Optimal Auxiliary Signal Design for a Lumped Tire-Road Friction System

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Abstract:
Active fault detection consists of finding an auxiliary input signal the use of which allows detection of the masked faults using a multi-model framework in continuous or discrete- time cases. In this paper, a modified approach to optimal auxiliary signal design in robust fault detection based on a multi-model formulation of healthy and faulty systems is used to study the problem of active fault detection for a class of systems with nonlinear coupled continuous state-space equations in the presence of uncertainties and disturbances. Due to the nonlinearity in the state-space equations, the traditional active fault detection approach is not straightforward to be employed. To overcome this difficulty, a modified solution is proposed in order to design an optimal auxiliary signal to guarantee robust fault detection for this class of nonlinear systems in the presence of uncertainties and disturbances. Finally, the proposed solution for optimal auxiliary signal design is applied to a Lumped Tire-Road Friction system.

Key Words: Active Fault Detection, Auxiliary Signal, Tire/Road Friction Model, Nonlinear Systems.

1. Introduction

Automobile manufacturers have made enormous efforts to increase safety and improve handling characteristics. Related to this propose, the accuracy of information obtained by direct measurement is crucial; however the appropriate sensors may be unreliable, out of the calibration, etc. Since some control systems such as Anti-lock Brake System[3], Traction Control System[4] or many variants of Electronic Stability Program[5] are strongly dependent on efficient transmission of the forces from vehicle wheels to the road, control and fault detection for ABS and tire-road friction is an essential task of vehicle dynamics control and is still a challenging issue for researchers [1-5].

However, the fault in the above mentioned systems would be revealed in special maneuvers. For example, in ABS, the fault in normal operation of system happens in high speed movement [1]. To have a better sense of masked faults, an example is given here. Suppose that you drive a car with a faulty braking system, and the controller masks this fault. You do not realize this until you push the brake pedal and encounter this problem. These kinds of faults are usually known as masked faults and incipient faults. Because of the robustness of controller, the masked faults are damped or not apparent in normal operation of the breaking system.

Model-based fault detection is one of the common approaches to fault detection. Therefore, using an accurate model of system is the fundamental part of trustable result for fault detection.

In this paper, in order to have a better fault detection, one of the accurate models which can introduce the behavior of ABS, is considered. In [3], a nonlinear model for tire-friction modeling is suggested. The mentioned model is very helpful to tire friction as well as ABS model. Therefore, a nonlinear tire-road friction model is used instead of ABS model in the present study.

On the other hand, choosing a proper fault detection strategy is another important factor for ABS system. By this way, two basic types are considered, passive fault detection[6] and active fault detection[7].

In PFD method, thanks to some reasons such as reliability, safety, and stability of system, the fault detection process has no interaction with the system. However, unfortunately, the PFD method, does not guarantee the detection of some kinds of faults known including masked faults such as a number of ABS system faults, happen but are masked in low speed.

Although, inputs and outputs are observed continuously by PFD system, this fault is not detected. In other words, merely observing input and output is not adequate to guarantee the correct decision between the faulty and healthy systems.

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3 .ABS: Anti-lock Brake System
4 .TCS: Traction Control System
5.ESP: Electronic Stability Program
6 .PDF: Passive Fault Detection
7 .AFD: Active Fault Detection

In contrast, injecting a proper signal into the system is useful for more accurate fault detection. Considering another aspect of his issue, in AFD approach the system is monitored for a short period of time, while monitoring in PFD is continuous.

In AFD, approach at least two models, healthy and faulty, are defined to describe the system behavior. In order to detect the abnormal behaviors of the system, an auxiliary signal “v” is injected into the system, as shown in Figure 1. Mathematically, this proves that if the proper auxiliary signal is injected into a multi-model system, the healthy and faulty models of system will be completely separated and detected [6]. Therefore, this problem of AFD leads to designing a proper auxiliary signal.

This paper, concentrates on designing an optimal auxiliary signal which is highly effective in detecting abnormal behaviors with minimally disruptive regular operations of the system. According to this goal, the problem is finding the auxiliary signal "v" in which the following conditions are satisfied; minimally disturbing regular system operation, being analytically computable, and guaranteeing the separation of system models.

