

Formation and Tracking Control of Quadrotors under a Leader-Follower Strategy

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Abstract—In recent decades, the researchers have been attracted in utilizing of the multi-agent systems due to the sophistication in industrial processes, the cost of performing them and increasing the reliability. One of the interesting problems in this field of study is formation control of agents. In this paper, we are going to design a decentralized control strategy for the formation control of a group of quadrotors. To be more specific, we simplify the nonlinear dynamic of a quadrotor by using motion approximation and feedback linearization. Then, we solve the formation control problem of quadrotors by the utilization of leader-follower strategy with a decentralized protocol. In this control strategy, only do a partial number of followers have access to the leader's information. This matter can reduce noticeably the energy consumption of the leader since it requires to send less amount of information. Thereafter, we will corroborate the convergence of quadrotors to the predefined formation and leader tracking mathematically. Finally, the simulation example will be presented in order to validate the theoretical results.

Index Terms—Multi-agent system, Formation control, Quadrotor, Leader-follower, Tracking.

I. INTRODUCTION

WITH the advent of time, the researchers have been intersected in the employing of the multi-agent systems especially in the field of coverage, formation, and consensus control [1]-[3]. Among these interesting fields, the formation problem of UAVs (Unmanned Aerial Vehicles) has attracted the attention of many researchers. This problem has a myriad of applications including gathering information, surveillance, traffic control, and etc., [2] and [4] Among many recent investigations, quadrotor is one of the interesting agents which has been used in many recent industrial and military projects and problems. However, this type of agent has its own challenges including under-actuated nonlinear coupled dynamic structure, which make it a sophisticated control problem. Despite these issues and encumbrances, quadrotor is widely used in many multi-agent systems especially in the formation problem due to its rational size, low cost of

manufacturing, and capable of equipping with sensors, cameras, communication systems or even lightweight weapons. So far, a lot of efforts have been made in order to facilitate or even obviate these challenges. In [5], the author employed classical linearization method in order to simplify the quadrotor's dynamic. However, based on the intrinsic limitation of this approach, the flawless performance and global stability of the closed-loop system cannot be guaranteed. Moreover, another interesting method was used in [6]. In this approach, the authors utilized feedback linearization method and transformed the nonlinear system into a linear decoupled system. On the other word, in the new dynamic model, the translational dynamic (position in 3-D space) and angular dynamic (heading angle) were decoupled and transformed into a fourth-order integrator and second-order integrator respectively. Then, by employing a leader-follower strategy, they solved the formation problem of a group of quadrotors.

In order to increase the robustness of the system, SMC (Sliding mode Control) was employed, [6]-[10]. In [7], the authors utilized SMC in order to cancel uncertainty and disturbance of single quadrotor. Furthermore, in [6] SMC was also employed to cancel the uncertainties in the dynamical model due to the use of feedback linearization. One of the main drawbacks of using SMC is the chattering problem. However, in [8], a QSMC (Quasi SMC) method and TSMC (Terminal SMC) method in [10] were employed in order to obviate the **problem of chattering**. It should be noted that defining appropriate bounds of uncertainty is also another kind of challenges, especially in practical experiments.

Moreover, some other strategies have been used to model and control the networked quadrotors [11]-[13]. As an instance, in [11], the authors employed a two-layer controlling procedure in order to stabilize and conduct the agents to a desired formation. In this approach, a linear MPC method (Model Predictive Control) was utilized in the top layer in order to produce an on-line trajectory planning as well as achieving a desired formation. On the other hand, in the bottom layer, a nonlinear

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control mechanism is employed in order to stabilize the quadrotors. However, this approach is not practical when the number of agents increases.

Another kind of strategy called potential field is also used to generate and achieve formation. This method was used in [14]-[17]. In this method, a potential field is defined over the domain of the whole system workspace such that the formation of agents can be achieved by attracting and repulsing forces among the agents. It is worth mentioning that this method was also used in flocking algorithm which the shape of formation cannot be defined beforehand, [18] and [19].

Dealing with the nonlinearity in the dynamic of quadrotor is a quite arduous task. However, in [6], this problem was relatively handled by using SMC and feedback linearization. In our proposed approach, we are going to employ the feedback linearization approach [6] in order to simplify the quadrotor's dynamic. Additionally, by employing a leader-follower strategy and partial accessibility of the followers to the leader's information, formation convergence and leader tracking of the followers will be shown mathematically. The interaction between the followers is defined by an undirected graph and also the leader sends its information to only a part of the followers with a unidirectional link. In this approach, there is no constraint on the number of agents and the shape of the final formation in comparison with the potential field strategy. Moreover, the state feedback control is used in order to adjust the leader at the desired height and heading angle. All in all, the main achievement of this paper can be summarized as follow:

1. Designing a decentralized leader-follower strategy for a formation of a group of quadrotors in which a part of the followers has access to the leader's information. In this approach, the shape of formation can be designed beforehand.
2. Designing a state feedback control in order to maintain the leader at a fixed and desired height and heading angle.

The rest of the paper is organized as follow. In Section 2, we will present some basic preliminaries regarding graph theory and a quick introduction to feedback linearization of MIMO (Multi-Input Multi-Output) systems. Next, in Section 3, we will introduce the nonlinear dynamical structure of a single quadrotor. Moreover, in Section 4, we will propose the control inputs in order to reach our goals. Finally, in the Section 5, simulation example will be presented in order to validate the theoretical results.

II. BACKGROUND AND PRELIMINARIES

In this section, we are going to present some basic preliminaries regarding graph theory and feedback linearization.

