$M_k(x_j) = \int_{-\infty}^{+\infty} (x_j - \mu_j)^k g_j(x_j) dx_j \quad , \quad k = 1,2,... \tag{2}$$

And $\lambda_{j,k}$ is defined as:

$$\lambda_{j,k} = \frac{M_k(x_j)}{\sigma_j^k} \tag{3}$$

where $\lambda_{j,3}$ and $\lambda_{j,4}$ are known as coefficients of skewness and kurtosis, respectively.
The expected value of the function can be calculated using the following equation:

$$\mu_Z = E(f(x)) \tag{4}$$

Using Taylor series, expanding the function at the expected values results in:

$$m_Z = f(m_1, m_2, ..., m_n) + \sum_{i=1}^{\infty} \sum_{m=1}^{n} \frac{1}{i!} \frac{\partial^i f}{\partial x_m^i} (m_1, m_2, ..., m_n) \, l_{m,i} s_m^i \tag{5}$$

Let $x_{j,k} = \mu_j + \xi_{j,k} \ (k = 1,2)$ be predefined concentration points and $p \ (k = 1,2)$ be the probability concentrations at points $x_{j,k}$. Considering these assumptions, the constants to be determined are $\xi_{j,k} \ (k = 1,2)$ and $p \ (k = 1,2)$. By estimating the mean of points, it can be said that:

$$\mu_Z = f(\mu_1, \mu_2, ..., \mu_n) . \sum_{m=1}^{n} \sum_{k=1}^{2} p_{m,k} + \sum \sum \frac{1}{i!} \frac{\partial^i f}{\partial x_m^i} (\mu_1, \mu_2, ..., \mu_n)(p_{m,1} . \xi_{m,1}^i + p_{m,2} . \xi_{m,2}^i) \sigma_m^i \tag{6}$$

Now, by making the first four orders of (5) and (6) equal, and considering the fact that the sum of the probability concentrations should be equal to one (7), the constant parameters can be derived:

$$\sum_{m=1}^{n} \sum_{k=1}^{3} p_{m,k} = 1 \tag{7}$$

$$\xi_{m,k} = \frac{\lambda_{m,3}}{2} + (-1)^{3-k} \sqrt{\lambda_{m,4} - \frac{3}{4} \lambda_{m,3}} \quad , k = 1,2 \tag{8}$$

$$p_{m,k} = \frac{(-1)^{3-k}}{\xi_{m,k} (\xi_{m,1} - \xi_{m,2})} \quad , \quad k = 1,2 \tag{9}$$

$$p_{m,3} = \frac{1}{n} - \frac{1}{\lambda_{m,4} - \lambda_{m,3}^2} \tag{10}$$

For the point constituted by the means of all random variables, just one run of function f is needed, as the other runs are repetitious. So we refine the probability of this point as the sum of all these repetitive points.

$$p_\mu = \sum_{m=1}^{n} p_{m,3} = 1 - \sum_{m=1}^{n} \frac{1}{\lambda_{m,4} - \lambda_{m,3}^2} \tag{11}$$

Finally, j^{th} raw moment of function Z of several random variables X can be found:

$$\mu_j = E\left[Z^j\right] \cong \sum_{m=1}^{n} \sum_{k=1}^{3} p_{m,k} (Z_{m,k})^j \tag{12}$$

where $Z_{m,k}$ is the output of the function for the point constituted by k^{th} point of m^{th} variable and the expected value of other random variables.
In this paper, the first few moments are used to estimate the output PDF using Gram-Charlier expansion method.

2.2. Integer Harmony Search Algorithm

HSA is one of the most powerful metaheuristic methods which uses stochastic random search instead of a gradient one. HS algorithm was invented analogous to music improvisation. It was first used to solve discrete optimization problems. Later, it was modified to solve continuous optimization problems as well.

In order to improvise a harmony which matches aesthetic standards, musicians try to adjust the instruments' pitches in each try. In optimization problems, the same process is followed. At each iteration decision variables are refined to improve the objective function.

Integer Harmony Search Algorithm (IHSA) has better performance than many other optimization methods for solving discrete optimization problems. It also has few parameters, easy implementation and simple concepts.