An auxiliary signal v guarantees fault detection if and only if $\mathcal{A}_0(v) \cap \mathcal{A}_1(v) = \emptyset$, where $\mathcal{A}_i(v)$is the set of outputs y with model i, healthy model ($i = 0$) and faulty model ($i = 1$), for a given input signal v [6].

The first idea for using auxiliary signal in fault detection was presented in [7] and later developed in [8, 9]. The view point of both researches was stochastic set up. Later, in [6], AFD was formulated in deterministic set up. Regarding that work, a number of linear application examples have been tested [9- 11].

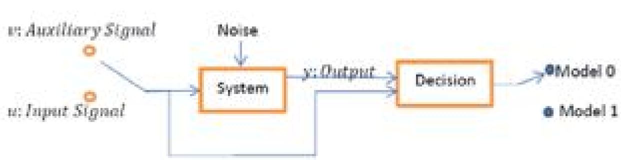

Figure 1. Active Fault Detection Diagram

Despite the fact that all of the existing publications have used PFD approach for fault detection and estimation of tire-road systems [12-16], PFD approach cannot guarantee the detection of masked and incipient faults [17]. Therefore, one of the main contributions of the present study is using AFD for ABS or tire-road friction system to boost the reliability and safety of ABS systems. Unfortunately, in spite of cons of AFD blaming its nonlinearity, uncertainty and complexity in ABS-system equations, using AFD is not straightforward. In this paper, a solution is presented to convert the model of system to a proper form and finally design the optimal auxiliary signal guaranteeing the fault detection.

The rest of this paper is organized as follows. A problem formulation is given in Section 2. In Section 3, preliminaries and problem solution are discussed. In Section 4, the auxiliary signal for a numerical example is designed. Finally, Section 5, concludes the research and give suggestions for future research.

2. Problem Formulation

In this section, the problem of auxiliary signal design for a lumped tire-road friction model and a general problem is formulated. Based on [3], the one-wheel model with lumped tire-road friction model is described as follows

$$J\dot{\omega} = -rF_n(\sigma_0 z + \sigma_1 \dot{z}) - \sigma_\omega \omega + u_r \qquad (1)$$

$$m\dot{v} = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r)F_n \qquad (1)$$

$$\dot{z} = v_r - \theta \frac{\sigma_0 |v_r|}{g(v_r)} z \qquad (2)$$

$$g(v_r) = \mu_c + (\mu_s - \mu_c)e^{-|v_r/v_s|^{1/2}} \qquad (3)$$

where m is 1/4 of the vehicle mass and J, r are the inertia and radius of the wheel, respectively. ω is the angular velocity of the wheel, u_r is the accelerating (or braking) torque, σ_ω is the viscous rotational damping, and parameters μ_c and μ_s are the normalized coulomb friction level and the normalized static friction so that $\mu_c \leq \mu_s \in [0,1]$. The parameters σ_0, σ_1 and σ_2 are the normalized rubber longitudinal lumped stiffness, normalized rubber longitudinal lumped damping, and the normalized viscous relative damping, respectively. The other parameters are v_s that is the Stribeck relative velocity, and the relative velocity is $v_r = (r\omega - v)$. F_n is the normal force and z is considered as internal friction state. The parameter θ denotes the parameter related to the unexpected changes in the road conditions, which can be interpreted as system fault. Hence, the system has two models, which are healthy model ($\theta = 1$) and faulty model ($\theta \neq 1$). It is assumed that, just parameter ω is measurable.

Regarding (1)-(4), two simple coordinate transformations are defined as follows

$$\gamma = J\omega + rF_n\sigma_1 z \qquad (4)$$

$$\delta = rmv + J\omega \qquad (5)$$

Hence, the system equation (1)-(4) are converted to

$$\begin{cases} \dot{\delta} = r\sigma_2 v_r F_n - \sigma_\omega \omega + u_r \\ \dot{\gamma} = -\frac{\sigma_0}{\sigma_1}\gamma + \left(J\frac{\sigma_0}{\sigma_1} - \sigma_\omega\right)\omega + u_r \\ \dot{z} = v_r - \theta\frac{\sigma_0 |v_r|}{g(v_r)} z \end{cases} \quad (6)$$

In [3], noise, uncertainty, and disturbance were not considered in the original model, but in this paper they are considered as an additive term in the system equation. Moreover, it is shown that they can be helpful in the fault detection.

