A Hierarchical Identification Method for SISO Fractional-order State-space Systems

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Abstract

This paper proposes a new hierarchical identification method for fractional-order systems. In this method, a SISO (single input, single output) state space model has been considered in which parameters and also state variables should be estimated. By using a linear transformation and a shift operator, the system will be transformed into a form appropriate for identification of a fractional-order system. Then, the unknown parameters will be identified through a recursive least squares method and the states will be estimated using a fractional order Kalman filter. This identification method is based on the hierarchical identification principle that reduces the computational burden and is easy to implement on computer. The promising performance of the proposed method is verified using two stable fractional-order systems.

Keywords: Fractional order systems; fractional order Kalman filter; recursive identification; hierarchical identification principle

1. Introduction

In 1695, the concept of fractional calculus was expressed for the first time by Leibniz and L'Hospital. In the late nineteenth century, Riemann and Liouville give the first definition of fractional derivatives. In recent years, because of their many applications, fractional-order systems (systems that contain fractional derivatives and fractional integrals) have received the attention of many researchers [1]. However, this idea began to be a topic of interest for engineers since 1960, especially insofar as they observed that certain actual systems in which fractional derivatives are used exhibit greater accuracy [2]. Modelling the behaviour of materials such as polymers and rubbers can be considered as an example [3]. Electrochemical processes and robots with flexible arms are also modelled by fractional-order systems [4]. Fractional calculus is also a useful tool for modelling traffic in information networks [5]. Another research topic in the area of fractional order systems, which is developing rapidly, is fractional order PID controllers [6]. More applications and examples for the fractional order systems and also the fractional calculus can be found in [7-11].

Fractional-order system identification methods can be broadly classified as techniques in the frequency domain and in the time domain. Due to their long-term memory behaviour, the identification of fractional-order models is more difficult than for integer-order models; therefore, different algorithms have been proposed in the frequency domain to solve this problem [4]. In [12-14], methods for the time domain identification of fractional-order discrete-time systems are presented based on non-recursive least squares. However, these methods can raise problems in computing, such as the creation of singular matrices, which hampers identification. Furthermore, online identification will be required for time-varying systems. Therefore, recursive identification is important to avoid the above mentioned problems.

State space system identification is important for controller design and pole placement. Some methods have been reported for linear and nonlinear systems [15-18]. Also, several papers have been presented in relation to identification of fractional order systems [19-25], but there is not any method of recursive identification of fractional order state space systems in the literature so far. The identification method provided in this paper is based on a recursive identification algorithm that has the capability of identifying the parameters of fractional order state space system recursively.

Therefore, in this paper a new hierarchical identification method will be introduced for SISO fractional order systems. Actually, hierarchical identification is based on a decomposition method and is widely used in state and parameter estimation [26, 27]. The basic idea is to use the hierarchical identification principle to decompose the system model into several sub-models with smaller dimensions and fewer variables, and then to identify the parameter vector of each sub-model.

This paper examines the problem of identification of a state space model for a SISO fractional-order

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system. A hierarchical approach will be proposed in which the unknown system parameters as well as the states of the system are estimated simultaneously. The parameters are calculated based on a recursive least squares method and the states are estimated using a fractional Kalman filter.

In addition, by decomposing the fractional state space system and using a hierarchical identification method, which is introduced in the present paper, a linear regression equation is derived including parameters and states of the system. This regression equation has the appropriate form for identification. Thus, by providing a recursive least squares algorithm, it is possible to identify the parameters of the fractional order state space system recursively.

The proposed hierarchical identification method has many applications in various fields such as system modeling, signal filtering and adaptive control. Another application is in state feedback in which the states are used for stabilization of unstable dynamical systems. An advantage of using the state space model is that it can be easily expanded to multi-input multi-output (MIMO) systems, while the identification methods that are based on a transfer function cannot be easily expanded to MIMO systems. In addition, the hierarchical identification method provided in this paper reduces the computational burden and significantly improves the execution time.

A canonical state space model has been used in this paper. This property can be exploited in system identification because the canonical models have a special form that makes them identifiable.

The rest of the paper is organized as follows. In Section 2, the problem formulation is presented and the identification model is given in Section 3. Section 4 provides the state estimation algorithm and the proposed hierarchical parameter and state estimation algorithm is given in Section 5. In Section 6, two examples are provided. Finally, Section 7 concludes the paper.

2. Problem formulation

Consider the following fractional-order discrete time linear stochastic state-space system [28].

$$\Delta^{\Upsilon} \mathrm{X}(k+1) = AX(k) + Bu(k) + w(k) \quad (1)$$

$$\mathrm{X}(k+1) = \Delta^{\Upsilon} \mathrm{X}(k+1) - \sum_{j=1}^{k+1} (-1)^j \binom{\Upsilon}{j} \mathrm{X}(k+1-j) \quad (2)$$

$$y(k) = HX(k) + v(k) \quad (3)$$

where Υ is the order of the fractional difference $(\Upsilon \in R^+)$ and $\mathrm{X}(k)$ is the state vector $(\mathrm{X}(k) \in R^n)$. $W(k) = [w_1(k) \quad w_2(k) \quad \dots \quad w_n(k)]$ and $n(k)$ are the process and measurement white Gaussian noises with zero mean, also $u(k)$ and $y(k)$ are the input and the output of the system, respectively. Furthermore, T denotes the matrix/vector transpose and the symbol $I(I_n)$ shows an identity matrix with appropriate size $(n \times n)$. The system matrices A, B are the unknown parameters to be estimated from input–output data $u(k)$ and $y(k)$.

Furthermore, $\binom{\Upsilon}{j}$ is defined as:

$$\binom{\Upsilon}{j} = \frac{\Gamma(\Upsilon+1)}{\Gamma(j+1)\Gamma(\Upsilon-j+1)} \quad (4)$$

Assuming that $v > 0$, Euler's function Γ is defined as:

$$\Gamma(x) = \int_0^{\infty} e^{-v} v^{x-1} dv \quad (5)$$

$$\Delta^{\Upsilon} \mathrm{X}(k+1) = \begin{bmatrix} \Delta^{g_1} x_1(k+1) \\ \vdots \\ \Delta^{g_n} x_n(k+1) \end{bmatrix} \quad (6)$$

where $g_1, \dots, g_n$ are the order of the system equation and n is the number of system equations.