At the first step, the optimization problem and algorithm parameters are initialized. The optimization problem is specified as follows:

$$\begin{aligned} &\textit{Minimize } f(x) \\ &\quad \textit{Subject to } x_i \in X_i \end{aligned} \tag{13}$$

where $f(x)$ is the objective function, x is the set of decision variables and X is the set of possible range for the values of x. The HSA parameters are also defined in this step. The first one is called Harmony Memory Size (HMS), which shows the number of solution vectors stored in the Harmony Memory (HM). Other parameters are the evolutionary ones which are Harmony Memory Consideration Rate (HMCR), Pitch Adjustment Rate (PAR), and Bandwidth (BW).
At the second step, HM is filled with a randomly generated set of decision variables in their corresponding possible ranges.

$$HM = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,N} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ x_{HMS,1} & x_{HMS,2} & \cdots & x_{HMS,N} \end{bmatrix} \tag{14}$$

Then fitness function is evaluated for each of these set of solutions.

In the third step, a new harmony is improvised using three rules (for each decision variable): 1) selecting a value from HM with the probability of HMCR, 2) selecting a value in a range (BW) near the value chosen from HM, with the probability specified by PAR. For integer variables, BW should be a set of integers around zero, like $BW = \{\ldots,-2,-1,0,+1,+2,\ldots\}$, 3) randomly generating a value in the permissible search space with a probability of (1-HMCR).

At the fourth step, the fitness function is evaluated for the new harmony. If the new harmony has a better fitness than the worst solution in the HM, it will take the place of the worst harmony in the HM.

At the final stage, the termination criterion is checked to see if the algorithm will stop or not. The common stopping criterion for HSA is the number of iterations which should be set at the first stage.

HSA steps are summarized in the flowchart shown in Fig.1. In this paper, IHSA is used to find the best location and size of capacitor units.

2.3. Gram-Charlier Expansion

In order to find the confidence interval of dependent parameters affecting the inequalities, Gram-Charlier expansion is used in this paper. According to Gram-Charlier expansion, PDF of many distributions can be formed as a series comprised of normal distribution PDF and its derivatives. Consider a random variable z with a continuous distribution and denote its mean and standard deviation as μ_z and σ_z, respectively. According to Gram-Charlier expansion, for the standardized variable $x=\frac{z-\mu_z}{\sigma_z}$, probability density function $f(x)$ can be written as follows:

$$f(x)=\varphi(x)+\frac{C_1}{1!}\varphi'(x)+\frac{C_2}{2!}\varphi''(x)+\frac{C_3}{3!}\varphi'''(x)+\ldots \quad (15)$$

where $\varphi(x)$ is the PDF of normal distribution. C_i are constant coefficients obtained from cumulants of random variable.

$$\begin{aligned} C_0 &= 1 \\ C_1 &= C_2 = 0 \\ C_3 &= -\frac{k_3}{\sigma^3} \\ C_4 &= \frac{k_4}{\sigma^4}-3 \end{aligned} \quad (16)$$

where k_i are cumulants of random variable z.