To formulate the problem of auxiliary signal design for one-wheel model with lumped tire-road friction, a general state-space system equation is presented which is used for some other systems such as Seri DC motors and some robotic models [18]. Therefore, proposed method will be useful for fault detection of these classes systems. The general state-space form of the system in (7) is as follows

$$\dot{x} = Ax + g(u, y) + B\, f(u, y, x)\, \theta \quad (7)$$

$$y = Cx \quad (8)$$

where $x = \begin{bmatrix}\delta \\ \gamma \\ z\end{bmatrix}, u = u_r, y = \omega$. A and C are the constant matrices with proper dimensions as follows

$$A = \begin{bmatrix} -\frac{F_n\,\sigma_2}{m} & 0 & 0 \\ 0 & -\frac{\sigma_0}{\sigma_1} & 0 \\ -\frac{1}{rm} & 0 & 0 \end{bmatrix}, \quad C = [0 \quad 1/J \quad -rF_n\,\sigma_1/J].$$

g is a linear function of input and output, which are u and y, respectively so that

$$g(u, y) = g(u, Cx) = B_u u + B_y y = B_u u + B_y Cx \quad (9)$$

where

$$B_u = [1 \quad 1 \quad 0]^T, \quad B_y = \begin{bmatrix} \frac{JF_n\,\sigma_2}{m} + r^2 F_n\,\sigma_2 - \sigma_\omega \\ J\frac{\sigma_0}{\sigma_1} - \sigma_\omega \\ r \end{bmatrix}.$$

The function f is the nonlinear function of input, output and state vector x. The nonlinear term of (8), can be pulled up and rewritten as

$$Bf(u, y, x) = \mathcal{F}_\theta B_\theta x \quad (10)$$

where

$$B_\theta = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}[0 \quad 0 \quad 1], \mathcal{F}_\theta = \sigma_0 . |V_r|/g(V_r).$$

It is straightforward that (8) and (9) can be rewritten as follows

$$\dot{x} = Ax + B_u u + B_y Cx + \mathcal{F}_\theta B_\theta x\theta \quad (11)$$

$$y = Cx \quad (12)$$

By using (12) and (13), the behavior of the multi-model system is

$$\dot{x}_i = A_i x_i + B_{u_i} u + B_{y_i} C_i x_i + \mathcal{F}_{\theta_i} B_\theta x_i \theta_i \quad (13)$$

$$y = C_i x_i \quad (14)$$

where healthy model and faulty model are shown by $(i = 0)$ and $(i = 1)$ respectively. Now, the general problem is defined as 'optimal auxiliary signal, v, design that guarantees fault detection,. Despite the fact that general problem has been formulated, it is not still applicable to apply to the algorithm of auxiliary signal design described in [6]. Therefore, in the next section, a solution is provided to solve this problem and then the auxiliary signal can be designed.

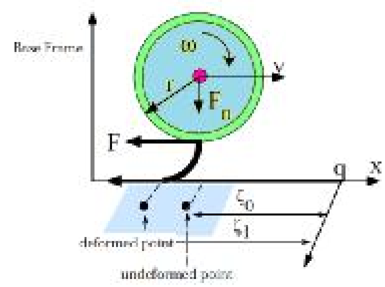

Figure 2. Lumped Tire-Road Friction System [3]

3. Strategy of Solution

In the previous section, the general problem was formulated but it is not appropriate for auxiliary signal design. In this section, at first, the standard model of auxiliary signal design will be discussed and finally, a solution for AFD design will be presented.

To consider the nonlinearity and inaccuracy in system modeling, the effect of the disturbance and noise on the actual systems, and the term of uncertainty should be considered in the model. Consequently, robust active fault detection[8] method is used for the sake of more accuracy in AFD.

