

# Reducing the Supervisory Control of Discrete-Event Systems under Partial Observation

Vahid Saeidi, Ali A. Afzalian, and Davood Gharavian

**Abstract**—Supervisor reduction procedure can be used to construct the reduced supervisor with a reduced number of states in discrete-event systems. However, it was proved that the reduced supervisor is control equivalent to the original supervisor with respect to the plant; it has not been guaranteed that the reduced supervisor and the original one are control equivalent under partial observation. In this paper, we extend the supervisor reduction procedure by considering partial observation; namely not all events are observable. A feasible supervisor which is constructed under partial observation becomes reduced based on control consistency of uncertainty sets of states, instead of the original supervisor. In order to construct a partial observation reduced supervisor, a partial observation control cover is constructed based on control consistency of uncertainty sets in the supervisor. Four basic functions are defined in order to capture the control and marking information on the uncertainty sets. In the resulting reduced supervisor, only observable events can cause state changes. The results are illustrated by some examples.

**Index Terms**—control consistency, control cover, discrete-event systems, partial observation, supervisor reduction.

## I. INTRODUCTION

The state size and the computational complexity of a monolithic supervisor increase with state sizes of the plant and the specification [1], and may lead to state explosion [2]. However, the application of this theory is restricted, some works are reported on application of this theory in practice, e.g. [3, 4]. Although modular [5, 6] and incremental [7, 8] approaches try to overcome the complexity of the supervisor synthesis, other approaches tend to reduce a supervisor for simple implementation. The supervisor reduction procedure, given by [9], is an evolution of the proposed method in [10]. This procedure reduces the redundant information in the supervisor synthesis without any effect on controlled behavior. A reduced supervisor has some advantages comparing to the original supervisor, such as simplicity. Although this procedure is a heuristic method, it has been extended to other applications, e.g. coordination planning for distributed agents [11], supervisor localization procedure with full observation [12], and supervisor localization procedure under partial observation [13]. In [13], the authors employed the concept of relative observability to compute a partial-observation monolithic supervisor, and then they designed a localization procedure using (feasible) partial-observation supervisor to decompose the supervisor into a set of local controllers.

In this paper, we extend supervisor reduction procedure [9],

to address the issue of partial observation. At first, we synthesize a partial-observation monolithic supervisor using the concept of relative observability [14]. Relative observability is stronger than observability [15, 16], weaker than normality [15, 16], and the supremal relative observable (and controllable) sublanguage of a given language exists. The supremal sublanguage may be effectively computed, and then implemented by a partial-observation (feasible and non-blocking) supervisor [13, 17]. Then, we suitably extend the supervisor reduction procedure in [9] to reduce a supervisor under partial observation.

In this paper, the partial-observation control cover is introduced. In particular, it is defined on the state set of the partial-observation supervisor; roughly speaking the latter corresponds to the power set of the full-observation supervisor's state set. As a result, a partial-observation reduced supervisor contains only observable state transitions.

The rest of the paper is organized as follows: In Section II, the necessary preliminaries are reviewed. Reducing the supervisory control under partial observation is proposed in Section III. In Section IV, five examples are given to clarify the proposed method. Finally, concluding remarks are given in Section V.

## II. PRELIMINARIES

A discrete-event system (DES) is represented by an automaton  $G = (Q, \Sigma, \delta, q_0, Q_m)$ , where  $Q$  is a finite set of states, with  $q_0 \in Q$  as the initial state and  $Q_m \subseteq Q$  being the marked states;  $\Sigma$  is a finite set of events ( $\sigma$ ) which is partitioned as a set of controllable events  $\Sigma_c$  and a set of uncontrollable events  $\Sigma_{uc}$ , where  $\Sigma = \Sigma_c \cup \Sigma_{uc}$ .  $\delta$  is a transition mapping  $\delta: Q \times \Sigma \rightarrow Q$ ,  $\delta(q, \sigma) = q'$  gives the next state  $q'$  is reached from  $q$  by the occurrence of  $\sigma$ .  $G$  is discrete-event model of the plant. In this context  $\delta(q_0, s)!$  means that  $\delta$  is defined for  $s$  at  $q_0$ .  $L(G) := \{s \in \Sigma^* | \delta(q_0, s)!\}$  is the closed behavior of  $G$  and  $L_m(G) := \{s \in L(G) | \delta(q_0, s) \in Q_m\}$  is the marked behaviour of  $G$  [17, 18].