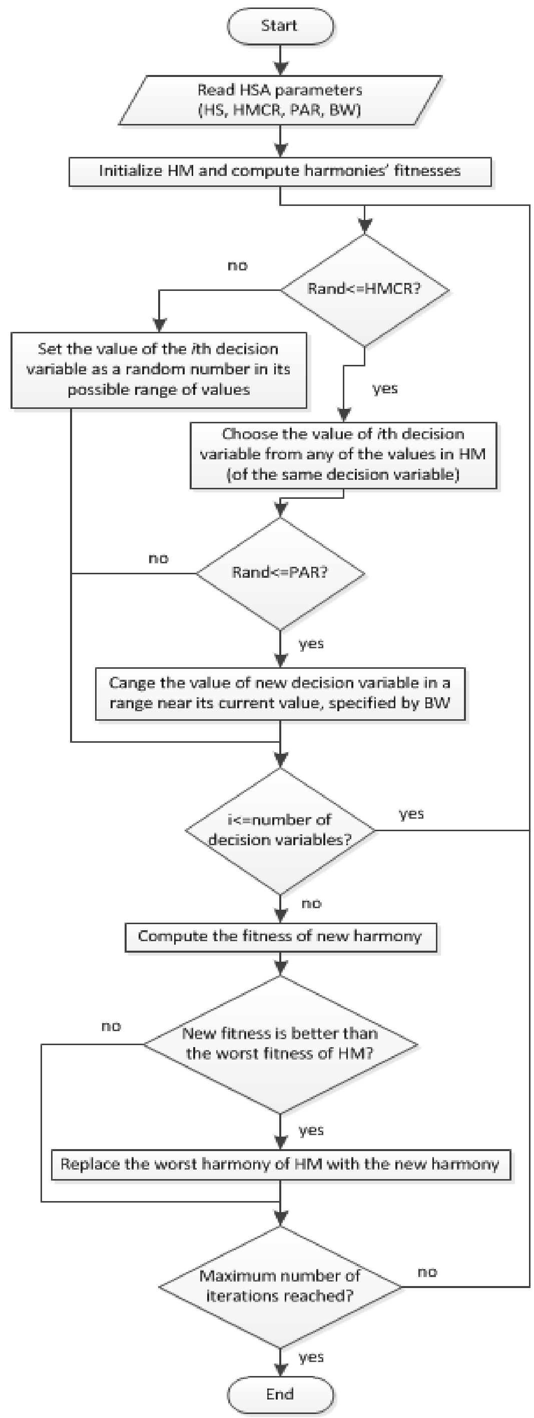

Fig.1. HSA Flowchart

Therefore, after finding moments of output variables using PE method, its cumulants are computed, and Gram-Charlier expansion can be utilized to rebuild the PDF of output variable.

3. Problem Formulation

Optimal capacitor placement is a mixed integer nonlinear optimization problem in which a defined objective function should be minimized while a set of equality and inequality constraints are met.

$$\begin{aligned} &Minimize \quad F_{obj}(X,C) \\ &\quad Subject\ to \\ &\quad G(X,C)=0 \\ &\quad H(X,C)\leq 0 \end{aligned} \tag{17}$$

where X and C are the state vector and control vector of the system, respectively. In the problem of optimal capacitor placement, state vector represents the magnitude and argument of the voltage of each busbar, and the control vector shows the sitting and sizing of capacitors at each busbar. F_{obj} , G(X,C) and H(X,C) are the objective function, and equality and inequality constraints. The deterministic solution of this problem is a well-studied subject of many papers in literature. However, when dealing with this problem as a probabilistic one, the definition of its structure will change as follows:

3.1. Probabilistic Multi-Objective Function

The multi-objective function used in this paper consists of three parts, namely capacitor cost, power loss, and voltage regulation. The last two parts are probabilistic dependent and state variables, respectively, and their expected value will be used in objective function. These objectives are detailed in the next subsections.

3.1.1. Expected Value of Power Loss

A remarkable amount of power used in distribution systems is due to line losses. So, a main objective of optimal capacitor placement problem is to minimize power loss. As a dependent variable, the expected value of power loss obtained from PE method is used in this paper. The formulation to obtain different moments of any function of several random variables is illustrated in section 2.1.

3.1.2. Expected Value of Voltage Regulation

One negative aspect of distribution systems is their considerable voltage drop at the end of long feeders. A common criterion to investigate the voltage quality in distribution systems is voltage regulation. In this paper, busbar voltages are state variables. So, their expected values can be used to calculate the voltage regulation index. For a system of n busbars, this index is defined as follows.

$$VR_{mean} = \sum_{i=1}^{n} \left|1 - V_{mean}^{pu}\right|^2 \tag{18}$$

A combination of these indices has been used to minimize the objective function. Weighting factors have been assigned to each index regarding the cost of saving energy on power loss and voltage drop of the system and the cost of capacitor units.

$$F_{obj} = C1.PL_{mean} + C2.VR_{mean} + C3.CC \tag{19}$$

Where, C1, C2 and C3 are weighting factors, PL_{mean} and VR_{mean} are the expected values of power loss and voltage regulation, respectively, and CC is the capacitor cost which is the product of number of capacitors and the unit cost.

3.2. Probabilistic Constraints

3.2.1. Power Equalities

In electrical systems, generated power should be equal to the demand. So the probabilistic power flow equations should be satisfied for any combination of active and reactive load demands, and capacitors placed at different buses:

$$P_{g_j} - P_{d_j} = 0 \tag{20}$$

$$Q_{g_j} - Q_{d_j} = -Q_{c_j} \tag{21}$$

where P_{g_j} and Q_{g_j} are active and reactive powers injected to bus j; P_{d_j} and Q_{d_j} are active and reactive power demands at bus j; and Q_{c_j} is the total reactive power injected to bus j by capacitor(s) placed at that bus.