Based on [10], suppose that the time period of injection auxiliary signal is $[0,T]$ and be relatively short. Moreover, there are two models of system, healthy $(i = 0)$ and faulty $(i = 1)$ models with model uncertainty and additive disturbance, which are

[8] RAFD: Robust Active Fault Detection

$$\dot{x}_i(t) = (A_i + \delta A_i)x_i(\mathrm{t})+(B_i + \delta B_i)\, v(t) + \bar{M}_i w_{ni}(t)$$
$$y(t) = (C_i + \delta C_i)x_i(t)+(D_i + \delta D_i)\, v(t) + \bar{N}_i w_{ni}(t) \quad (15)$$

Here, x_i(t) and $y(t)$ are state variables and measured output of system, respectively. $v(t)/u(t)$is an auxiliary signal/input signal and $w_{ni}(t)$ is additive disturbance such as measurement error, noise, and disturbance. Parameters $(\delta A_i , \delta B_i , \delta c_i , \delta D_i)$, are represent the model uncertainties. To generalize the model, the model uncertainties is considered as follows

$$\begin{pmatrix} \delta A_i & \delta B_i \\ \delta C_i & \delta D_i \end{pmatrix} = \begin{pmatrix} \tilde{M}_i \\ \tilde{N}_i \end{pmatrix} \Delta (H_i \quad G_i) \qquad (16)$$

where matrices $\tilde{M}_i, \tilde{N}_i, H_i, G_i$ are providing structure and weights of the uncertainty terms. The matrix, "Δ" is parameter uncertainty matrix in which $\|\Delta\|_\infty \le 1$.

Another uncertainty with an energy-type constraint composed of additive uncertainty (w_{ni}) and initial condition ($x_i(0)$) is

$$x_i(0)^T P_{0,i} x_i(0) + \int_0^T \|w_{ni}(t)\|^2\, dt < 1 \qquad (17)$$

where $P_{0,i}$ is a weighting matrix on initial condition. In what follows, let

$$\begin{cases} \xi_{a,i=\Delta \xi_{c,i}} \\ \xi_{b,i} = w_{ni} \end{cases} \qquad (18)$$

Now, by removing the output "y", rewrite (16) as follows

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i\, v(t) + (\tilde{M}_i \quad \bar{M}_i) \begin{pmatrix} \xi_{a,i}(t) \\ \xi_{b,i}(t) \end{pmatrix} \\ \quad 0 = G_i x_i(t) + H_i\, v(t) - \xi_{c,i}(t) \\ y(t) = C_i x_i(t) + D_i\, v(t) + (\tilde{N}_i \quad \bar{N}_i) \begin{pmatrix} \xi_{a,i}(t) \\ \xi_{b,i}(t) \end{pmatrix} \end{cases} \qquad (19)$$

By relaxing and adding the uncertainty constraint, for $0 \le s \le T$, regarding to (19), (18) can be rewritten as

$$x_i(0)^T P_{0,i} x_i(0) + \int_0^s \left\|\xi_{a,i}(t)\right\|^2 + \left\|\xi_{b,i}(t)\right\|^2 - \left\|\xi_{c,i}(t)\right\|^2 dt < 1 \qquad (20)$$

We define the $E = \begin{pmatrix} E_0 \\ E_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{I} \end{pmatrix}$, $x = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$, $\xi = \begin{pmatrix} \xi_0 \\ \xi_1 \end{pmatrix}$, where $\xi_i = \left(\xi_{a,i}{}^T \quad \xi_{b,i}{}^T \quad \xi_{c,i}{}^T\right)^T$. Hence, the augmented model can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(\mathrm{t}) + Bv(t) + M\xi(t) \\ Ey(t) = Cx(t) + Dv(t) + N\xi(t) \end{cases} \qquad (21)$$

Now, (21) can be described as

$$S_v^i = (x_i(0), \xi_i, s) = x_i(0)^T P_{0,i} x_i(0) + \int_0^s \xi_i{}^T J_i\ \xi_i dt < 1, \quad \forall s \in [0, T]$$
(22)

The problem is finding the optimal auxiliary signal v with the minimum energy, in which the output sets of the healthy model and faulty model are separated completely. To satisfy this condition, there should be no simultaneous solution to (22) and (23) for $i = 0, 1$. In other words, the optimization problem for standard AFD form is

$$\inf_{\substack{\xi_0, \xi_1, y \\ x_0, x_1}} \max(S_v^0, S_v^1)$$

Subject to (22) for $i = 0, 1$.