A set of all control patterns is denoted with  $\Gamma = \{\gamma \in Pwr(\Sigma) | \gamma \supseteq \Sigma_{uc}\}$ . A supervisory control for  $G$  is any map  $V: L(G) \rightarrow \Gamma$ , where  $V(s)$  represents the set of enabled events after the occurrence of the string  $s \in L(G)$ . The pair  $(G, V)$  is written  $V/G$ , to suggest " $G$  under the supervision of  $V$ ". A behavioral constraint on  $G$  is given by specification language  $E \subseteq \Sigma^*$ . Let  $K \subseteq L_m(G) \cap E$  be the supremal controllable sublanguage of  $E$  w.r.t.  $L(G)$  and  $\Sigma_{uc}$ , i.e.  $K = \sup C(L_m(G) \cap E)$  [17]. If  $K \neq \emptyset$ , it can be shown as a DES,  $SUP =$

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$(X, \Sigma, \xi, x_0, X_m)$ , which is the recognizer for  $K$ . If  $G$  and  $E$  are finite-state DES, then  $K$  is regular language. Write  $|\cdot|$  for the state size of DES. Then  $|\text{SUP}| \leq |G||E|$ . In applications, engineers want to employ RSUP, which has a fewer number of states (i.e.  $|\text{RSUP}| \ll |\text{SUP}|$ ) and is control equivalent to SUP w.r.t.  $G$  [9], i.e.

$$L_m(G) \cap L_m(\text{RSUP}) = L_m(\text{SUP}), \quad (1)$$

$$L(G) \cap L(\text{RSUP}) = L(\text{SUP}). \quad (2)$$

The natural projection is a mapping  $P: \Sigma^* \rightarrow \Sigma_0^*$  where (1)  $P(\epsilon) := \epsilon$  ( $\epsilon$  is the empty string), (2) for  $s \in \Sigma^*$ ,  $\sigma \in \Sigma$ ,  $P(s\sigma) := P(s)P(\sigma)$ , and (3)  $P(\sigma) := \sigma$  if  $\sigma \in \Sigma_0$  and  $P(\sigma) := \epsilon$  if  $\sigma \notin \Sigma_0$ . The effect of an arbitrary natural projection  $P$  on a string  $s \in \Sigma^*$  is to erase the events in  $s$  that do not belong to observable events set,  $\Sigma_0$ . The natural projection  $P$  can be extended and denoted with  $P: \text{Pwr}(\Sigma^*) \rightarrow \text{Pwr}(\Sigma_0^*)$ . For any  $L \subseteq \Sigma^*$ ,  $P(L) := \{P(s) | s \in L\}$ . The inverse image function of  $P$  is denoted with  $P^{-1}: \text{Pwr}(\Sigma_0^*) \rightarrow \text{Pwr}(\Sigma^*)$  for any  $L \subseteq \Sigma_0^*$ ,  $P^{-1}(L) := \{s \in \Sigma^* | P(s) \in L\}$ . The synchronous product of languages  $L_1 \subseteq \Sigma_1^*$  and  $L_2 \subseteq \Sigma_2^*$  is defined by  $L_1 \parallel L_2 = P_1^{-1}(L_1) \cap P_2^{-1}(L_2) \subseteq \Sigma^*$ , where  $P_i: \Sigma^* \rightarrow \Sigma_i^*$ ,  $i = 1, 2$  for the union  $\Sigma = \Sigma_1 \cup \Sigma_2$  [19].