3.2.2. Bus Voltage Limit

In electrical systems, one of the main targets is to feed the loads while keeping the frequency and voltage in an admissible range. When dealing with uncertainties of loads, bus voltages will be probabilistic variables as well. So, the maximum and minimum allowable voltage amplitude at all buses should be considered in optimal capacitor planning problem as follow:

$$V_{min} + \alpha_{R\%} < V < V_m - \beta_{R\%} \tag{22}$$

Where $\alpha_{R\%}$ and $\beta_{R\%}$ are two values found from the voltage CDF of each bus that guarantee bus voltage to be in the admissible range with a probability of $R\%$.

3.2.3. Maximum Line Current

Feeders of power system can handle different current amplitudes based on their gender, operating conditions, weather conditions, etc. So, as a safety rule, current amplitude in a line, which is a probabilistic variable, should not exceed its maximum allowable limit.

$$I_j \le I_{j\,max} - \gamma_{R\%} \tag{23}$$

where $\gamma_{R\%}$ is a value found from the current amplitude CDF of each line that guarantees line current amplitude to be less than the maximum allowable value with a probability of $R\%$.

3.3. Assessment Procedure of Probabilistic Inequalities

In each iteration of HSA, the validity of new harmony should be checked. In probabilistic optimal capacitor problem, probabilistic inequalities should be met for each harmony (set of capacitor size and site). The steps to assess the probabilistic inequalities of optimal capacitor placement problem introduced in this paper are mentioned here consecutively:

- For new harmony, find the bus voltage and line current moments, using PE method.
- Using GC expansion, find the bus voltage and line current CDFs.
- Find the voltage and current corresponding to the required confidence interval ($a_{R\%}, b_{R\%}, g_{R\%}$).
- Check if the voltage and current found in the previous step meet the probabilistic inequalities.

4. Simulation Results and Discussion

The proposed procedure of probabilistic optimal capacitor placement has been tested on a modified 33-bus radial distribution system illustrated in Fig. 2.

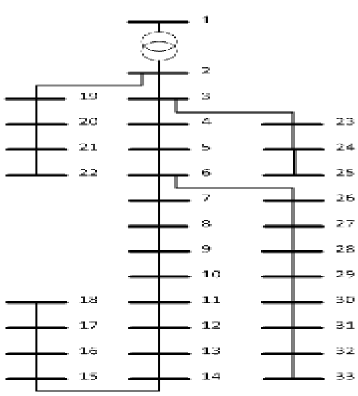

Fig.2. 33-bus Radial Distribution System

The load demands are considered to have Generalized Extreme Value (GEV) distribution. Location and scale parameters have been assumed to have the same value as the original load demands, and 10 percent of it, respectively. Details of GEV distribution are presented in Appendix. Shape parameter of all load demands are supposed +0.3. This corresponds to consumers that have light load most of the time and heavy load, rarely. Two different scenarios have been considered.

Scenario A: the optimal capacitor placement problem has been solved for two cases. In the first case, the deterministic optimal capacitor problem has been solved in which, load demands were supposed to have deterministic values equal to the expected value of their GEV PDF with parameters mentioned in the previous paragraph. In the second case, load demands are supposed to be probabilistic variables with the same GEV parameters. In this case, the probabilistic optimal capacitor placement problem has been solved for a confidence interval of 0.97, meaning that all probabilistic inequalities (bus voltages and line currents) should be satisfied 97 percent of the time.

Scenario B: in this scenario two different cases were also considered; in the first case, an approximate confidence interval of $\pm 3\sigma$ has been considered to counter probabilistic inequalities. In this case, the bus voltages and line currents were assumed to have a normal PDF in which $\pm 3\sigma$ indicates a confidence interval of 99.87 percent. Considering this, in the second case of scenario B, the same problem of probabilistic optimal capacitor placement has been solved using the Gram-Charlier method with a confidence interval of 0.9987.

In both scenarios mentioned above, the maximum permissible current and minimum permissible voltage have been considered 0.025 and 0.9 per unit, respectively. The results of each scenario have been compared and the profits of the method mentioned in this paper have been discussed.