Now, the above optimization problem to design optimal auxiliary signal could be solved by an algorithm described in Appendix A [6]. In spite of this method, applying the mentioned nonlinear model to this standard form is not straightforward. Therefore, in what follows, a general nonlinear model will be introduced and converted to AFD standard form.

4. Auxiliary Signal Design

First of all, the output and state disturbance are added to the system equation defined by (12) and (13) and for more generality, the uncertainty in input is considered. Therefore, the system model is described as

$$\dot{x} = Ax + (B_u + B_{u\Delta})u + B_y Cx + \mathcal{F}_\theta B_\theta x\theta + B_y\, N w_{n_1} + M w_{n_2} \qquad (23)$$
$$y = Cx + N w_{n_1} \qquad (24)$$

Next, to simplify the computation without losing generality, some assumptions are considered in disturbance weightings, M and N. Can be considered to satisfy the proper disturbance weighting so that satisfying the following relation

$$\begin{cases} M w_{n_2} + B_y\, N w_{n_1} = \bar{M} w_n \\ \qquad N w_{n_1} = \bar{N} w_n \end{cases} \qquad (25)$$

Moreover, due to technical reasons, it is assumed that matrix $\bar{N}$ is full row rank. Regarding (26), the system model (24)-(25) can be rewritten as

$$\dot{x} = \left(A + \mathcal{F}_\theta\, B_\theta\, \theta + B_y C\right)x + (B_u + B_{u\Delta})\, u + \bar{M} w_n$$
(26)

$$y = Cx + \bar{N} w_n$$
(27)

With regard to prior discussions, two models should be defined. To obtain these two faulty and normal models, the nonlinear term "$\mathcal{F}_\theta$" is decomposed to a constant and nonlinear term.

Hence, by using Teylor-Series and eliminating the small and ignorable terms, the outcome is

$$\mathcal{F}_\theta = \frac{\sigma_0|V_r|}{g(V_r)} = \frac{\sigma_0|V_r|}{\mu_c+(\mu_s-\mu_c)\exp(-|V_r/V_s|^{\frac{1}{2}})} \cong K\big(1+\Delta_\theta(V_r)\big) \leq K(1+\Delta_{\theta max}) \quad (29)$$

where K is a positive constant, and V_r is an unknown limited positive constant. It means that this assumption helps find a bound for the worse case by taking a first order of derivation on $K\big(1+\Delta_\theta(V_r)\big)$ subject to V_r and equal to zero. In this way the maximum value of V_r and $\Delta_\theta(V_r)$can be calculated.

Based on the discussion above, the bound of uncertainty, Δ_θ, is obtained. In the next step, by changing the variables, the Eq. (26) is transformed to

$$\dot{x} = \big(A + B_\theta.K.\theta + B_y.C + \widetilde{M}\Delta G\big)x + (B_u + B_{u\Delta})u + \overline{M}w_n \quad (30)$$

where

$$\begin{cases} B_\theta.K.\Delta_\theta.\theta = \widetilde{M}\Delta G \\ B_{u\Delta} = \widetilde{M}\Delta H \end{cases} \quad (31)$$

Finally, the faulty model ($\theta = \theta_1$) and healthy model ($\theta = \theta_0$) are considered. Therefore the State-Space of the augmented system is

$$\begin{cases} \dot{x}_i = \Big(A_i + B_{\theta_i}K\theta_i + B_{y_i}C_i + \widetilde{M}_i\Delta_i G_i\Big)x_i + \big(B_{u_i} + \widetilde{M}_i\Delta_i H_i\big)u + \overline{M}w_n \\ y = C_i x_i + \overline{N}w_n \end{cases} \quad (32)$$

Regarding the recent equations, the model of system is transformed to the desired form. Now, the auxiliary signal can be designed. In the next section, the solution of the present study will be applied to a numerical example.