Let  $\text{SUP} = (X, \Sigma, \xi, x_0, X_m)$  be the recognizer of  $K$ ,  $\Sigma_0 \subseteq \Sigma$  and  $P: \Sigma^* \rightarrow \Sigma_0^*$  be the natural projection. For  $s \in \Sigma^*$ , observation of  $P(s)$  results in uncertainty as to the state of SUP given by the "uncertainty set"  $U(s) := \{\delta(q_0, s') | P(s') = P(s), s' \in \Sigma^*\} \subseteq Q$ . Uncertainty sets can be used to obtain a recognizer for the projected language  $P(K)$ . By definition of uncertainty set, each pair of states  $x, x' \in X$ , reachable by  $s, s'$ , are control consistent, if there exists a non-blocking supervisor  $V$  such that  $P(s') = P(s) \Rightarrow V(s') = V(s)$ .  $V$  is called a feasible supervisor [17]. Each pair of states  $x, x' \in X$  in a monolithic supervisor can be considered one state in the feasible supervisor by self-looping an unobservable event  $\sigma$ , which occurs between states  $x, x'$ .

It was defined in [14], that  $K$  is relative observable w.r.t.  $\bar{C}$ ,  $G$  and  $P$  (or  $(\bar{C}, G, P)$ -observable) for  $K \subseteq C \subseteq L_m(G)$ , where  $\bar{K}$  and  $\bar{C}$  are prefix closed languages, if for every pair of strings  $s, s' \in \Sigma^*$  such that  $P(s) = P(s')$ , the following two conditions hold,

$$(\forall \sigma \in \Sigma) s\sigma \in \bar{K}, s' \in \bar{C}, s'\sigma \in L(G) \Rightarrow s'\sigma \in \bar{K}, \quad (4)$$

$$s \in K, s' \in \bar{C} \cap L_m(G) \Rightarrow s' \in K. \quad (5)$$

In the special case, if  $C = K$ , then the relative observability property is tighten to the observability property. An observation property called normality was defined in [16], that is stronger than the relative observability.  $K$  is said to be normal w.r.t.  $(L(G), P)$ , if  $P^{-1}P(\bar{K}) \cap L(G) = \bar{K}$ , where  $L(G)$  is a prefix closed language and  $P$  is a natural projection.

### III. REDUCING THE SUPERVISORY CONTROL UNDER PARTIAL OBSERVATION

Similar to the procedure, proposed in [9], to reduce the state size of the supervisory control with full observation, we propose a method to reduce the state size of the supervisory control under partial observation.

Let  $G = (Q, \Sigma, \delta, q_0, Q_m)$ , be the plant,  $\Sigma_0 \subseteq \Sigma$  be the subset of observable events, and  $P: \Sigma^* \rightarrow \Sigma_0^*$  be the corresponding natural projection. Also let  $\text{SUP} = (X, \Sigma, \xi, x_0, X_m)$  be the recognizer of supervisor  $K$ . Under partial observation, if  $s \in L(\text{SUP})$  occurs, then  $P(s)$  is observed. Let  $U(s)$  be the subset of states that may be reached by some  $s'$  that looks like  $s$ , i.e.

$$U(s) = \{x \in X | (\exists s' \in \Sigma^*) P(s) = P(s'), x = \xi(x_0, s')\}.$$

Let  $\mathcal{U}(X)$  be the set of uncertainty sets of all states in  $X$ , associated with strings in  $L(\text{SUP})$ , i.e.

$$\mathcal{U}(X) := \{U(s) \subseteq X | s \in L(\text{SUP})\}$$

The transition function associated with  $\mathcal{U}(X)$  is  $\hat{\xi}: \mathcal{U}(X) \times \Sigma_0 \rightarrow \mathcal{U}(X)$ .  $\hat{\xi}$  is given by

$$\hat{\xi}(U, \sigma) = \bigcup \{\xi(x, u_1 \sigma u_2) | x \in U, u_1, u_2 \in \Sigma_{u_0}^*\}$$