Assumption 1: $v(k)$ and $W(k)$ are two independent white noises with zero mean and covariance matrixes $R(k)$ and $Q(k)$, respectively. In other words, we have:

$$E[w(k)] = 0, E[v(k)] = 0 \quad (7)$$

$$E\left[W(k)W^T(j)\right] = Q(k)d(k-j) \quad (8)$$

$$E\left[v(k)v^T(j)\right] = R(k)d(k-j) \quad (9)$$

$$E\left[w(k)v^T(j)\right] = 0, \forall k, j \quad (10)$$

Assumption 2: $X(0)$ is uncorrelated with $v(k)$ and $W(k)$, and

$$E[\mathrm{X}(0)] = \hat{X}(0) \quad (11)$$

$$E\left[(\mathrm{X}(0) - \hat{X}(0))(\mathrm{X}(0) - \hat{X}(0))^T\right] = P_0 \quad (12)$$

By combining the equations (1) and (2), the following equation is obtained:

$$\mathrm{X}(k+1) = AX(k) + Bu(k) + w(k) + \sum_{j=1}^{k+1} C_j \mathrm{X}(k+1-j) \quad (13)$$

$$C_j = (-1)^{j+1} diag\left[\binom{g_1}{j} \quad \dots \quad \binom{g_n}{j}\right] \quad (14)$$

where the matrices A, B and the state vector $\mathrm{X}(k)$ are defined as follows:

$$A=\begin{bmatrix}0&1&0&\dots&0\\0&0&1&\dots&0\\\vdots&\vdots&\ddots&\ddots&\vdots\\0&0&0&0&1\\a_1&a_2&a_3&\dots&a_n\end{bmatrix}_{n\times n}\quad B=\begin{bmatrix}b_1\\b_2\\b_3\\\vdots\\b_n\end{bmatrix}_{n\times 1}$$

$$X=\begin{bmatrix}x_1&x_2&\cdots&x_n\end{bmatrix}^T$$

$$C_j=\begin{bmatrix}c_{11,j}&&&0\\&c_{22,j}&&\\&&\ddots&\\0&&&c_{nn,j}\end{bmatrix}_{n\times n},H=\begin{bmatrix}1&0&\dots&0\end{bmatrix}_{1\times n}$$

In addition, we also assume that:

$$\begin{aligned}u(k)&=0 \quad k\le 0\\ y(k)&=0 \quad k\le 0\\ \mathrm{X}(k)&=0 \quad k\le 0\end{aligned}\tag{15}$$

In the state space model of the system, A and B matrices have unknown parameters which should be estimated using input-output data points. Block diagram of the fractional-order state-space system model is shown in Figure 1.

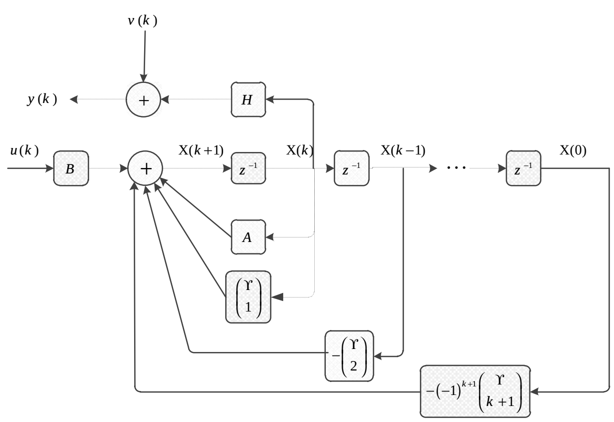

Fig. 1.: Block diagram of the fractional-order state-space system model.

3. The model used for identification

The main topic that is discussed in this paper is the identification problem. In this section, a linear regression equation will be derived in which the unknown parameters can be estimated using a recursive least squares method. Therefore, a hierarchical method is applied to have a proper form of the regression equation.

Expanding equation (13) gives:

$$\begin{aligned}\mathrm{X}(k+1)&=AX(k)+Bu(k)+C_1\mathrm{X}(k)+C_2\mathrm{X}(k-1)+C_3\mathrm{X}(k-2)\\&+\dots+C_{k+1}\mathrm{X}(0)+w(k)\end{aligned}\tag{16}$$

In addition, by opening equation (16), we obtain:

$$\begin{aligned}x_1(k+1)&=x_2(k)+b_1u(k)+c_{11,1}x_1(k)+c_{11,2}x_1(k-1)\\&+\dots+c_{11,k+1}x_1(0)+w_1(k)\\x_2(k+1)&=x_3(k)+b_2u(k)+c_{22,1}x_2(k)+c_{22,2}x_2(k-1)\\&+\dots+c_{22,k+1}x_2(0)+w_2(k)\\&\vdots\\x_{n-1}(k+1)&=x_n(k)+b_{n-1}u(k)+c_{(n-1)(n-1),1}x_{n-1}(k)\\&+c_{(n-1)(n-1),2}x_{n-1}(k-1)+\dots\\&+c_{(n-1)(n-1),k+1}x_{n-1}(0)+w_{n-1}(k)\\x_n(k+1)&=a_1x_1(k)+a_2x_2(k)+\dots+b_nu(k)+c_{nn,1}x_n(k)\\&+c_{nn,2}x_n(k-1)+\dots+c_{nn,k+1}x_n(0)+w_n(k)\end{aligned}\tag{17}$$

The shift operator is defined as $z^{-1}\mathrm{X}(k)=X(k-1)$. Applying the shift operator to equation (17) results in:

$$\begin{aligned}(z^{-1})\times x_1(k+1)&=(z^{-1})\times(x_2(k)+b_1u(k)+c_{11,1}x_1(k)\\&+c_{11,2}x_1(k-1)+\dots+c_{11,k+1}x_1(0)+w_1(k))\\(z^{-2})\times x_2(k+1)&=(z^{-2})\times(x_3(k)+b_2u(k)+c_{22,1}x_2(k)\\&+c_{22,2}x_2(k-1)+\dots+c_{22,k+1}x_2(0)+w_2(k))\\&\vdots\\(z^{-n+1})\times x_{n-1}(k+1)&=(z^{-n+1})\times(x_n(k)+b_{n-1}u(k)\\&+c_{(n-1)(n-1),1}x_{n-1}(k)+c_{(n-1)(n-1),2}x_{n-1}(k-1)\\&+\dots+c_{(n-1)(n-1),k+1}x_{n-1}(0)+w_{n-1}(k))\\(z^{-n})\times x_n(k+1)&=(z^{-n})\times(a_1x_1(k)+a_2x_2(k)\\&+\dots+b_nu(k)+c_{nn,1}x_n(k)+c_{nn,2}x_n(k-1)+\dots\\&+c_{nn,k+1}x_n(0)+w_n(k))\end{aligned}$$