A. Graph Theory

A graph is an ordered pair $G(V, E)$ where V represents the node set of the graph and E is the subset of $E \subset V \times V$ which

represents the edge set of the graph. We denote an edge between the node i and j as a pair. A directed path between two nodes i and j is defined as a connected sequence of edges $\{(i, e), (e, t), \dots, (h, j)\}$ where e, t, \dots, h are the middle nodes of the path. An undirected path can be traversed both directions (whether from i to j or j to i). An undirected graph is connected if at least there is one undirected path between every pair of its nodes. The neighborhood of node i is defined as:

$$N_i = \{j \in V \mid (i, j) \in E\}$$

Also, $\Delta_i = |N_i|$ is the degree of a node i and the degree matrix is defined as $D = \text{diag}(\Delta_1, \Delta_2, \dots, \Delta_N)$.

The adjacency matrix $A \in \mathbb{R}^{N \times N}$ of a given graph $G(V, E)$ is defined as:

$$a_{ij} = \begin{cases} 1, & (i, j) \in E \\ 0, & \text{otherwise} \end{cases}$$

where a_{ij} 's are the entries of the adjacency matrix A . A graph is called symmetric if $a_{ij} = a_{ji}$, $\forall i, j$. Moreover, the Laplacian matrix L is defined as $L = D - A$, [20].

Lemma 1. [21]

Consider L as the Laplacian matrix and H_a as the accessibility vector in a leader-follower strategy. Then, the matrix $L + \bar{H}_a$ ($\bar{H}_a = \text{diag}(H_a)$) is positive definite if and only if L is semi-positive definite and H_a is nonzero.

Remark 1.

Lemma 1 indicates that the topology should be connected and at least one of the followers has access to the leader's information.

B. Feedback Linearization

In this section, we are going to introduce feedback linearization method in the control of a nonlinear system with multiple inputs. In this technique, the nonlinear system is transformed to a linear system by designing and applying proper control laws.

Definition 1. [22]

Let $h: \mathbb{R}^n \rightarrow \mathbb{R}$ be a smooth scalar function, and $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a smooth vector field on \mathbb{R}^n , then the lie derivative of h with respect to f is a scalar function defined by $L_f h = \nabla h \cdot f$.

Now, Consider a nonlinear system defined in a neighborhood Ω of the point x_0 with the following form:

$$\begin{cases} \dot{x} = f(x) + G(x)u \\ y = h(x) \end{cases} \quad (1)$$

wh

ere $x \in \mathbb{R}^n$ is state vector, $u \in \mathbb{R}^m$ is control input vector, $y \in \mathbb{R}^m$ is output vector of the system, $G \in \mathbb{R}^{m \times m}$ is a matrix whose columns are smooth vector fields $g_i \in \mathbb{R}^m$, and $f(x), h(x)$ are also smooth vector fields.

Assume that r_i is the smallest integer such that at least one of the inputs appears in $y_i^{(r_i)}$, then:

$$y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^m [L_{g_j} L_f^{r_i-1} h(i)] u_j$$

With $L_{g_j} L_f^{r_i-1} h(i) \neq 0$ for at least one j , in a neighborhood Ω_i of the point x_0 . By repeating the abovementioned procedure for each output, we will have, [22]:

$$\begin{bmatrix} y_1^{(r_1)}(x) \\ y_2^{(r_2)}(x) \\ \vdots \\ y_m^{(r_m)}(x) \end{bmatrix} = b(x) + \Delta(x) \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

where $b(x) \in \mathbb{R}^m$ and $\Delta(x) \in \mathbb{R}^{m \times m}$ are a vector and an invertible matrix respectively with the following forms, [22]:

$$\Delta(x) = \begin{bmatrix} L_{g_1}^1 L_f^{r_1-1} h_1 & L_{g_2}^1 L_f^{r_1-1} h_1 & \cdots & L_{g_m}^1 L_f^{r_1-1} h_1 \\ L_{g_1}^1 L_f^{r_2-1} h_2 & L_{g_2}^1 L_f^{r_2-1} h_2 & \cdots & L_{g_m}^1 L_f^{r_2-1} h_2 \\ \vdots & \vdots & \ddots & \vdots \\ L_{g_1}^1 L_f^{r_m-1} h_m & L_{g_2}^1 L_f^{r_m-1} h_m & \cdots & L_{g_m}^1 L_f^{r_m-1} h_m \end{bmatrix}$$

$$b(x) = \begin{bmatrix} L_f^{r_1} h_1(x) \\ L_f^{r_2} h_2(x) \\ \vdots \\ L_f^{r_m} h_m(x) \end{bmatrix}$$

Now, by defining the following control input, [22]:

$$u = \Delta^{-1}(x)(v - b(x)) \quad (2)$$

where $v \in \mathbb{R}^m$ is control input vector. The system (1) is transformed into a linear system with the following form:

$$y_i^{(r_i)}(x) = v_i, i = 1, 2, \dots, m$$

Remark 2. [22]

In the case $r = n$ (where $r = r_1 + r_2 + \dots + r_m$ is the total relative degree of the system), there is no internal dynamics. On the other word, the control input (2) guarantees the stability of the system (1) without any worry regarding the stability of the internal dynamics.

III. DYNAMIC MODEL OF A QUADROTOR

In this section, we are going to introduce the dynamic of the quadrotor. Its dynamic is defined with 6-DOF (Degree of

Freedom) with the $\alpha = [\phi, \theta, \psi]^T$ as its angular coordinates and $p = [x, y, z]^T$ as its Cartesian coordinates. Moreover, the mass of each rotor is $m_i, i = 1, 2, 3, 4$ and the total mass of quadrotor is M , Fig. 1 shows a quadrotor in 3-D space.