Table 1: Optimal Capacitor Location and Size for Deterministic Case of Scenario A

Installed KVAR Capacity	Bus Number
50	8, 11, 16, 29, 30, 33
100	10
150	18, 31, 32
200	14

Table 2: Optimal Capacitor Location and Size for probabilistic Case of Scenario A with a confidence interval of 0.97

Installed KVAR Capacity	Bus Number
50	9, 10, 11, 30
100	12, 14, 15
150	16, 18
200	31, 33

Scenario A: tables 1 and 2 represent the sitting and sizing of capacitor units for deterministic and probabilistic cases, respectively. It is clear that when load demands are considered constant and no confidence intervals are taken into account, fewer capacitor banks are needed to minimize the objective function while assuring the equality and inequality constraints. However, in case of stochastic load demands, more capacitor units should be placed in the network in order to satisfy the probabilistic inequality constraints considering a predefined confidence interval.

Bus voltages for both cases of scenario A are well compared in Fig. 3. Continuous lines show the exact value (deterministic case) and the expected value (probabilistic case) of bus voltages. The confidence intervals of probabilistic case are also drawn. Fig. 3 shows the effect of considering stochastic nature of load demands on optimal capacitor placement problem for a certain degree of confidence. A similar discussion can be made for line currents and corresponding figures can be plotted. However, for the sake of conciseness, only bus voltages are compared.

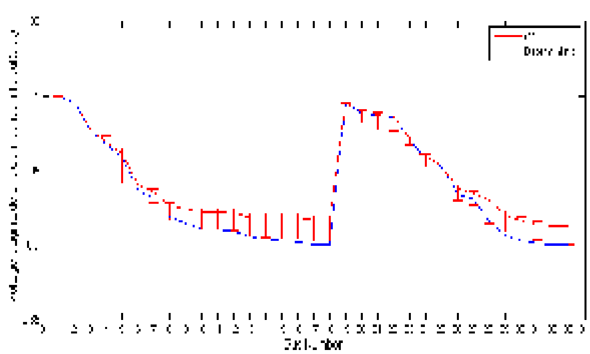

Fig.3. Comparison of bus voltages obtained from deterministic method (Blue Line) and expected values of bus voltages obtained from GC method (Red Line)

Fig. 4 shows the evolution of best fitness found by HSA method in both cases. It is found from Fig. 4 that there is no major difference between deterministic and probabilistic cases' fitness. However, in probabilistic case, a sensible confidence interval has been met. This is one of the merits of using the method proposed in this paper. Table 3 too, demonstrates the best objective values found by HSA. Probabilistic method also has lower loss and voltage deviation because of its greater capacitor investment.

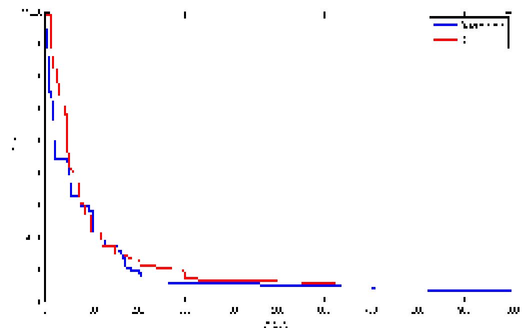

Fig.4. HSA best fitness evolution for deterministic case (Blue Line) and GC method (Red Line)

Table 3: Comparison of Deterministic and GC-Based Methods Used in Scenario A

	Deterministic	GC for CI=0.97	GC to Deterministic Ratio
Loss (Expected Value)	145.1826	139.9067	0.9637
Voltage Deviation (Expected Value)	0.1764	0.1446	0.8197
Total Installed KVAR Capacity	1050	1200	1.1429

Scenario B: in this scenario, the probabilistic optimal capacitor placement is solved for both methods of 3sigma and GC. Tables 4 and 5 show the best sitting and sizing of capacitor banks for 3sigma and GC methods, respectively. There is also a comparison of these methods' best objectives found by HSA in table 6. It should be noted that although the expected values of loss and voltage deviation are lower in 3sigma method, the capacitor investment is much lower in GC method. The reason behind this is that by correctly estimating the bus voltage and line current PDF and CDF, the exact confidence intervals can be found. In this way, the excess capacitor investment to meet the excess confidence interval of 3sigma would be reduced, while meeting the probabilistic inequalities of bus voltage and line currents.