or

$$\begin{aligned}x_1(k)&=x_2(k-1)+b_1u(k-1)+c_{11,1}x_1(k-1)\\&+c_{11,2}x_1(k-2)+\dots+c_{11,k+1}x_1(-1)+w_1(k-1)\\x_2(k-1)&=x_3(k-2)+b_2u(k-2)+c_{22,1}x_2(k-2)\\&+c_{22,2}x_2(k-3)+\dots+c_{22,k+1}x_2(-2)+w_2(k-2)\\&\vdots\\x_{n-1}(k-n+2)&=x_n(k-n+1)+b_{n-1}u(k-n+1)\\&+c_{(n-1)(n-1),1}x_{n-1}(k-n+1)+c_{(n-1)(n-1),2}x_{n-1}(k-n)\\&+\dots+c_{(n-1)(n-1),k+1}x_{n-1}(-n+1)+w_{n-1}(k-n+1)\\x_n(k-n+1)&=a_1x_1(k-n)+a_2x_2(k-n)\\&+\dots+b_nu(k-n)+c_{nn,1}x_n(k-n)\\&+c_{nn,2}x_n(k-1-n)+\dots+c_{nn,k+1}x_n(-n)+w_n(k-n)\end{aligned}\tag{18}$$

Therefore, by combining equations (18), a linear regression equation is obtained as:

$$y(k)=x_1(k)+v(k)=j^T(k)q+N(k)+v(k)\tag{19}$$

Where

$$q=\begin{bmatrix}a_1&a_2&\dots&a_n&b_1&b_2&\dots&b_n\end{bmatrix}^T\tag{20}$$

$$j(k) = [x_1(k-n) \quad x_2(k-n) \quad \dots \quad x_n(k-n) \quad u(k-1) \quad u(k-2) \quad \dots \quad u(k-n)]^T \tag{21}$$

$$\begin{aligned} N(k) &= c_{11,1}x_1(k-1) + \dots + c_{11,k+1}x_1(-1) + c_{22,1}x_2(k-2) \\ &+ c_{22,2}x_2(k-3) + \dots + c_{22,k+1}x_2(-2) \\ &+ \dots + c_{\mathrm{nn},1}x_n(k-n) + c_{\mathrm{nn},2}x_n(k-1-n) + \dots \\ &+ c_{\mathrm{nn},k+1}x_n(-n) + w_1(k-1) + \dots + w_n(k-n) \end{aligned} \tag{22}$$

In $N(k)$, the coefficients $c_{11,1},\dots,c_{nn,k+1}$ are known (because orders are known) and the states will be estimated using the fractional-order Kalman filter (as described in Section 4). For this reason, $N(k)$ can be considered as an additive term in the regression equation. $\hat{q}_k$ denotes the estimate of q at time k.

To estimate the vector $\hat{q}$, the following objective function should be minimized [2]:

$$\hat{q}_{k+1} = \arg\min \frac{1}{k}\sum_{i=1}^{k}[y(i) - \hat{y}(i,q)]^2 \tag{23}$$

The vector $\hat{q}$ which minimizes (23) is given as:

$$\hat{q}_k = \left[\sum_{i=1}^{k} j(i-1) j^T(i-1)\right]^{-1} \sum_{i=1}^{k} j(i-1) y(i) \tag{24}$$

Note that, to ensure that the system has the identifiability property, the $\left[\sum_{i=1}^{k} j(i-1) j^T(i-1)\right]^{-1}$ matrix must exist.

The recursive least squares identification algorithm is as follows [29]:

$$\hat{q}_{k+1} = \hat{q}_k + F_k j(\mathrm{k}) e(\mathrm{k+1}) \tag{25}$$

$$\mathrm{F}_{k+1} = F_k - \frac{F_k j(k) j^T(k) F_k}{1 + j^T(k) F_k j(k)} \tag{26}$$

$$e(k+1) = \frac{y(k+1) - \hat{q}_k^T j(k) - N(k)}{1 + j^T(k) F_k j(k)} \tag{27}$$

Here, F_k is the adaptation gain matrix, and its initial value is equal to:

$$F_0 = \frac{1}{d} I, 0 < d < 1 \tag{28}$$

4. The state estimation algorithm

The purpose of this section is to estimate the states of a fractional-order system. Since j is composed of the unknown states of the system, a fractional-order Kalman filter will be used to estimate it. Then the estimated j value will be used in the final hierarchical identification method.

The Kalman filter is an optimal state vector estimator based on the knowledge of the system model and also the use of inputs and outputs to perform the estimate [30].

An optimal estimation is given by minimizing the following objective function, [31]:

$$\hat{X}(k) = \arg\min_X [(\breve{X}(\mathrm{k}) - X)\breve{P}_k^{-1}(\breve{X}(\mathrm{k}) - X)^T + (y(k) - HX)R_k^{-1}(y(k) - HX)^T] \tag{29}$$

where $\breve{X}(k) = E\left[\mathrm{X(k)} \mid z_{k-1}^*\right]$ is a state vector prediction at time k, defined as expectation of the random variable $X(k)$ conditioned on the measurements z_{k-1}^* and $\hat{X}(k) = E\left[\mathrm{X(k)} \mid z_k^*\right]$ is the state vector estimate at time k, defined as expectation of the random variable $X(k)$ conditioned on the measurements z_k^* [32].

The measurement z_k^* includes measuring all the outputs $y(0), y(1), \dots, y(k)$ and all the input signals $u(0), u(1), \dots, u(k)$.

In addition, $\breve{P}_k = E\left[(\breve{X}(\mathrm{k}) - X(k))(\breve{X}(\mathrm{k}) - X(k))^T\right]$ is the prediction error covariance matrix and $P_k = E\left[(\hat{X}(\mathrm{k}) - X(k))(\hat{X}(\mathrm{k}) - X(k))^T\right]$ is the estimation error covariance matrix.