Where  $\Sigma_{u_0} = \Sigma - \Sigma_0$ . If there exist  $u_1, u_2 \in \Sigma_{u_0}^*$  such that  $\xi(x, u_1 \sigma u_2)!$  then  $\hat{\xi}(U, \sigma)$  is defined and denoted as  $\hat{\xi}(U, \sigma)!$ . Having  $\mathcal{U}(X)$  and  $\hat{\xi}$ , partial observation monolithic supervisor SUPO can be defined. It is a feasible supervisor, and its synchronization by the plant is control equivalent to the original supervisor w.r.t. the plant. SUPO is defined as follows,

$$\text{SUPO} = (\mathcal{U}(X), \Sigma_0, \hat{\xi}, U_0, \mathcal{U}_m)$$

Where  $U_0 = U(\epsilon)$  and  $\mathcal{U}_m = \{U \in \mathcal{U}(X) | U \cap X_m \neq \emptyset\}$ . It is known [13], that  $L(\text{SUPO}) = P(L(\text{SUP}))$  and  $L_m(\text{SUPO}) = P(L_m(\text{SUP}))$ .

Let  $U \in \mathcal{U}(X)$ ,  $x \in U$  be any state in SUP and  $\alpha \in \Sigma_c$  be a controllable event. We know that 1.  $\alpha$  is enabled at  $x \in U$ , if  $\xi(x, \alpha)!$ , or 2.  $\alpha$  is disabled at  $x \in U$ ,  $\neg \xi(x, \alpha)!$  and  $(\exists s \in \Sigma^*) [\xi(x_0, s) = x \ \& \ \hat{\xi}(U_0, Ps) = U] \ \& \ \delta(q_0, s\sigma)!$  or 3.  $\alpha$  is not defined at  $x \in U$ , if  $\neg \xi(x, \alpha)!$  and  $\neg \xi(x, \alpha)!$  and  $(\exists s \in \Sigma^*) [\xi(x_0, s) = x \ \& \ \hat{\xi}(U_0, Ps) = U] \Rightarrow \neg \delta(q_0, s\sigma)!$ . Under partial observation, the control actions after string  $s \in L(\text{SUP})$  depend on the uncertainty set  $U(s) \in \mathcal{U}(X)$ , i.e. the state of SUPO. It was proved that, if  $\alpha$  is enabled at  $x \in U$ , then for all  $x' \in U$ , either  $\alpha$  is also enabled at  $x' \in U$ , or  $\alpha$  is not defined at  $x' \in U$ . On the other hand, if  $\alpha$  is disabled at  $x \in U$ , then for all  $x' \in U$ , either  $\alpha$  is also disabled at  $x' \in U$ , or  $\alpha$  is not defined at  $x' \in U$  [13].

In order to propose a supervisor reduction procedure under partial observation, consider the following four functions which capture the control and marking information on the uncertainty sets. Define  $E: \mathcal{U}(X) \rightarrow \text{Pwr}(\Sigma_0)$  according to

$$E(U) = \{\sigma \in \Sigma_0 | (\exists x \in U) \xi(x, \sigma)!\}$$

$E(U)$  denotes the set of events enabled at state  $U$ . Also define  $D: \mathcal{U}(X) \rightarrow \text{Pwr}(\Sigma_0)$  according to

$$D(U) = \{\sigma \in \Sigma_0 | (\exists x \in U) \neg \xi(x, \sigma)! \ \& \ (\exists s \in \Sigma^*) [\xi(x_0, s) = x \ \& \ \delta(q_0, s\sigma)!]\}$$

$D(U)$  is the set of events, which are disabled at state  $U$ . Next, define  $M: \mathcal{U}(X) \rightarrow \{0, 1\}$  according to

$$M(U) = \begin{cases} 1, & \text{if } (U \in \mathcal{U}_m), \\ 0, & \text{otherwise.} \end{cases}$$