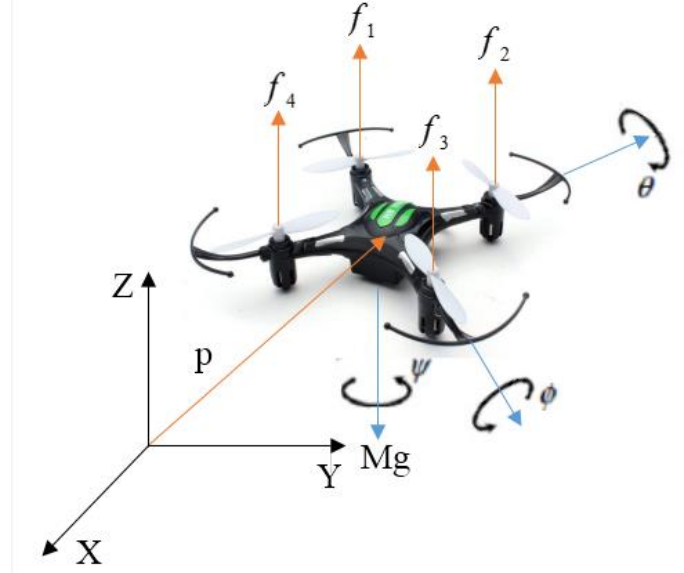


Fig. 1. A single quadrotor in 3-D space.

By assuming the fact that quadrotor is not allowed to have agile motion, which means $|\theta|, |\phi|$ are so small, the dynamic model of a quadrotor can be represented as follows, [23]:

$$\begin{aligned} \dot{x} &= v_x, & \dot{v}_x &= -\frac{A_1}{M} u^1 \\ \dot{y} &= v_y, & \dot{v}_y &= -\frac{A_2}{M} u^1 \\ \dot{z} &= v_z, & \dot{v}_z &= -\frac{A_3}{M} u^1 + g \\ \dot{\phi} &\approx \eta, & \dot{\eta} &= \frac{I_{yy} - I_{zz}}{I_{xx}} \beta \gamma + \frac{1}{I_{xx}} u^2 \\ \dot{\theta} &\approx \beta, & \dot{\beta} &= \frac{I_{zz} - I_{xx}}{I_{yy}} \eta \gamma + \frac{1}{I_{yy}} u^3 \\ \dot{\psi} &\approx \gamma, & \dot{\eta} &= \frac{I_{xx} - I_{yy}}{I_{zz}} \eta \beta + \frac{1}{I_{zz}} u^4 \end{aligned} \quad (3)$$

where, p is the position of the quadrotor, $p = [x, y, z]^T$ is control $U = [u^1, u^2, u^3, u^4]^T$ are Euler angels, $\alpha = [\phi, \theta, \psi]^T$ input signal, I_{xx}, I_{yy}, I_{zz} are the moments of inertia along x, y , and z axis respectively, g is gravitational acceleration constant, and A_1, A_2, A_3 are defined as:

$$\begin{cases} A_1 = \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ A_2 = \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ A_3 = \cos \phi \cos \theta \end{cases} \quad \begin{cases} \ddot{p}_i = \bar{r}_i, \\ \ddot{\psi}_i = r_i^4, \quad i = 1, 2, \dots, N \end{cases} \quad (7)$$

Now, by utilizing the procedure performed in [6], the dynamic of the system can be rewritten as:

$$\dot{\Pi} = F(\Pi) + \sum_{k=1}^4 G^k(\Pi)u^k \quad (4)$$

Where $\Pi = [x, y, z, v_x, v_y, v_z, \zeta, \xi, \phi, \theta, \psi, \eta, \beta, \gamma]^T$, $u^1 = \zeta$, $u^2 = \xi$, and $F_{14 \times 1}$ is a vector with the following form:

$$F = \left[v_x, -\frac{A_1}{M} \zeta, v_y, -\frac{A_2}{M} \zeta, v_z, g - \frac{A_3}{M} \zeta, \xi, 0, \eta, \frac{I_{yy} - I_{zz}}{I_{xx}} \beta \gamma, \beta, \frac{I_{zz} - I_{xx}}{I_{yy}} \eta \gamma, \gamma, \frac{I_{xx} - I_{yy}}{I_{zz}} \eta \beta \right]^T$$

Moreover, $G_{14 \times 1}^k, k = 1, 2, 3, 4$ are vectors whose all the entries are zero except $G_8^1 = 1, G_{10}^2 = \frac{1}{I_{xx}}, G_{12}^3 = \frac{1}{I_{yy}}, G_{14}^4 = \frac{1}{I_{zz}}$.

According to [6], the system (4) can be transformed into a linear system by utilizing feedback linearization method. This goal can be achieved by the following control input:

$$U = \Delta^{-1}(-b + r) \quad (5)$$

By applying control input (5) to the system (4), the linearized form of the system (4) has the following form:

$$\begin{cases} \ddot{p} = \bar{r} \\ \ddot{\psi} = r^4 \end{cases} \quad (6)$$

where $r = [\bar{r}^T, r^4]^T$ is a desired control input which will be designed later.

IV. PROBLEM STATEMENT AND MAIN RESULTS

In this part, we are attempting to design a leader-follower control strategy such that a group of networked quadrotor converge to an optional predefined formation as well as tracking their leader. In this problem, the dynamical equations (6) are employed as the dynamics of each agent. Moreover, the problems of converging to formation and tracking will be shown mathematically. Then, a control strategy will be proposed to fix the leader at a desired height and heading angle.