For the fractional-order random discrete-time state-space system, the fractional-order Kalman filter is as follows [28]:

1. The system and measurement equations are given by Eqs (1)-(3).

$$\Delta^{\Upsilon} \mathrm{X}(k+1) = AX(k) + Bu(k) + w(k)$$

$$\mathrm{X}(k+1) = \Delta^{\Upsilon} \mathrm{X}(k+1) - \sum_{j=1}^{k+1}(-1)^j \binom{\Upsilon}{j} \mathrm{X}(k+1-j)$$

$$y(k) = HX(k) + v(k)$$

2. Initialization

$$\hat{X}(0) = E[X(0)]$$

$$P_0 = E\left[(X(0) - \hat{X}(0))(X(0) - \hat{X}(0))^T\right]$$

3. For each time instance *k=1,2,3,...,* execute the following equations to update the state estimate:

$$\Delta^{\Upsilon} \breve{\mathrm{X}}(k+1) = A(k)\hat{\mathrm{X}}(k) + B(k)u(k) \tag{30}$$

$$\breve{\mathrm{X}}(k+1) = \Delta^{\Upsilon} \breve{\mathrm{X}}(k+1) + \sum_{j=1}^{k+1} C_j \hat{\mathrm{X}}(k+1-j) \tag{31}$$

$$\hat{\mathrm{X}}(k) = \breve{X}(k) + K(k)\left(y(k) - H\breve{X}(k)\right) \tag{32}$$

$$P_k = (I - K(k)H)\breve{\mathrm{P}}_k \tag{33}$$

$$K(k) = \breve{P}_k H^T (H\breve{P}_k H^T + R(k))^{-1} \tag{34}$$

$$\breve{P}_{k+1} = (A(k) + C_1)\mathrm{P}_k(A(k) + C_1)^T + \sum_{j=2}^{k+1} C_j P_{k+1-j} C_j^T + Q(k) \tag{35}$$

The Kalman filter proposed in this section is appropriate for state estimation in a fractional order system. It can also be used to estimate the elements of the unknown vector $j(k)$ which is used in the regression equation (19).

5. The Parameter and State Estimation Algorithm

In this section, we combine the recursive least squares identification algorithm (Section 3) and the fractional-order Kalman filter (Section 4) to provide a unit algorithm.

The new hierarchical identification method that will be proposed in this section is a new method for concurrent identification and estimation of parameters and states in fractional systems. According to the regression equation (19), the vectors j and N consist unknown state variables $x(k-i)\left(i=1,2,...,n\right)$ and vector q consists unknown parameters. So, in this algorithm, vector q_k (in equation (19)) will be identified with the provided RLS algorithm in Section 3. Furthermore, vector $X(k+1)$ will be estimated with the provided fractional order Kalman filter in Section 4. Therefore, first, we assume that the states are specified, and the parameters are estimated using the least squares method. Next, using the fractional-order Kalman filter algorithm, new states are estimated and used in the next step. This is repeated for the later stages.

In the resulting method, the system parameters ($\hat{q}$) and states ($\hat{X}(k+1)$) are estimated correctly at the same time. The parameters identification steps have been provided in equations (37)-(39) and the parameters estimation step has been expressed in equations (44).

$$J(q) := \sum_{j=1}^{t}\left[y(j) - j^{T}(j)q - N(k)\right]^{2} \tag{36}$$

$$\hat{q}_{k+1} = \hat{q}_k + F_k \hat{j}(\mathrm{k}) e(\mathrm{k}+1) \tag{37}$$

$$\mathrm{F}_{k+1} = F_k - \frac{F_k \hat{j}(k)\hat{j}^{T}(k)F_k}{1+\hat{j}^{T}(k)F_k\hat{j}(k)} \tag{38}$$

$$e(k+1) = \frac{y(k+1) - \hat{q}_k^{T}\hat{j}(k) - \hat{N}(k)}{1+\hat{j}^{T}(k)F_k\hat{j}(k)} \tag{39}$$

$$\hat{j}(k) = [\hat{x}_1(k-n) \quad \hat{x}_2(k-n) \quad \mathbf{L} \quad \hat{x}_n(k-n) \quad u(k-1) \quad u(k-2) \quad ... \quad u(k-n)]^{T} \tag{40}$$

$$\begin{aligned}\hat{N}(k) &= c_{11,1}\hat{x}_1(k-1) + ... + c_{11,k+1}\hat{x}_1(-1) + c_{22,1}\hat{x}_2(k-2) \\ &+ c_{22,2}\hat{x}_2(k-3) + ... + c_{22,k+1}\hat{x}_2(-2) \\ &+ ... + c_{\mathrm{nn},1}\hat{x}_n(k-n) + c_{\mathrm{nn},2}\hat{x}_n(k-1-n) + ... \\ &+ c_{\mathrm{nn},k+1}\hat{x}_n(-n) + w_1(k-1) + ... + w_n(k-n)\end{aligned} \tag{41}$$

The identified parameters are used to form $\hat{A},\hat{B}$ and $\hat{q}$ as follows:

$$\hat{q} = \left[\hat{a}_1 \quad \hat{a}_2 \quad ... \quad \hat{a}_n \quad \hat{b}_1 \quad \hat{b}_2 \quad ... \quad \hat{b}_n\right]^{T} \tag{42}$$

$$\hat{A} = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ \mathrm{M} & \mathrm{M} & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ 0 & 0 & 0 & ... & 1 \\ \hat{a}_1 & \hat{a}_2 & \hat{a}_3 & ... & \hat{a}_n \end{bmatrix}_{n\times n}, \hat{B} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \\ \mathrm{M} \\ \hat{b}_n \end{bmatrix}_{n\times 1} \tag{43}$$

In consequence, $\hat{A}$ and $\hat{B}$ matrices are used in the fractional-order Kalman filter as follows[2]:

$$\begin{aligned}&\Delta^{\Upsilon}\tilde{X}(k+1) = \hat{A}(k)\hat{X}(k) + \hat{B}(k)u(k) \\ &\tilde{X}(k+1) = \Delta^{\Upsilon}\tilde{X}(k+1) + \sum_{j=1}^{k+1} C_j \hat{X}(k+1-j) \\ &\hat{X}(k) = \tilde{X}(k) + K(k)\left(y(k) - H\tilde{X}(k)\right) \\ &P_k = (I - K(k)H)\tilde{P}_k \\ &K(k) = \tilde{P}_k H^{T}(H\tilde{P}_k H^{T} + R(k))^{-1} \\ &\tilde{P}_k = (\hat{A}(k) + C_1)\mathrm{P}_{k-1}(\hat{A}(k) + C_1)^{T} + \sum_{j=2}^{k+1} C_j P_{k-j} C_j^{T} + Q(k-1)\end{aligned} \tag{44}$$

The above algorithm is implementable according to the instructions below:

1. Initialization at time instance $k = 1$
2. Forming the $\hat{j}(k)$ and collecting the inputs and outputs.
3. Calculation of F_{k+1} and $e(k+1)$
4. Updating the parameter identification vector $\hat{q}_k$
5. Reading $\hat{a}_i, \hat{b}_{ij}$ parameters from $\hat{q}_k$ vector using the original definition of parameter identification vector $\hat{q}_k$
6. Forming $\hat{A},\hat{B}$ matrices
7. Calculating covariance matrix P_k and gain vector $K(k)$
8. Estimating the state vector $\hat{X}(k+1)$
9. Increasing k and going to step 2.