$M(U) = 1$  if  $U$  is marked in SUPO, i.e.  $U$  contains a marked state

of SUP. Finally define  $T: \mathcal{U}(X) \rightarrow \{0,1\}$  according to

$$T(U) = \begin{cases} 1, & \text{if } (\exists s \in \Sigma^*) \xi(x_0, s) \in U \\ \xi(U_0, Ps) = U, \delta(q_0, s) \in Q_m & \\ 0, & \text{otherwise.} \end{cases}$$

$T(U) = 1$  if  $U$  contains some states that correspond to a marked state of  $G$ , i.e.  $U$  contains a marked state of  $G$ . Now, the control consistency relation  $\mathcal{R}_U \subseteq \mathcal{U}(X) \times \mathcal{U}(X)$  can be defined.  $U, U' \in \mathcal{U}(X)$  are control consistent, i.e.  $(U, U') \in \mathcal{R}_U$ , if

$$E(U) \cap D(U') = E(U') \cap D(U) = \emptyset, \quad (6)$$

$$T(U) = T(U') \Rightarrow M(U) = M(U'). \quad (7)$$

Thus a pair of uncertainty sets  $(U, U')$  satisfies  $(U, U') \in \mathcal{R}_U$ , if (i) each event is enabled at least at one state of  $U$ , but is not disabled at any state of  $U'$ , and vice versa; (ii)  $U, U'$  both contain marked states of SUP (both do not contain) provided that they both contain states corresponding to some marked states of  $G$  (both do not contain). It is easily verified that  $\mathcal{R}_U$  is generally not transitive, thus it is not an equivalence relation. This leads to the partial-observation control cover. Let  $I$  be some index set, and  $\mathcal{C}_U = \{\mathcal{U}_i \subseteq \mathcal{U}(X) | i \in I\}$  be a cover on  $\mathcal{U}(X)$ .  $\mathcal{C}_U$  is a partial observation control cover, if

$$(i) (\forall i \in I) (\forall U, U' \in \mathcal{U}_i) (U, U') \in \mathcal{R}_U, \\ (ii) (\forall i \in I) (\forall \sigma \in \Sigma_0) (\exists U \in \mathcal{U}_i) \xi(U, \sigma)! \Rightarrow [(\exists j \in I) (\forall U' \in \mathcal{U}_j) \xi(U', \sigma)! \Rightarrow \xi(U', \sigma) \in \mathcal{U}_j], \quad (8)$$

A partial observation control cover  $\mathcal{C}_U$  lumps the uncertainty sets  $U \in \mathcal{U}(X)$  into cells  $\mathcal{U}_i \in \mathcal{C}_U, i \in I$  such that (i) the uncertainty sets  $U$  that reside in the same cell  $\mathcal{U}_i$  must be pairwise control consistent, (ii) for every observable event  $\sigma \in \Sigma_0$ , the uncertainty set that is reached from any uncertainty set  $U' \in \mathcal{U}_i$  by one-step transition  $\sigma$ , must be covered by the same cell  $\mathcal{U}_j$ . Obviously, two uncertainty sets  $U$  and  $U'$  belong to a common cell of  $\mathcal{C}_U$ , if and only if  $U$  and  $U'$  are control consistent, and two future uncertainty sets that can be reached respectively from  $U$  and  $U'$  by a given observable string are again control consistent.  $\mathcal{C}_U$  is called a partial-observation control congruence if  $\mathcal{C}_U$  happens to be a partition on  $\mathcal{U}(X)$ , namely its cells are pairwise disjoint. Having  $\mathcal{C}_U$ ,  $U_0 = U(\epsilon)$  and  $x_0 \in U_0$ , a generator  $J = (I, \Sigma_0, \zeta, i_0, I_m)$  can be defined over  $\Sigma_0$  as follows,

$$i_0 \in I \text{ such that } U_0 \in \mathcal{U}_{i_0}, \\ I_m := \{i \in I | (\exists U \in \mathcal{U}_i) X_m \cap U \neq \emptyset\} \\ \zeta: I \times \Sigma_0 \rightarrow I \text{ with } \zeta(i, \sigma) = j \\ \text{if } (\exists U \in \mathcal{U}_i), \xi(U, \sigma) \in \mathcal{U}_j; \quad (9)$$