A. Formation Control of Quadrotor

Consider a network of N agents as the followers modeled by an undirected graph $G(V, E)$. In this modeling, each node represents an agent and the edge set E represents their communication links. Additionally, some of the followers access the information of their leader through an undirected link. The accessibility of followers to the leader is defined by a vector called accessibility vector H_a .

The dynamics of each follower have the following form:

where i is the agent number and $r_i = [\bar{r}_i, r_i^4]^T$ is the control input signal of the follower i . Similarly, the dynamics of the leader has also the following form:

$$\ddot{p}_L = \bar{r}_L, \quad \ddot{\psi}_L = r_L^4 \quad (8)$$

where $r_L = [\bar{r}_L, r_L^4]^T$ is the control input signal of the leader. Before the proposing the control input signal, the formation should be defined first. Therefore, we define the relative distance between each follower and the leader which is represented by d_{iL}^p . This parameter is fixed and indicates that each follower reaches this relative distance at the end while the formation is creating. It is worth mentioning that each agent requires 3 components in order to determine its relative distance in 3-D space. These components are:

$$d_{iL}^p = [d_{iL}^x, d_{iL}^y, d_{iL}^z]^T$$

Now, we redefine the states of the system as $S_i^1 = p_i, S_i^2 = \dot{p}_i, S_i^3 = \ddot{p}_i, S_i^4 = \ddot{\psi}_i$ and $s_i^1 = \psi_i, s_i^2 = \dot{\psi}_i$ where $i = 1, 2, \dots, N$ is used to show the numbered agent is a follower and $i = L$ is used to show the numbered agent is the leader (It should be noted that $S_i^k \in \mathbb{R}^3$, for instance $S_1^1 = [x, y, z]^T$). Based on the aforementioned discussions, the proposed control inputs have the following form:

$$\begin{aligned} \bar{r}_i = & -\frac{\alpha_p^1}{K_i} \left[H_a^i (S_i^1 - S_L^1 - d_{iL}^p) + \sum_{j=1}^N a_{ij} (S_i^1 - S_j^1 - d_{ij}^p) \right] \\ & - \frac{1}{K_i} \sum_{k=2}^4 \alpha_p^k \left[H_a^i (S_i^k - S_L^k) + \sum_{j=1}^N a_{ij} (S_i^k - S_j^k) \right] \end{aligned} \quad (9)$$

$$\begin{aligned} r_i^4 = & -\frac{1}{K_i} \sum_{k=1}^2 \alpha_\psi^k \left[H_a^i (s_i^k - s_L^k) + \sum_{j=1}^N a_{ij} (s_i^k - s_j^k) \right] \\ & + \frac{1}{K_i} \left[H_a^i s_L^2 + \sum_{j=1}^N a_{ij} s_j^2 \right] \end{aligned} \quad (10)$$

where $\alpha_\psi^k, k = 1, 2$ and $\alpha_p^k, k = 1, 2, 3, 4$ are positive constant numbers, a_{ij} are the entries of the adjacency matrix, and

$$K_i = H_a^i + \sum_{j=1}^N a_{ij}. \quad \text{Additionally, according to (8),}$$

$\dot{s}_L^4 = \bar{r}_L, \dot{s}_L^2 = r_L^4$. The accessibility vector $H_a = [H_a^1, H_a^2, \dots, H_a^N]^T$ is also defined such that $H_a^i = 1$

whenever agent i has access to the leader's information and

$H_a^i = 0$ whenever an agent i does not have access to the leader's information.

Remark 3.

To clarify the control laws (9) and (10), it should be noted that we employ the consensus control laws with some modifications. On the other words, the quadrotors converge to desired formation rather than a single point by applying nonzero relative distance among quadrotors. Additionally, the time derivatives of leader's states are also added to the control inputs to guarantee that the tracking error will converge to zero.

Remark 4.

In the proposed control inputs (9) and (10), three main goals are considered. First one is to navigate all the agents to the predefined formation. The second one is that the higher order time derivatives of all agents' position converge to a common value in order to the guarantee the endurance of formation. Finally, the third one is that the followers follow the leader.

B. Stability Analysis and Leader Tracking

To corroborate that the systems (7) and (8) under the control inputs (9), (10), and r_L guarantee the goals stated in Remark 3, first the following theorem is presented and then it will be proved.

Theorem 1.

Consider a multi-agent system with N follower with the dynamic of (7) under control inputs (9) and (10), and one leader with the dynamic of (8) under control input r_L . The follower interacts through an undirected graph $G(V, E)$. Additionally, the accessibility of the follower to the leader's information is determined by the accessibility vector H_a . Now, if:

1. The Graph $G(V, E)$ is connected.
2. The accessibility vector $H_a \neq 0_{N \times 1}$, which means at least one follower has access to leader's information.
3. The polynomials assigned to $(\alpha_p^1, \alpha_p^2, \alpha_p^3, \alpha_p^4)$ and $(\alpha_\psi^1, \alpha_\psi^2)$ are Hurwitz.

Then the system (7) and (8) under control inputs (9) and (10), and r_L will converge to the predefined formation and track the leader.

Proof.

Before starting the proof, it should be noted that the translational and angular dynamics of the quadrotor are separated since these two parts were decoupled by feedback linearization. Therefore, the stability (converging to the formation and tracking the leader) of each part will be proved separately.