A flowchart of the proposed method is given in Figure 2:

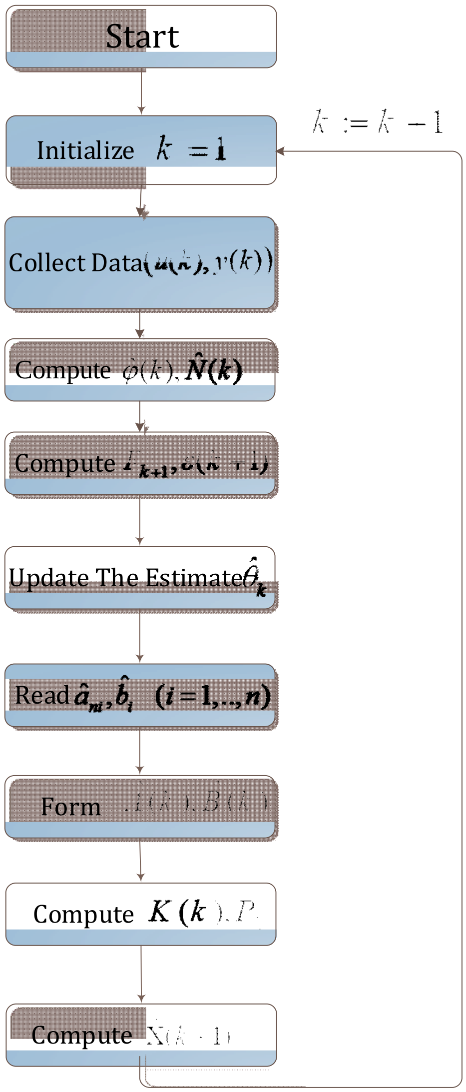

Fig. 2. Flowchart of the proposed identification method.

6. Simulation Results

To show the accuracy of the algorithm expressed in Section 5, two fractional-order systems with one input and one output have been used. The first system has two state and the second one contains four state variables.

System 1:

Consider a SISO fractional-order state-space system with the following matrices:

$a_{21} = -0.5, \quad a_{22} = -0.9$

$b_1 = -0.3, \quad b_2 = -0.7$

$g_1 = 0.9, \quad g_2 = 0.3$

$$A = \begin{bmatrix} 0 & 1 \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, H = [1 \quad 0], \quad \Upsilon = [g_1 \quad g_2]$$

The number of elements in equation (2) should be limited – here, the value is equal to *L*, which would simplify and reduce the number of calculations. Although it will cause a bit of error, by considering a reasonable value for *L*, the error value would be very small and negligible. Accordingly, equation (2) can be written as follows:

$$X(k+1) = \Delta^{\Upsilon} X(k+1) - \sum_{j=1}^{L} (-1)^j \binom{\Upsilon}{j} X(k+1-j) \qquad (45)$$

The state equations are then given as:

$$\Delta^{\Upsilon} X(k+1) = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.9 \end{bmatrix} X(k) + \begin{bmatrix} -0.3 \\ -0.7 \end{bmatrix} u(k) + w(k)$$

$$X(k+1) = \Delta^{\Upsilon} X(k+1) - \sum_{j=1}^{L} (-1)^j \begin{pmatrix} \binom{0.9}{j} & 0 \\ 0 & \binom{0.3}{j} \end{pmatrix} X(k-1+j)$$

$$y(k) = [1 \quad 0] X(k) + v(k)$$

and

$$E\left[n(k) n^T(k)\right] = 0.1, \quad E\left[w(k) w^T(k)\right] = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Here, $u(k)$ and $y(k)$ are the input and the output of the system, respectively. The information vector and the parameters vector are formed as follows:

$$j(k) = [x_1(k-2) \quad x_2(k-2) \quad u(k-1) \quad u(k-2)]^T$$

$$q = [a_{21} \quad a_{22} \quad b_1 \quad b_2]^T$$

In this simulation, the input $\{u(k)\}$ is an uncorrelated signal with variance 1.

The initial values at time *k=1* are considered as follows:

$$\hat{q}_0 = 10^{-6}, P_0 = 10^6, \hat{X}(1) = 0$$

$$R = [0.1], Q = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

First, we assume that the states are specified, and the parameters are estimated using the least squares method. Next, using the fractional-order Kalman filter algorithm, new states are estimated and used in the next step. This is repeated for the later stages. In this example, it is assumed that $L = 10, 25$ and 50.

The estimated parameters are shown in Tables 1, 2 and 3 for *L=50, 20* and *10*, respectively. As it can be seen in these tables, the estimation error is negligible. However, decreasing the value of *L* results in an increased error value in Tables 2 and 3. Furthermore, Figures 3 and 4 show the identified parameters' evolution through time. As it is seen, the identified parameters have converged to the true values rapidly. The original and the estimated state variables x_1 and x_2 are also shown in Figures 5 and 6, respectively. As it is seen, the proposed method can estimate the state variables accurately. It is assumed that $L = 50$ in these figures.

Tab. 1. The estimated parameters for $L = 50$.

No. of iterations	100	300	500	1000	2000
$a_{21} = -0.5$	-0.471	-0.493	-0.496	-0.497	**-0.498**
$a_{22} = -0.9$	-0.834	-0.885	-0.891	-0.893	**-0.894**
$b_1 = -0.3$	-0.287	-0.293	-0.296	-0.297	**-0.309**
$b_2 = -0.7$	-0.656	-0.682	-0.690	-0.694	**-0.695**

Tab. 2. The estimated parameters for $L = 25$.

No. of iterations	100	300	500	1000	2000
$a_{21} = -0.5$	-0.372	-0.409	-0.447	-0.463	**-0.477**
$a_{22} = -0.9$	-0.700	-0.755	-0.792	-0.836	**-0.854**
$b_1 = -0.3$	-0.199	-0.238	-0.255	-0.279	**-0.290**
$b_2 = -0.7$	-0.571	-0.601	-0.632	-0.664	**-0.681**

Tab. 3. The estimated parameters for $L = 10$ **.**

No. of iterations	100	300	500	1000	2000
$a_{21} = -0.5$	-0.333	-0.359	-0.389	-0.415	**-0.450**
$a_{22} = -0.9$	-0.654	-0.695	-0.737	-0.763	**-0.7[illegible]**
$b_1 = -0.3$	-0.169	-0.211	-0.226	-0.241	**-0.2[illegible]**
$b_2 = -0.7$	-0.533	-0.582	-0.609	-0.633	**-0.6[illegible]**