Note that, overlapping of some states results that  $i_0$  and  $\zeta$  may not be uniquely determined, and  $J$  may not be unique. If  $\mathcal{C}_U$  is partition on  $\mathcal{U}(X)$ ,  $J$  can be determined uniquely and it can be selected as the reduced supervisor,  $\text{RSUP}_p$ .

We prove in Theorem 1,  $\text{RSUP}_p$  is control equivalent to SUP w.r.t.  $G$ .

**Theorem 1:**  $\text{RSUP}_p$  is control equivalent to SUP w.r.t.  $G$ , i.e.

$$L(G) \cap L(\text{RSUP}_p) = L(\text{SUP}), \quad (10)$$

$$L_m(G) \cap L_m(\text{RSUP}_p) = L_m(\text{SUP}). \quad (11)$$

*Proof:* We prove the claim in two steps, a.  $\subseteq$ , b.  $\supseteq$ .

a. As it was assumed that  $L_m(\text{SUP})$  is not empty, it follows that  $L(G)$  and  $L(\text{RSUP}_p)$  are not empty, and as they are closed, the

empty string  $\epsilon$  belongs to each. Now, suppose that  $s \in L(G) \cap L(\text{RSUP}_p)$  implies that  $s \in L(\text{SUP})$  and  $s\sigma \in L(G) \cap L(\text{RSUP}_p)$  such that  $\sigma \in \Sigma$ . We must prove that  $s\sigma \in L(\text{SUP})$ . If  $\sigma \in \Sigma - (\Sigma_c \cup \Sigma_0)$ , then  $s\sigma \in L(\text{SUP})$ , because  $L(\text{SUP})$  is controllable and observable. Now, assume  $\sigma \in \Sigma_c \cap \Sigma_0$  and  $s\sigma \in L(G) \cap L(\text{RSUP}_p)$ . Since  $U$  and  $U'$  belong to the same cell  $\mathcal{U}_i$ , by definition of partial-observation control cover, they must be control consistent, i.e.  $(U, U') \in \mathcal{R}_U$ . Thus,  $E(U) \cap D(U') = \emptyset$  which implies that  $D(U') = \emptyset$ . It means that all controllable and observable  $\sigma$  that is enabled at  $U$ , cannot be disabled at  $U'$ . Thus,  $\forall x \in U'$ , either  $\xi(x, \sigma)!$  or  $(\forall t \in \Sigma^*) [\xi(x_0, t) = x \ \& \ \neg \delta(q_0, t\sigma)!]$ . Note that,  $s\sigma \in L(G)$ . Thus  $\neg \delta(q_0, t\sigma)!$  is not true. Therefore,  $\xi(x, \sigma)!$  is true, i.e.  $s\sigma \in L(\text{SUP})$ .

Now, assume  $s \in L_m(G) \cap L_m(\text{RSUP}_p)$ . It means that  $\zeta(i_0, s) \in I_m$ . From (9), it is obvious  $\xi(x_0, s) \in X_m$ , i.e.  $s \in L_m(\text{SUP})$ .

b. Suppose that  $s \in L(\text{SUP})$  implies that  $s \in L(G) \cap L(\text{RSUP}_p)$ . Assume  $s\sigma \in L(\text{SUP})$ . If  $\sigma \in \Sigma - \Sigma_0$ , then it is a self-loop transition at some states in  $\text{RSUP}_p$ . thus,  $s \in L(G) \cap L(\text{RSUP}_p) \Rightarrow s\sigma \in L(G) \cap L(\text{RSUP}_p)$ . If  $\sigma \in \Sigma_0$ , then (9) implies that  $\zeta(i, \sigma) = j$ . Thus,  $s\sigma \in L(G) \cap L(\text{RSUP}_p)$ .