In order to investigate the stability of translational dynamics of the quadrotor, we define a new variable called position error with the following form, [24]:

$$e_i = H_a^i (\tilde{S}_i^1 - S_L^1) + \sum_{j=1}^N a_{ij} (\tilde{S}_i^1 - \tilde{S}_j^1), \quad i = 1, 2, \dots, N \quad (11)$$

where $\tilde{S}_i^1 = S_i^1 - d_{iL}^p$. Now, based on this definition, the position error dynamic of the system is rewritten. By computing the fourth-order time derivative of the error variable, we will obtain:

$$\begin{aligned} \ddot{e}_i &= H_a^i (\ddot{\tilde{S}}_i^1 - \ddot{S}_L^1) + \sum_{j=1}^N a_{ij} (\ddot{\tilde{S}}_i^1 - \ddot{\tilde{S}}_j^1) \\ &= H_a^i (\ddot{\bar{r}}_i - \ddot{S}_L^4) + \sum_{j=1}^N a_{ij} (\ddot{\bar{r}}_i - \ddot{S}_j^4) \\ &= -\sum_{j=1}^N a_{ij} \ddot{S}_j^4 - H_a^i \ddot{S}_L^4 + \left(H_a^i + \sum_{j=1}^N a_{ij} \right) \ddot{\bar{r}} \end{aligned} \quad (12)$$

By choosing $K_i = H_a^i + \sum_{j=1}^N a_{ij}$, the (12) can be simplified into:

$$\ddot{e}_i = K_i \ddot{\bar{r}} - \sum_{j=1}^N a_{ij} \ddot{S}_j^4 - H_a^i \ddot{S}_L^4, \quad i = 1, 2, \dots, N \quad (13)$$

By substituting (9), (11), and first-order to third-order time derivatives of (11) into (13), we will obtain:

$$\ddot{e}_i = -\alpha_p^1 e_i - \alpha_p^2 \dot{e}_i - \alpha_p^3 \ddot{e}_i - \alpha_p^4 \ddot{e}_i, \quad i = 1, 2, \dots, N \quad (14)$$

Eq. (14) is a differential equation expressing the error dynamic (convergence of the followers to the formation and tracking the leader) of the system. In order to show that the error variable in differential equation (14) will converge to zero, the polynomial assigned to $(\alpha_p^1, \alpha_p^2, \alpha_p^3, \alpha_p^4)$ should be Hurwitz. By holding this assumption, we can conclude that:

$$e_i = 0, \quad \dot{e}_i = 0, \quad \ddot{e}_i = 0, \quad \ddot{e}_i = 0 \quad i = 1, 2, \dots, N$$

It can be shown that (11) can be written into the following form:

$$e = \left[(L + \bar{H}_a) \otimes I_{3 \times 3} \right] \tilde{S}^1 - (H_a \otimes I_{3 \times 3}) S_L^1 \quad (15)$$

where $\bar{H}_a = \text{diag}(H_a)$, L is Laplacian matrix, and $\tilde{S}^1 = \left[[\tilde{S}_1^1]^T, [\tilde{S}_2^1]^T, \dots, [\tilde{S}_N^1]^T \right]^T$. According to $e = 0$, we will have $\left[(L + \bar{H}_a) \otimes I_{3 \times 3} \right] \tilde{S}^1 = (H_a \otimes I_{3 \times 3}) S_L^1$. In order to simplify this equation, we should investigate the invertibility of $L + \bar{H}_a$. In order to satisfy this condition, two constraints should hold. Firstly, the interaction topology between followers should be connected (Laplacian matrix L should have the rank of $N - 1$) and secondly at least one follower has access to leader's information ($\bar{H}_a \neq 0_{N \times N}$). Then, according to Lemma 1, $L + \bar{H}_a$ is full rank and invertible. Based on these assumptions and (16), we will have:

$$\begin{aligned}
\tilde{S}^1 &= \left[(L + \bar{H}_a) \otimes I_{3 \times 3} \right]^{-1} (H_a \otimes I_{3 \times 3}) S_L^1 \\
&= \left[(L + \bar{H}_a)^{-1} \otimes I_{3 \times 3} \right] (H_a \otimes I_{3 \times 3}) S_L^1 \\
&= \left((L + \bar{H}_a)^{-1} H_a \otimes I_{3 \times 3} \right) S_L^1
\end{aligned} \quad (16)$$

where \otimes is kronecker product. It can be easily shown that $(L + \bar{H}_a) \mathbf{1}_{N \times 1} = H_a$. Then, according to the inversibility of $L + \bar{H}_a$, we will have $(L + \bar{H}_a)^{-1} H_a = \mathbf{1}_{N \times 1}$. By substituting this result into (16), we will obtain:

$$\tilde{S}^1 = (\mathbf{1}_{N \times 1} \otimes I_{3 \times 3}) S_L^1 \quad (17)$$

The (17) implies that the position error of all the followers ($\tilde{S}_i^1 = \tilde{S}_i^1 - d_{iL}^p$, $i = 1, 2, \dots, N$) will converge to the leader's position. On the other word, the desired formation will be achieved. Additionally, it can be shown that

$$S^k = (\mathbf{1}_{N \times 1} \otimes I_{3 \times 3}) S_L^k, \quad (S^k = \left[\begin{matrix} [S_1^k]^T \\ [S_2^k]^T \\ \dots \\ [S_N^k]^T \end{matrix} \right]^T),$$

$k = 2, 3, 4$ which they imply that the other states of followers will track leader's states. On the other word, formation and tracking will be guaranteed.

So far, we have shown the stability of translational dynamics of the quadrotor. Similarly, we can show the stability of angular dynamics of the quadrotor.