5. Simulation Results

In this section, the present approach is applied to one-wheel with lumped tire-road friction model to design an auxiliary signal. Based on [19], the following values for one-wheel model parameter are taken

$\sigma_0 = 40$(1/m), $\sigma_1 = 4.9487$ (s/m), $\sigma_2 = 0.0018$(s/m),
$\sigma_\omega = 0.0001(Kgm^2/s)$, $\mu_c = 0.5$, $\mu_s = 0.9$,
$V_s = 12.5$ (m/s), $r = 0.25\ (m)$, $m = 50(Kg)$,
$J = 0.2344(Kgm^2)$, $F_n = 14(Kgm^2/s^2)$

$$A_{0,1} = \begin{bmatrix} -0.005 & 0 & 0 \\ 0 & -8.0923 & 0 \\ -0.8 & 0 & 0 \end{bmatrix},$$

$$B_{\theta_{0,1}} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}[0 \quad 0 \quad 1],\ B_{u_{0,1}} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$

$$B_{y_{0,1}} = \begin{bmatrix} \frac{JF_n\,\sigma_2}{m} + r^2F_n\,\sigma_2 - \sigma_\omega \\ J\frac{\sigma_0}{\sigma_1} - \sigma_\omega \\ r \end{bmatrix},$$

$C_{0,1} = [0 \quad 4.2662 \quad -73.8927]$,

$\theta_0 = 1$, $\theta_1 = 0.8$, $\overline{N} = [1]$, $\widetilde{N} = 0_{1\times5}$,

$\overline{M} = 0_{3\times1}$, $\widetilde{M} = [I_{3\times3} \vdots I_{3\times3}]$,

$$H_i = W_H\begin{bmatrix} 0_{4\times2} \\ \alpha_i I_{2\times2} \end{bmatrix}, \alpha_0 = 0.15, \alpha_1 = 0.18,$$

$$\widetilde{M}\Delta G_i = W_G\Delta\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K\big(1+\Delta_{\theta_i}\big) \end{bmatrix},$$

$$\Delta G_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K\big(1+\Delta_{\theta_0=1}\big) \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times1} \end{bmatrix},$$

$$\Delta G_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K\big(1+\Delta_{\theta_1=0.8}\big) \\ 0_{3\times1} & 0_{3\times1} & 0_{3\times1} \end{bmatrix},$$

$$\Delta = \begin{bmatrix} \Delta^1_{3\times3} & \vdots & \Delta^2_{3\times3} = 0_{3\times3} \\ \dots\dots\dots\dots. & + & \dots\dots\dots\dots\dots\dots \\ \Delta^3_{3\times3} & \vdots & \Delta^4_{3\times3} = 0_{3\times3} \end{bmatrix}, \|\Delta\| \leq 1$$

By using (29) the parameter 'K' is calculated as $K = 2500$ and $\Delta_{\theta_i} = 0.28 \times \theta_i$.

As it was mentioned in the appendix A, the maximum injection time-period should be determined experimentally. For this case the injection time period has been assumed as 0.4s.

In this example, the auxiliary signal is designed for variety values of uncertainty weighting. To have a better sense of uncertainty effect on auxiliary signal, the uncertainty weighting matrices W_G and W_H are normalized. As mentioned in Table 1, W_G is given a constant value equal to 1 and W_H has different values between 0 and 1. This specifies that by increasing the value of uncertainty weighting W_H, the energy of optimal auxiliary signal decreases. The related simulation results are demonstrated in Fig.2. In the next step, as shown in Table 2, the inverse condition is considered, W_G is given as a constant value equal to 0.6 and W_H has different values between 0 and 1. The simulation result shows that the increasing uncertainty value leads to decreasing energy of auxiliary signal. The second simulation results are shown in Fig. 3 and Fig. 4.

In Table 3, both of the uncertainty weighting matrices W_G and W_H are varied and the results confirm our previous claim, stating that more uncertainty in model cause to smaller auxiliary signal design. Additionally, by analyzing the results of Table 1 1-Table 3, it is concluded that the auxiliary signal is more sensitive to parameter W_G than W_H. The simulation related to Table 3 is depicted in Fig.5.