Table 4: Optimal Capacitor Location and Size for 3sigma Case of Scenario B (CI=0.9987)

Installed KVAR Capacity	Bus Number
50	11, 12, 13, 15, 16, 23, 26, 29, 30
100	10, 31
150	8, 14, 33

300	7
350	17

Table 5: Optimal Capacitor Location and Size for GC Case of Scenario B (CI=0.9987)

Installed KVAR Capacity	Bus Number
50	8, 9, 14, 16, 28
100	33
150	18
200	13
250	17
400	32

Table 6: Comparison of 3sigma and GC-Based Methods Used in Scenario B

	3sigma	GC for CI=0.9987	GC to 3sigma Ratio
Loss (Expected Value)	137.7265	141.0444	1.0241
Voltage Deviation (Expected Value)	0.0942	0.1220	1.2951
Total Installed KVAR Capacity	1750	1350	0.7714

Fig. 5 and 6 show the expected value of bus voltage after optimal placement of capacitors found by HSA for 3sigma and GC method, respectively. The difference between estimated confidence intervals found by 3sigma and GC methods is well indicated in these two figures. Fig 7 also demonstrates the evolution of the best fitness found by HSA for GC and 3sigma methods.

So, by estimating the near-real CDF of bus voltage and line currents, lower capacitor investment is needed while inequalities are all met.

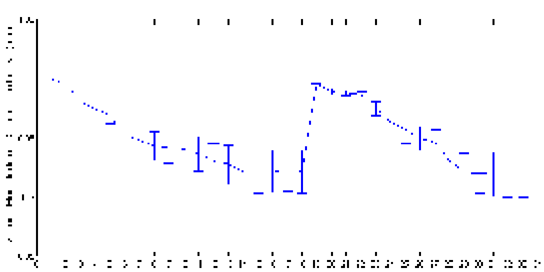

Fig.5. Expected value of bus voltage magnitudes and confidence intervals for 3sigma method

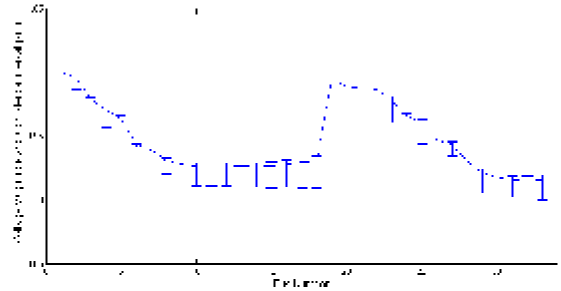

Fig.6. Expected value of bus voltage magnitudes and confidence intervals for GC method. (CI=0.9987)

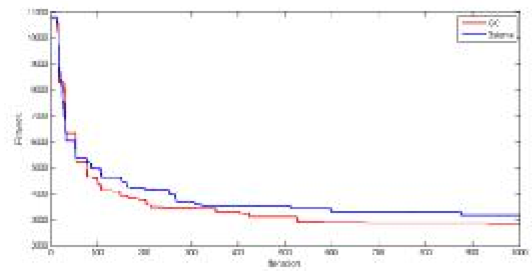

Fig.7. HSA best fitness evolution for 3sigma (Blue Line) and GC method (Red Line)

5. Conclusion

In this paper, a new method to solve the probabilistic optimal capacitor placement problem was presented. It was assumed that load demands have stochastic nature. Therefore, a probabilistic method was proposed to handle the stochastic aspect of the optimal capacitor placement problem.

In addition, Harmony Search Algorithm was exploited to solve the optimization problem. In order to speed up the optimization problem, Point Estimation method was utilized instead of Monte Carlo method to find the bus voltage and line current moments. Then, Gram-Charlier expansion has been used to find the near-real CDF of variables concerning the inequalities. It was shown that by having the CDF of state and dependent variables, the probabilistic optimal capacitor problem can be solved for an arbitrary amount of confidence interval. The proposed method was compared with 3-sigma method where commonly used in literature. Furthermore, it is assumed that load demands have Generalized Extreme Value distribution to clearly demonstrate the merits of this method. The method is verified by testing on a 33-bus radial distribution system. The results show that the proposed method can be used to assess the probabilistic aspects of systems with many uncertain energy sources, like wind, solar, etc. Besides, it can be used to analyze such systems up to an arbitrary confidence interval.

6. Appendix

The probability density function for the generalized extreme value with scale parameter k , scale parameter σ and location parameter μ is defined as:

$$f(x\,|\,k,\sigma,\mu)=\frac{1}{\sigma}\left(1+k\frac{x-\mu}{\sigma}\right)^{-1-\frac{1}{k}}exp\left(-\left(1+k\frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}\right)$$

Fig. 8 presents a sample PDF of variable x for $k=0.3$, $\mu=80$, and $\sigma=8$.