In a similar way, we define a new variable called angle error with the following form:

$$c_i = \sum_{j=1}^N a_{ij} (s_i^1 - s_j^1) + H_a^i (s_i^1 - s_L^1), \quad i = 1, 2, \dots, N \quad (18)$$

Now, based on this definition, the angular error dynamic of the system is rewritten. By performing the same procedure, it can be obtained that:

$$\ddot{c}_i = -\alpha_{\psi}^1 c_i - \alpha_{\psi}^2 \dot{c}_i, \quad i = 1, 2, \dots, N \quad (19)$$

Eq. (19) is a differential equation expressing the error dynamic (tracking the leader) of the system. In order to show that the error will converge to zero, the polynomial assigned to $(\alpha_{\psi}^1, \alpha_{\psi}^2)$ should be Hurwitz. By holding this assumption, we can conclude that $c_i = 0$, $\dot{c}_i = 0$ $i = 1, 2, \dots, N$.

Now, it can be shown that (18) can be written into the following form:

$$c = (L + \bar{H}_a) s^1 - H_a s_L^1 \quad (20)$$

where $s^1 = [s_1^1, s_2^1, \dots, s_N^1]^T$. By performing the same procedure and holding the mentioned assumptions, we will have:

$$s^1 = \mathbf{1}_{N \times 1} s_L^1$$

The (20) implies that the heading angle of all the followers will converge to the leader's heading angle. Additionally, it can be

shown that $s^2 = \mathbf{1}_{N \times 1} s_L^2$ ($s^2 = [s_1^2, s_2^2, \dots, s_N^2]^T$), which it implies that the other states of followers will track leader's states.

All in all, we showed that all the followers track the leader as well as converging to the formation. ■

Remark 4.

According to the aforementioned proof, there is no constraint on the number of agents and the shape of formation. Therefore, any formation with any number of agents can be achievable by the proposed control input.

Remark 5.

Although the Theorem 1 states that the followers interact through an undirected graph, the necessary condition is that the topology should be connected. Additionally, based on the proof, there is no limitation on the direction of communications. It is only necessary that the lemma 1 holds. Therefore, this proposed approach can be used when the followers interact through a directed graph. In this case the graph should contain a spanning tree.

C. Leader Control

So far, we have discussed about the controlling of followers. In this section, we are going to design a control input signal $r_L = [r_L^1, r_L^2, r_L^3, r_L^4]^T$ so that the leader converges to a fixed desired height and fixed desired heading angle as well as tracking a desired path in the XY plane. In order to achieve these goals, we utilize a state feedback method for r_L^3, r_L^4 .

Therefore, we propose the following control laws:

$$\begin{aligned}
r_L^3 &= -\lambda_z^1 (z_L^1 - h_d) - \lambda_z^2 z_L^2 - \lambda_z^3 z_L^3 - \lambda_z^4 z_L^4 \\
r_L^4 &= -\lambda_{\psi}^1 (s_L^1 - \psi_d) - \lambda_{\psi}^2 s_L^2
\end{aligned} \quad (21)$$

where z_L^k is the third component of

$$S_L^k = [x_L^k, y_L^k, z_L^k]^T, \quad k = 1, 2, 3, 4.$$

Additionally, positive constant numbers λ_z^k , $k = 1, 2, 3, 4$ and λ_{ψ}^k , $k = 1, 2$ should be chosen such that the polynomial assigned to $(\lambda_z^1, \lambda_z^2, \lambda_z^3, \lambda_z^4)$

and $(\lambda_{\psi}^1, \lambda_{\psi}^2)$ are Hurwitz respectively. By holding these

conditions, the leader will converge to the desired height $h_d \in (0, \infty)$ and desired heading angle $\psi_d \in \left[0, \frac{\pi}{2}\right]$.

Moreover, in order to navigate the system in the XY plane, control input signal r_L^1, r_L^2 can be just an appropriate function of time, $r_L^1(t), r_L^2(t)$.

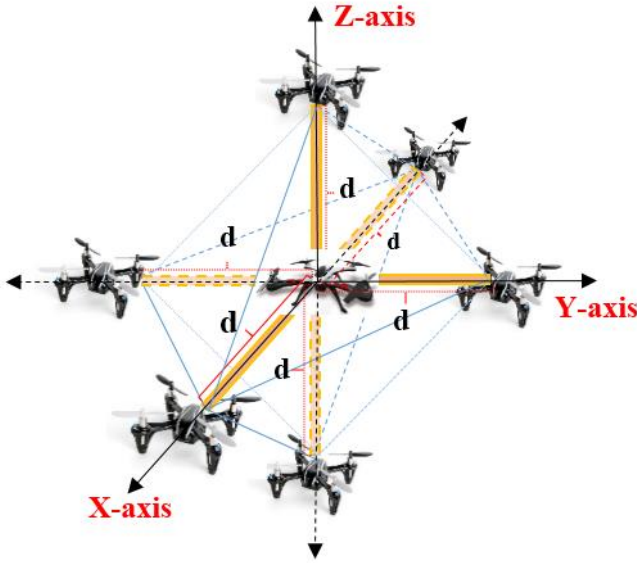


Fig. 2. Desired formation.

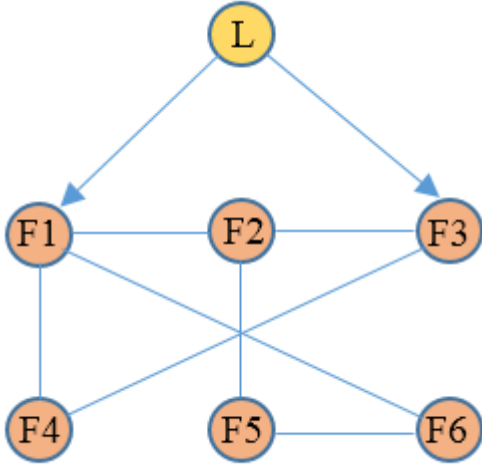


Fig. 3. Interaction topology among the followers and the leader.