Table 1: Auxiliary Signal Energy for $\boldsymbol{W_G = 1,\ 0 \leq W_H \leq 1}$

Item	W_G	W_H	$\|v\|$
1	1	0.0	124.44
2	1	0.2	107.70
3	1	0.4	77.87
4	1	0.6	63.90
5	1	0.8	41.28
6	1	1.0	11.45

Table 2. Auxiliary Signal Energy for $\boldsymbol{W_H = 0.6,\ 0 \leq W_G \leq 1}$

Item	W_G	W_H	$\|v\|$
1	0.0	0.6	4400.50
2	0.2	0.6	1875.23
3	0.4	0.6	74.50
4	0.6	0.6	65.19
5	0.8	0.6	64.52
6	1.0	0.6	63.90

Table 3: Auxiliary Signal Energy for $\boldsymbol{0.2 \leq W_H \leq 0.6, 0.0 \leq W_G \leq 0.6}$

Item	W_G	W_H	$\|v\|$
1	0	0.2	5200.00
2	0	0.6	4400.50
3	0.2	0.2	2230.48
4	0.2	0.6	1875.23
5	0.6	0.2	120.27
6	0.6	0.6	65.19

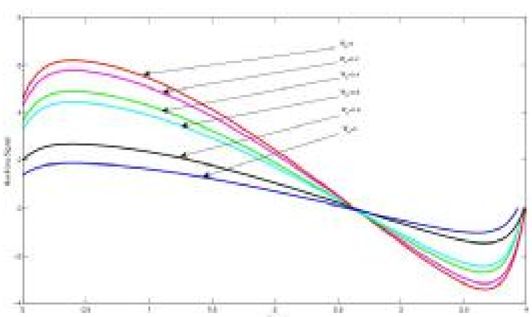

Figure 2. Auxiliary Signal for $\boldsymbol{W_G = 1,\ 0 \leq W_H \leq 1}$

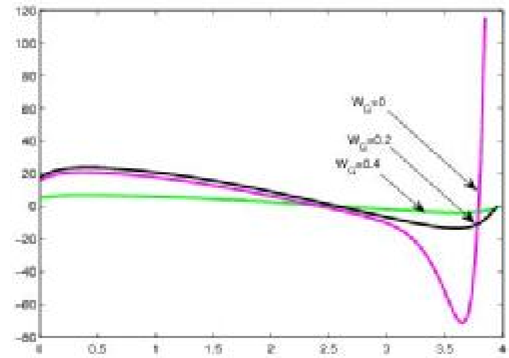

Figure 3. Auxiliary Signal for $\boldsymbol{W_H = 0.6,\ 0.0 \leq W_G \leq 0.4}$

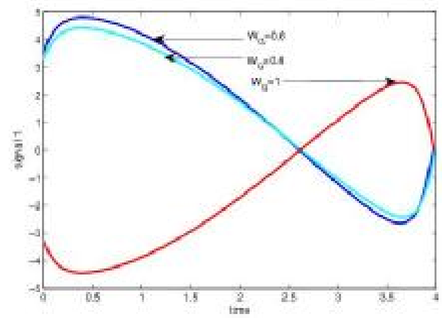

Figure 4. Auxiliary Signal for $\boldsymbol{W_H = 0.6,\ 0.6 \leq W_G \leq 1}$

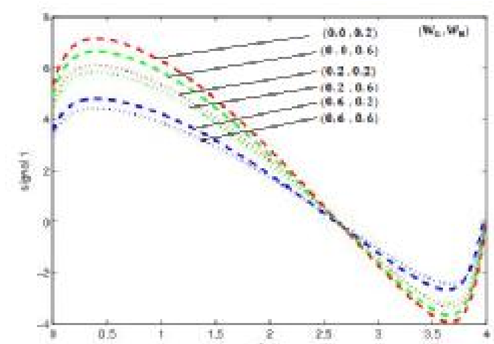

Figure 5. Auxiliary Signal Energy for $\boldsymbol{0.2 \leq W_H \leq 0.6, 0.0 \leq W_G \leq 0.6}$

6. Conclusion and Future Works

This paper presents a solution to design an auxiliary signal for a one-wheel model with lumped tire-road friction. Although, this solution is limited for this application, it can be generalized for a class of nonlinear systems. Moreover, the simulation results proved the theoretical claim of the study, saying that the effect of uncertainty in model is useful in designing optimal auxiliary signal. For future works, the designed auxiliary signals will be used for improving fault detection, especially to detect the masked faults in the closed-loop control system for general class of uncertain nonlinear systems.