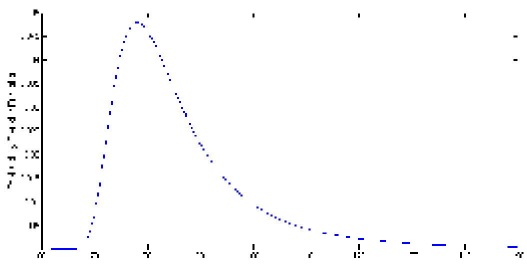

Fig.8. A sample PDF of variable x for $k=0.3$, $\mu=80$, and $\sigma=8$

References

[1] H. Ng, M. Salama, and A. Chikhani, “Classification of Capacitor Allocation Techniques,” *IEEE Transactions on Power Delivery,* vol. 15, no. 1, pp. 387-392, 2000.

[2] C.-F. Chang, “Reconfiguration and Capacitor Placement for Loss Reduction of Distribution Systems by Ant Colony Search Algorithm,” *IEEE Transactions on Power Systems,* vol. 23, no. 4, pp. 1747-1755, 2008.

[3] Y.-C. Huang, H.-T. Yang, and C.-L. Huang, “Solving the Capacitor Placement Problem in a Radial Distribution System Using Tabu Search Approach,” *IEEE Transactions on Power Systems,* vol. 11, no. 4, pp. 1868-1873, 1996.

[4] A. Eajal, and M. El-Hawary, “Optimal Capacitor Placement and Sizing in Unbalanced Distribution Systems with Harmonics Consideration Using Particle Swarm Optimization,” *IEEE Transactions on Power Delivery,* vol. 25, no. 3, pp. 1734-1741, 2010.

[5] A. Etemadi, and M. Fotuhi-Firuzabad, “Distribution System Reliability Enhancement Using Optimal Capacitor Placement,” *IET Generation, Transmission & Distribution,* vol. 2, no. 5, pp. 621-631, 2008.

[6] B. A. De Souza, H. D. N. Alves, and H. A. Ferreira, “Microgenetic Algorithms and Fuzzy Logic Applied to the Optimal Placement of Capacitor Banks in Distribution Networks,” *IEEE Transactions on Power Systems,* vol. 19, no. 2, pp. 942-947, 2004.

[7] A. Dukpa, B. Venkatesh, and L. Chang, “Fuzzy Stochastic Programming Method: Capacitor Planning in Distribution Systems with Wind Generators,” *IEEE Transactions on Power Systems,* vol. 26, no. 4, pp. 1971-1979, 2011.

[8] B. Venkatesh, and R. Ranjan, “Fuzzy Ep Algorithm and Dynamic Data Structure for Optimal Capacitor Allocation in Radial Distribution Systems,” in *IEE Proceedings- Generation, Transmission and Distribution*, pp. 80-88, 2006.

[9] H. Ng, M. Salama, and A. Chikhani, “Capacitor Allocation by Approximate Reasoning: Fuzzy Capacitor Placement,” *IEEE Transactions on Power Delivery,* vol. 15, no. 1, pp. 393-398, 2000.

[10] M. Kalantari, and A. Kazemi, “Placement of Distributed Generation Unit and Capacitor Allocation in Distribution Systems Using Genetic Algorithm,” in *10th International Conference on Environment and Electrical Engineering*, pp. 1-5, 2011.

[11] G. Levitin, A. Kalyuzhny, A. Shenkman, and M. Chertkov, “Optimal Capacitor Allocation in Distribution Systems Using a Genetic Algorithm and a Fast Energy Loss Computation Technique,” *IEEE Transactions on Power Delivery,* vol. 15, no. 2, pp. 623-628, 2000.

[12] M.-R. Haghifam, and O. Malik, “Genetic Algorithm-Based Approach for Fixed and Switchable Capacitors Placement in Distribution Systems with Uncertainty and Time Varying Loads,” *IET Generation, Transmission & Distribution,* vol. 1, no. 2, pp. 244-252, 2007.

[13] M. A. Masoum, M. Ladjevardi, A. Jafarian, and E. F. Fuchs, “Optimal Placement, Replacement and Sizing of Capacitor Banks in Distorted Distribution Networks by Genetic Algorithms,” *IEEE Transactions on Power Delivery,* vol. 19, no. 4, pp. 1794-1801, 2004.