V. SIMULATION

Before starting the simulation, it should be noted that, first of all, the nonlinear dynamics of quadrotor (3) is transformed into the linear dynamic (6) by applying control input (5) (In this control input, $I_{xx} = I_{yy} = 0.03 \text{ kg.m}^2$, $I_{zz} = 0.04 \text{ kg.m}^2$ and $M = 1.5 \text{ kg}$ were considered). Then, in the second step, by considering the dynamics of the followers as (7) and the leader as (8) and applying control inputs (9), (10), and r_L , the simulation will be executed.

Moreover, in order to control the leader, we will employ control inputs (21). We choose λ_c^k , $k = 1, 2, 3, 4$ as $[0.0052, 0.0797, 0.4475, 1.1]$, λ_ψ^k , $k = 1, 2$ as $[1, 2]$,

$r_L^1(t) = 0.0002$, and $r_L^2(t) = 0$. Also, we adjust the desired height at $h_d = 12$ and heading angle at $\psi_d = \frac{\pi}{4}$.

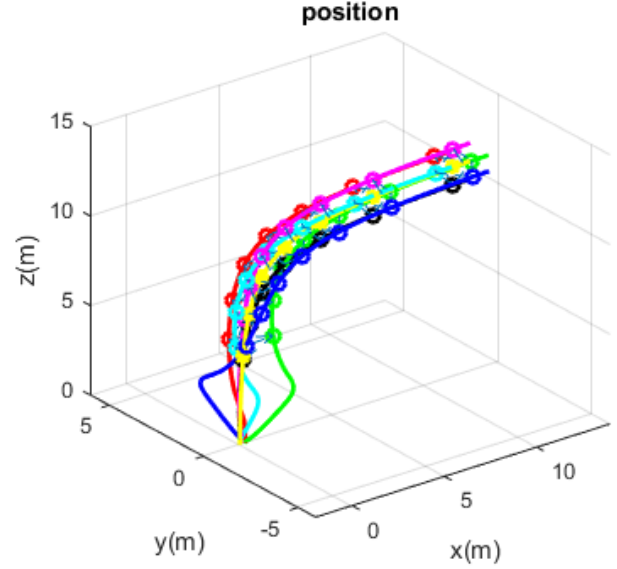


Fig. 4. Path of the agents moving in 3-D space.

Now, consider six followers and one leader is available. The interaction topology among the followers and the leader is depicted in Fig. 3. Additionally, we define the formation with the following relative distances ($d = 1 \text{ m}$), Fig. 2.

$$d_{1L}^p = [0, d, 0]^T, \quad d_{2L}^p = [d, 0, 0]^T, \quad d_{3L}^p = [0, -d, 0]^T$$

$$d_{4L}^p = [-d, 0, 0]^T, \quad d_{5L}^p = [0, 0, d]^T, \quad d_{6L}^p = [0, 0, -d]^T$$

The parameters of the control inputs (10) and (11), i.e. α_p^k , $k = 1, 2, 3, 4$ are selected as $[0.0938, 0.7813, 2.1875, 2.5]$ and α_ψ^k , $k = 1, 2$ are selected as $[0.4, 1.3]$.

By performing the simulation, the path of moving the agents is depicted in Fig. 4. As it can be observed in this figure, all the agents converged to the desired formation and the followers are tracking the leader as well. Additionally, the leader has converged to the desired height. In Fig. 5, the first-order time derivative of all the followers' position are shown and as it can be seen they have converged to the leader as well as tracking it. Moreover, in Fig. 6 and Fig. 7, the higher-order time derivative of all the followers' position are depicted. As it can be observed all of them converged to the leader's trajectory. In Fig. 8, the heading angle and its time derivative are depicted. As it is obvious in this figure, all the followers' heading angles are tracking the leader's heading angle and they have converged to the desired heading angle.

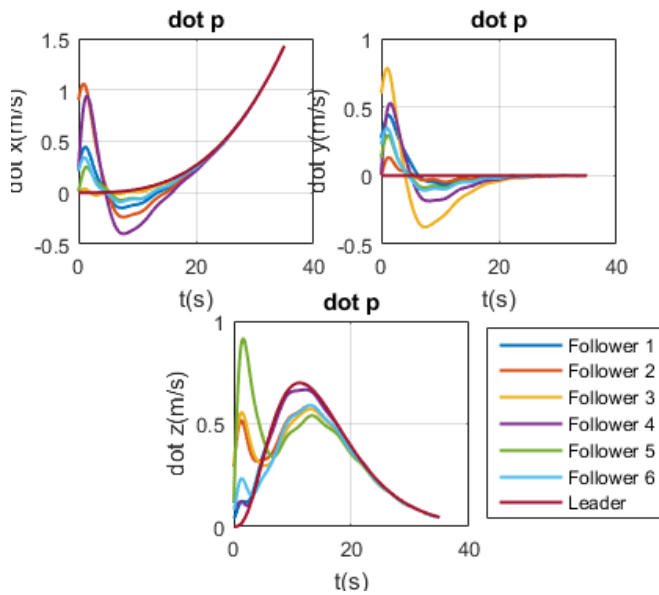


Fig. 5. The first-order time derivative of all agents' position.