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Appendix A

In this section, the construction of an optimal proper auxiliary signal is summarized [6]. The general models are in the form of

$$\begin{cases} \dot{x}_i(t) = A_i x_i(\mathrm{t}) + B_i v_i(t) + M_i \xi_i(t) \\ E_i y(t) = C_i x_i(\mathrm{t}) + D_i v_i(t) + N_i \xi_i(t) \end{cases} \tag{A.1}$$

where i = 0, 1 correspond to healthy and faulty system models respectively. The v is the auxiliary signal which is computed prior to the test while y are outputs that become known during the test.
Since v, y are known they are common to both models. However, y cannot be used to design v since
v is computed before the test.

The unknown initial conditions x_i (0) and uncertainty parameter v are assumed to satisfy the bounds

$S_i = x_i(0)^T P_{0,i} x_i(0) + \int_0^s \xi_i^T J_i \ \xi_i dt < 1, \forall s \in [0, T]$ (A.2)

where the J_i's are signature matrices. That is, J_i is a diagonal matrix with 1 and −1 on the diagonal and also assume that $v_i \in L_2$.

This formulation includes a number of different problems. For example, it includes the case of purely additive noise where $E_i = I$ and $J_i = I$. In that case we need only consider s = T, since the integrand is non-negative and the maximum value of the integral occurs at s = T. Now, suppose we have y, given a v, consistent with one of the models. We seek an optimal v for which observation of y provides enough information to decide from which model y has been generated.

That is, we seek an optimal proper v. The first step is to characterize the proper v. That is those v for which there exist no solution to (A.1) and (A.2) for i = 0 and 1 simultaneously.

The optimality criterion of minimizing is the L_2 norm of the auxiliary signal. In the other words, the solution is

$$\min_{v} |||v|||^2$$
$$subject\ to \quad \mathcal{A}_0(v) \cap \mathcal{A}_1(v) = \emptyset$$

where $|||v|||^2 = \int_0^T \|v\|^2 dt$ and $\mathcal{A}_i(v)$is the set of outputs y with model i, healthy model ($i = 0$) and faulty model ($i = 1$), for a given auxiliary signal v.

The goal of the algorithms is to find the minimum proper signal, v, where a proper signal is one which makes the two output sets disjoint. This signal is then input into the system during a short test period and the output is measured. Based on the measurement, a decision is made whether the system is healthy or faulty.

If (x_0, ξ_0, x_1, ξ_1) satisfies the models (A.1) and the uncertainty bound (A.2), then v, guarantees that the output of each model will be distinct. Else, if (x_0, ξ_0, x_1, ξ_1) satisfy the models and v is a proper auxiliary signal, but the outputs are still not distinct, then (x_0, ξ_0, x_1, ξ_1) must not satisfy the uncertainty bound (A.2) for i = 0, 1. Thus, if (x_0, ξ_0, x_1, ξ_1) satisfies the models and v is a proper auxiliary signal, but an output y is still in both output sets, it must be that

$x_i(0)^T P_{0,i} x_i(0) + \int_0^s \xi_i^T J_i \ \xi_i dt \geq 1.$ (A.3)

Since this is true for all (x_0, ξ_0, x_1, ξ_1) that satisfy the models but do not have distinct outputs, it is sufficient to ensure it is true for the minimum of them. Thus,

$$\min_{x_i, \xi_i, y_0 = y_1} max\{S_0, S_1\} \geq 1 \qquad (A.4)$$

The algorithm is to find the minimum proper signal v such that (A.4) holds.