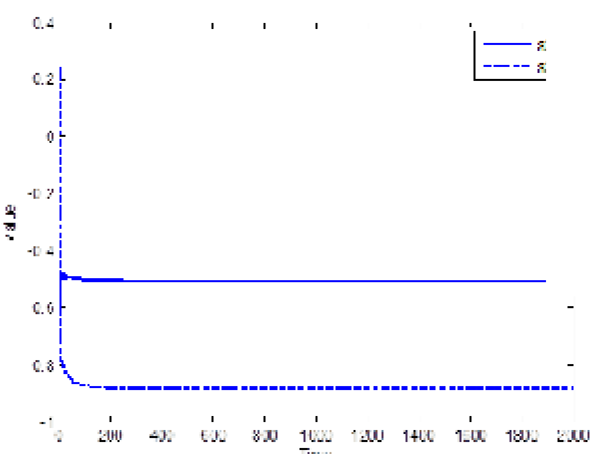

Fig. 3. The identified parameters a_{21}, a_{22} for $L = 50$ **.**

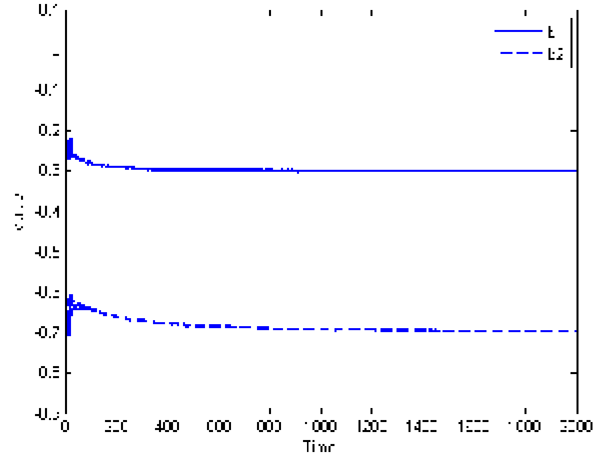

Fig. 4. The identified parameters b_1, b_2 for $L = 50$.

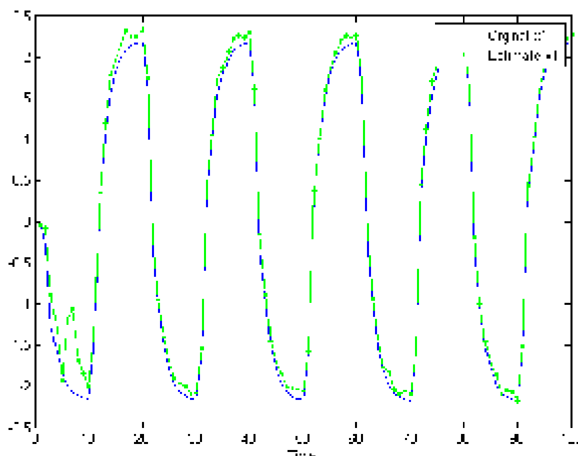

Fig. 5. Original and estimated state variable x_1 for $L = 50$.

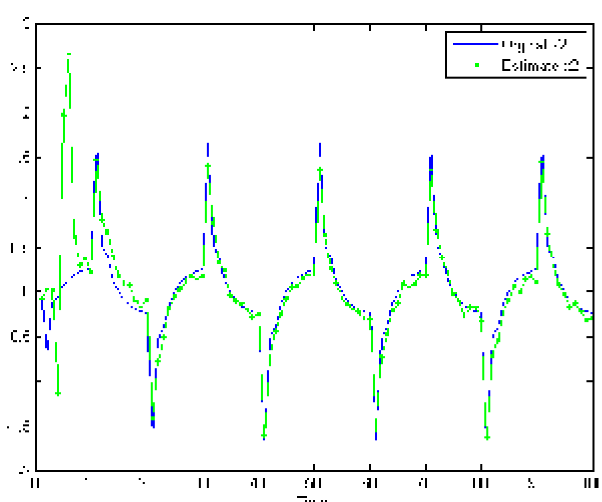

Fig. 6. Original and estimated state variable x_2 for $L = 50$ **.**

Another technique for state estimation in a fractional order state space system with unknown parameters is using of a fractional-order discrete-time transfer function. In this method, the parameters of the transfer function are identified first and then, the transfer function will be converted to a state-space form in which the states are estimated. This method has more computational burden in comparison with the method presented in the paper and therefore, the execution time increases. Furthermore, this method is not easily executable for MIMO systems. In the following, performance of the proposed method will be compared with a transfer function based method.

To identify the parameters of a discrete-time fractional state space mode, the corresponding transfer function can be obtained as follows[33]:

$$G(z)=\frac{Y(z)}{U(z)}=H\left(I(z\Delta^{\Upsilon}(z))-A\right)^{-1}B$$

$$=H\left(\begin{bmatrix} z\Delta^{g_1}(z) & 0 \\ 0 & z\Delta^{g_2}(z)\end{bmatrix}-\begin{bmatrix} 0 & 1 \\ a_{21} & a_{22}\end{bmatrix}\right)^{-1}B$$

$$=H\left(\begin{bmatrix} z\Delta^{g_1}(z) & -1 \\ -a_{21} & z\Delta^{g_2}(z)-a_{22}\end{bmatrix}\right)^{-1}B$$

therefore,

$$G(z)=\frac{\begin{bmatrix}1 & 0\end{bmatrix}\left(\begin{bmatrix} z\Delta^{g_2}(z)-a_{22} & 1 \\ a_{21} & z\Delta^{g_1}(z)\end{bmatrix}\right)\begin{bmatrix} b_1 \\ b_2\end{bmatrix}}{z^2\Delta^{g_1+g_2}(z)-a_{22}z\Delta^{g_1}(z)-a_{21}}$$

$$=\frac{b_1 z\Delta^{g_2}(z)+(-a_{22}b_1+b_2)}{z^2\Delta^{g_1+g_2}(z)-a_{22}z\Delta^{g_1}(z)-a_{21}}$$

$$=\frac{b_1 z^{-1}\Delta^{g_2}(z)+(-a_{22}b_1+b_2)z^{-2}}{\Delta^{g_1+g_2}(z)-a_{22}z^{-1}\Delta^{g_1}(z)-a_{21}z^{-2}}$$

where

$$\Delta^{g_1+g_2}y_k-a_{22}\Delta^{g_1}y_{k-1}-a_{21}y_{k-2}=b_1\Delta^{g_2}u_{k-1}+(-a_{22}b_1+b_2)u_{k-2}$$

Defining

$$j_k=\begin{bmatrix}\Delta^{g_1}y_{k-1} & y_{k-2} & \Delta^{g_2}u_{k-1} & u_{k-2}\end{bmatrix}$$

$$q^T=\begin{bmatrix}a_{22} & a_{21} & b_1 & (-a_{22}b_1+b_2)\end{bmatrix}$$

$$Y_k=\begin{bmatrix}\Delta^{g_1+g_2}y_k\end{bmatrix}$$

Results in a regression equation as

$$Y_k=j_k q$$

where

$$\Delta^g y_k=\sum_{j=0}^{k}(-1)^j\binom{g}{j}y_{k-j}$$

$$\Delta^g u_k=\sum_{j=0}^{k}(-1)^j\binom{g}{j}u_{k-j}$$

As it can be seen in the above equations, to form the vectors j_k and Y_k, it is necessary to gather much information. Furthermore, after identification of the system's parameters, the transfer function must be converted to the state-space form so that the states can be estimated. Therefore, it can be concluded that estimating the parameters and states of a fractional state space system using the hierarchical identification method presented in this paper, results in a considerable reduction in the execution time.