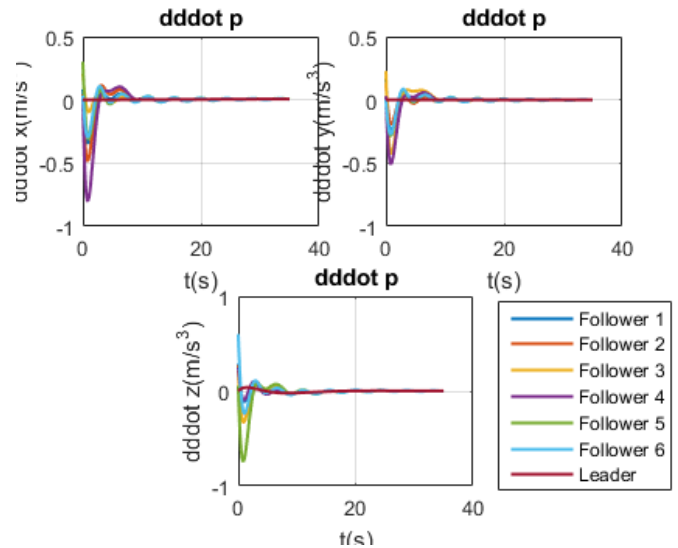


Fig. 7. The third-order time derivative of all agents' position.

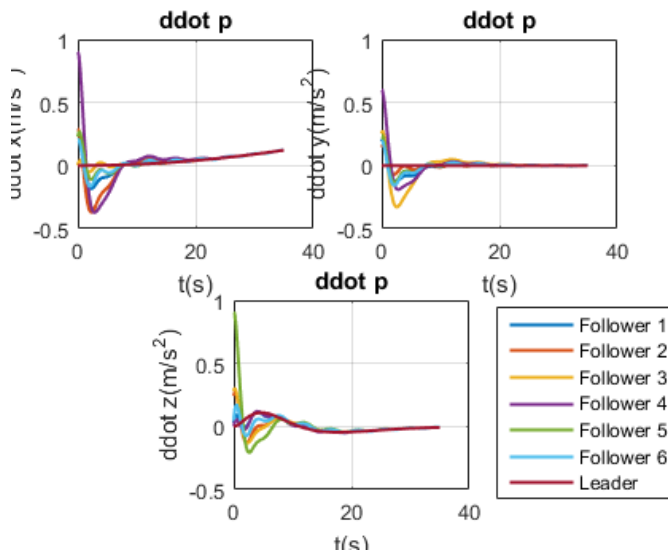


Fig. 6. The second-order time derivative of all agents' position.

VI. CONCLUSION

In this paper, the formation and tracking problem of a group of quadrotors were investigated. Under the leader-follower strategy, a control input signals were designed such that the closed-loop system achieve a formation and track their leader. The convergence of the followers and leader to the predefined formation and leader tracking were corroborated through a mathematical proof. It should be noted that, by employing this control strategy, **any kind of formation is feasible**. Moreover, a height and heading angle regulation were also designed to improve the free movement of the leader in the XY plane.

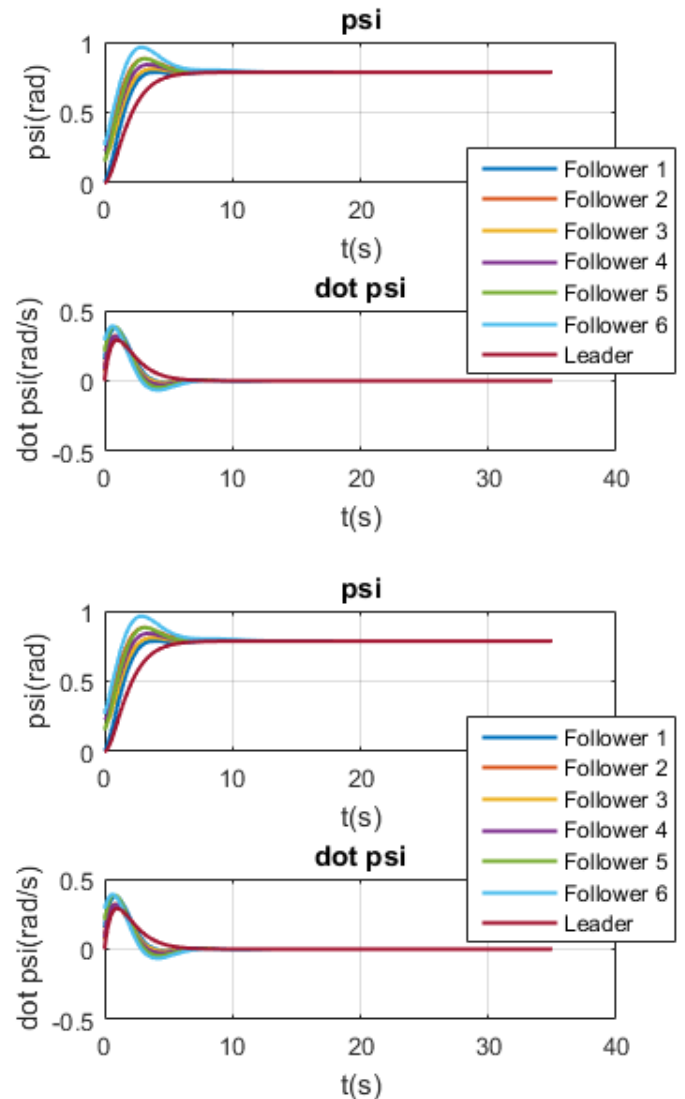


Fig. 8. Heading angle of all agents and their first-order time derivative.

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