[14] T. Abdel-Salam, A. Chikhani, and R. Hackam, “A New Technique for Loss Reduction Using Compensating Capacitors Applied to Distribution Systems with Varying Load Condition,” *IEEE Transactions on Power Delivery,* vol. 9, no. 2, pp. 819-827, 1994.

[15] K. Nekooei, M. M. Farsangi, H. Nezamabadi-Pour, and K. Y. Lee, “An Improved Multi-Objective Harmony Search for Optimal Placement of Dgs in Distribution Systems,” *IEEE Transactions on Smart Grid,* vol. 4, no. 1, pp. 557-567, 2013.

[16] R. Sirjani, A. Mohamed, and H. Shareef, “Optimal Allocation of Shunt Var Compensators in Power Systems Using a Novel Global Harmony Search Algorithm,” *International Journal of Electrical Power & Energy Systems,* vol. 43, no. 1, pp. 562-572, 2012.

[17] R. S. Rao, K. Ravindra, K. Satish, and S. Narasimham, “Power Loss Minimization in Distribution System Using Network Reconfiguration in the Presence of Distributed Generation,” *IEEE*

Transactions on Power Systems, vol. 28, no. 1, pp. 317-325, 2013.

[18] D. Singh, and R. Misra, "Effect of Load Models in Distributed Generation Planning," *IEEE Transactions on Power Systems,* vol. 22, no. 4, pp. 2204-2212, 2007.

[19] M. E. Baran, and F. F. Wu, "Optimal Sizing of Capacitors Placed on a Radial Distribution System," *IEEE Transactions on Power Delivery,* vol. 4, no. 1, pp. 735-743, 1989.

[20] M. E. Baran, and F. F. Wu, "Optimal Capacitor Placement on Radial Distribution Systems," *IEEE Transactions on Power Delivery,* vol. 4, no. 1, pp. 725-734, 1989.

[21] J.-Y. Park, J.-M. Sohn, and J.-K. Park, "Optimal Capacitor Allocation in a Distribution System Considering Operation Costs," *IEEE Transactions on Power Systems,* vol. 24, no. 1, pp. 462-468, 2009.

[22] R. Dashti, S. Afsharnia, and H. Ghasemi, "A New Long Term Load Management Model for Asset Governance of Electrical Distribution Systems," *Applied Energy,* vol. 87, no. 12, pp. 3661-3667, 2010.

[23] G. Verbič, and C. Canizares, "Probabilistic Optimal Power Flow in Electricity Markets Based on a Two-Point Estimate Method," *IEEE Transactions on Power Systems,* vol. 21, no. 4, pp. 1883-1893, 2006.

[24] M. Mohammadi, A. Shayegani, and H. Adaminejad, "A New Approach of Point Estimate Method for Probabilistic Load Flow," *International Journal of Electrical Power & Energy Systems,* vol. 51, pp. 54-60, 2013.

[25] C. Delgado, and J. Domínguez-Navarro, "Point Estimate Method for Probabilistic Load Flow of an Unbalanced Power Distribution System with Correlated Wind and Solar Sources," *International Journal of Electrical Power & Energy Systems,* vol. 61, pp. 267-278, 2014.

[26] G. Carpinelli, C. Noce, D. Proto, A. Russo, *et al.*, "Single-Objective Probabilistic Optimal Allocation of Capacitors in Unbalanced Distribution Systems," *Electric Power Systems Research,* vol. 87, pp. 47-57, 2012.

[27] T. Williams, and C. Crawford, "Probabilistic Load Flow Modeling Comparing Maximum Entropy and Gram-Charlier Probability Density Function Reconstructions," *IEEE Transactions on Power Systems,* vol. 28, no. 1, pp. 272-280, 2013.

[28] F. J. Ruiz-Rodriguez, M. Gómez-González, and F. Jurado, "A Method for Reliability Optimization of Distributed Generation Using Meta-Heuristic and Probabilistic Techniques," *Electric Power Components and Systems,* vol. 43, no. 1, pp. 32-43, 2015.

[29] M. Kabir, Y. Mishra, and R. Bansal, "Probabilistic Load Flow for Distribution Systems with Uncertain Pv Generation," *Applied Energy,* vol. 163, pp. 343-351, 2016.