In Table 4, the estimated parameters through the transfer function method are compared with the proposed method of the paper. As can be seen in both cases, the parameter estimation accuracy is good. However, simulation results show that the run-time of the method presented in this paper is 2.26 seconds, while the run-time of the method described above (the transfer function based method) is 5.3 seconds. The CPU used in the simulation was a 2.20 GHz one with 6 GB of RAM.

Tab. 4. The estimated parameters by two methods for $L=50$.

No. of iterations		500	1000	2000
$a_{21}=-0.5$	The proposed method	-0.496	-0.497	-0.498
	Transfer function method	-0.495	-0.498	-0.499
$a_{22}=-0.9$	The proposed method	-0.891	-0.893	-0.894
	Transfer function method	-0.888	-0.892	-0.893
$b_1=-0.3$	The proposed method	-0.296	-0.297	-0.309
	Transfer function method	-0.292	-0.296	-0.298
$b_2=-0.7$	The proposed method	-0.690	-0.694	-0.695
	Transfer function method	-0.693	-0.695	-0.697

System 2:

In this example, a SISO fractional-order state-space system with four state variables and the following matrices is considered.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44}\end{bmatrix}, B=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4\end{bmatrix},$$

$$H=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}, \Upsilon=\begin{bmatrix}g_1 & g_2 & g_3 & g_4\end{bmatrix}$$

$a_{41} = -0.2,\quad a_{42} = -0.4,\quad a_{43} = -0.3,\quad a_{44} = -0.45$
$b_1 = -0.3,\quad b_2 = -0.2,\quad b_3 = -0.15,\quad b_4 = -0.25$
$g_1 = 0.4,\quad g_2 = 0.15,\quad g_3 = 0.3,\quad g_4 = 0.1$

The state equations are as follows:

$$\Delta^{\Upsilon} X(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.2 & -0.4 & -0.3 & -0.45 \end{bmatrix} X(k) + \begin{bmatrix} -0.3 \\ -0.2 \\ -0.15 \\ -0.25 \end{bmatrix} u(k) + w(k)$$

$$X(k+1) = \Delta^{\Upsilon} X(k+1) - \sum_{j=1}^{L} (-1)^j \begin{pmatrix} \binom{0.4}{j} & 0 & 0 & 0 \\ 0 & \binom{0.15}{j} & 0 & 0 \\ 0 & 0 & \binom{0.3}{j} & 0 \\ 0 & 0 & 0 & \binom{0.1}{j} \end{pmatrix} X(k-1+j)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} X(k) + v(k)$$

$$E\left[n(k)n^T(k)\right] = 0.05,$$

$$E\left[w(k)w^T(k)\right] = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix}$$

The initial values at time *k=1* are considered as follows:

$$\hat{q}_0 = 10^{-6}, P_0 = 10^6, \hat{X}(1) = 0$$

$$R = [0.05], Q = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix}$$

In this example, the information vector and the parameters vector are formed as follows:

$$j(k) = [x_1(k-4) \quad x_2(k-4) \quad x_3(k-4) \quad x_4(k-4) \quad u(k-1) \quad u(k-2) \quad u(k-3) \quad u(k-4)]^T$$

$$q = [a_{41} \quad a_{42} \quad a_{43} \quad a_{44} \quad b_1 \quad b_2 \quad b_3 \quad b_4]^T$$

The estimated parameters are shown in Tables 5, 6 and 7 for *L=50, 20* and *10*, respectively. As it can be seen in these tables, the estimation error is negligible. Furthermore, Figures 9 and 10 show that the identified parameters' evolution through time converge to the true values rapidly. The original and the estimated state variables x_1, x_2, x_3, x_4 are also shown in Figures 9, 10, 11 and 12, respectively. As it is seen in these figures, the proposed method can estimate the state variables accurately.

Tab. 5. The estimated parameters for $L = 50$.

No. of iterations	100	300	500	1000	2000
$a_{41} = -0.2$	-0.245	-0.227	-0.222	-0.216	**-0.213**
$a_{42} = -0.4$	-0.370	-0.391	-0.402	-0.412	**-0.416**
$a_{43} = -0.3$	-0.335	-0.311	-0.307	-0.291	**-0.280**
$a_{44} = -0.45$	-0.551	-0.499	-0.483	-0.467	**-0.456**
$b_1 = -0.3$	-0.239	-0.277	-0.289	-0.297	**-0.306**
$b_2 = -0.2$	-0.166	-0.185	-0.194	-0.198	**-0.203**
$b_3 = -0.15$	-0.132	-0.142	-0.148	-0.154	**-0.152**
$b_4 = -0.25$	-0.233	-0.240	-0.241	-0.247	**-0.248**

Tab. 6. The estimated parameters for $L = 25$.

No. of iterations	100	300	500	1000	2000
$a_{41} = -0.2$	-0.293	-0.263	-0.248	-0.232	**-0.222**
$a_{42} = -0.4$	-0.353	-0.398	-0.408	-0.416	**-0.420**
$a_{43} = -0.3$	-0.344	-0.315	-0.303	-0.285	**-0.276**
$a_{44} = -0.45$	-0.571	-0.505	-0.486	-0.475	**-0.463**
$b_1 = -0.3$	-0.209	-0.252	-0.281	-0.302	**-0.321**
$b_2 = -0.2$	-0.120	-0.144	-0.153	-0.169	**-0.180**
$b_3 = -0.15$	-0.085	-0.111	-0.122	-0.131	**-0.137**
$b_4 = -0.25$	-0.179	-0.206	-0.215	-0.229	**-0.236**

Tab. 7. The estimated parameters for $L = 10$ **.**

No. of iterations	100	300	500	1000	2000
$a_{41} = -0.2$	-0.299	-0.278	-0.261	-0.247	**-0.236**
$a_{42} = -0.4$	-0.343	-0.386	-0.411	-0.422	**-0.431**
$a_{43} = -0.3$	-0.348	-0.311	-0.291	-0.278	**-0.262**
$a_{44} = -0.45$	-0.560	-0.516	-0.493	-0.479	**-0.471**
$b_1 = -0.3$	-0.233	-0.268	-0.290	-0.302	**-0.310**
$b_2 = -0.2$	-0.143	-0.164	-0.173	-0.185	**-0.191**
$b_3 = -0.15$	-0.126	-0.135	-0.137	-0.139	**-0.142**
$b_4 = -0.25$	-0.218	-0.229	-0.233	-0.239	**-0.241**

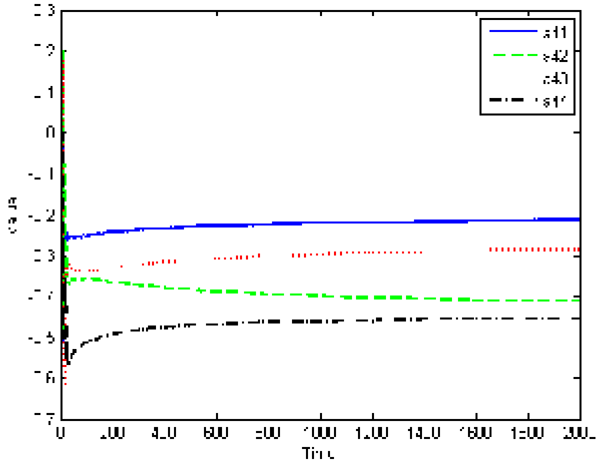

Fig. 7. The identified parameters $a_{41}, a_{42}, a_{43}, a_{44}$ for $L = 50$.

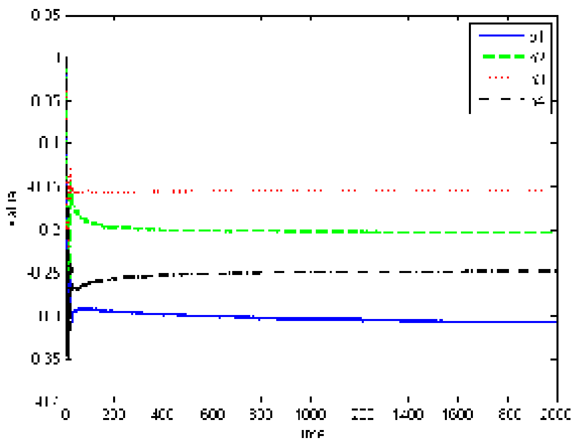

Fig. 8. The identified parameters b_1, b_2, b_3, b_4 for $L = 50$.

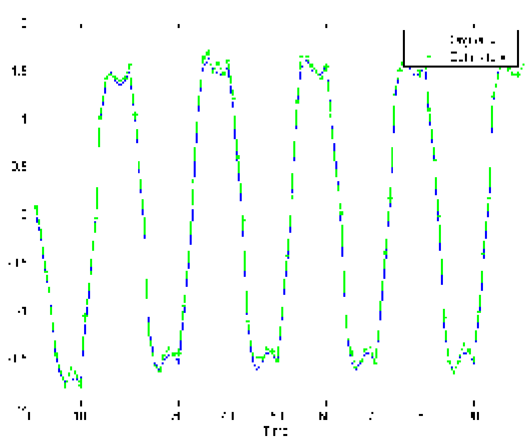

Fig. 9. Original and estimated state variable x_1 for $L = 50$.

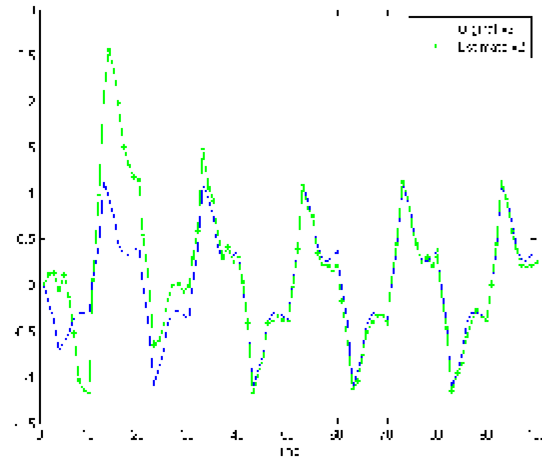

Fig. 10. Original and estimated state variable x_2 for $L = 50$.

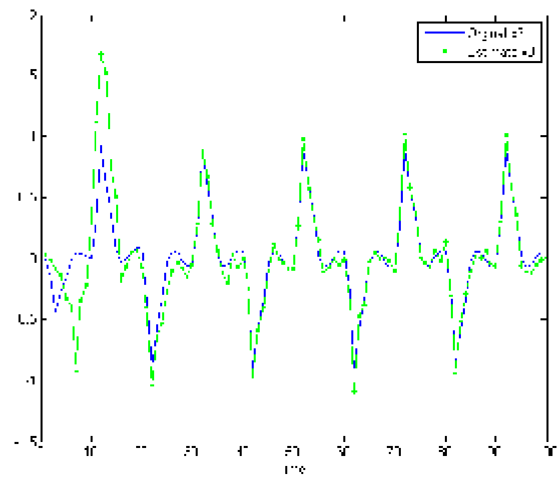

Fig. 11. Original and estimated state variable x_3 for $L = 50$.

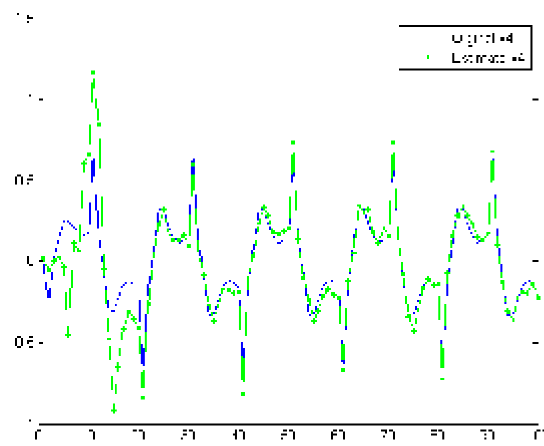

Fig. 12. Original and estimated state variable x_4 for $L = 50$.

7. Conclusion

This paper proposed a novel identification approach for canonical fractional-order state-space systems. An advantage of the proposed method is that not only the unknown parameters of the system but also its states are estimated. Promising performance of the proposed method was verified using two examples. Concluding the simulation results, it is clear that the proposed algorithm is able to successfully perform the identification and estimation hierarchically.

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