Now, assume  $s \in L_m(\text{SUP})$ . It means that  $\xi(x_0, s) \in X_m$ . From (9), we can write  $\zeta(i_0, s) \in I_m$ . Namely,  $s \in L_m(G) \cap L_m(\text{RSUP}_p)$ . The proof is complete.

**Corollary 1:** Let  $G$  be a non-blocking plant, described by closed and marked languages  $L(G), L_m(G) \subseteq \Sigma^*$ , and  $\text{SUP} = (X, \Sigma, \xi, x_0, X_m)$  be the recognizer of the supervisor  $K$ , i.e.  $K = L_m(\text{SUP})$ . Let  $\text{RSUP}_p$  be the reduced supervisor under partial observation. If  $K$  is relatively observable w.r.t.  $(\bar{C}, G, P)$ , where  $P: \Sigma^* \rightarrow \Sigma_0^*$ , and  $K \subseteq C \subseteq L_m(G)$ , then  $P(\text{RSUP}_p)$  is control equivalent to  $P(\text{SUP})$  w.r.t.  $G$ , i.e.

$$L_m(G) \cap P^{-1}(L_m(P(\text{RSUP}_p))) = L_m(G) \cap P^{-1}(L_m(P(\text{SUP}))), \\ L(G) \cap P^{-1}(L(P(\text{RSUP}_p))) = L(G) \cap P^{-1}(L(P(\text{SUP}))).$$

In order to clarify the proposed method for reducing a supervisor under partial observation, some examples are illustrated in the next section.

#### IV. EXAMPLES

In this section, we consider examples in order to verify the extended theory in Section III. The model construction and supervisor synthesis are carried out by TCT software [20]. A brief description of TCT procedures, which are used in this paper, is given in the Appendix.

**Example 1:** Let  $\Sigma = \{1, 2, 3\}$  and  $G, \text{SUP}$  be the plant and the recognizer of supervisor, respectively (Fig. 1). Obviously, we can find  $\mathcal{C}_1$  and  $\mathcal{C}_2$  such that  $K = L_m(\text{SUP})$  is relatively observable w.r.t.  $(\bar{C}_1, G, P_1)$ , where  $P_1: \Sigma^* \rightarrow \Sigma_1^*$  and  $\Sigma_1 = \{1, 3\}$  and  $K$  is relatively observable w.r.t.  $(\bar{C}_2, G, P_2)$ , where  $P_2: \Sigma^* \rightarrow \Sigma_2^*$  and  $\Sigma_2 = \{2, 3\}$ . But, we cannot find any  $C$  such that  $K$  is relatively observable w.r.t.  $(\bar{C}, G, P_0)$ , where  $P_0: \Sigma^* \rightarrow \Sigma_0^*$  and  $\Sigma_0 = \{3\}$ . We can find uncertainty sets  $\mathcal{U}_1(X)$  and  $\mathcal{U}_2(X)$  corresponding to  $P_1$  and  $P_2$ , respectively.

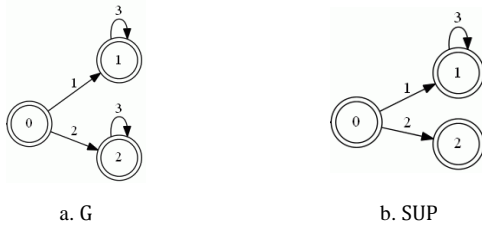


Fig. 1. The plant G and the corresponding supervisor, SUP

Fig. 2. Feasible reduced supervisors RSUP<sub>1</sub> and RSUP<sub>2</sub>

$\mathcal{U}_1(X) = \{\{0,1\}, \{2\}\}$  and  $\mathcal{U}_2(X) = \{\{0,2\}, \{1\}\}$  can be constructed. The partial observation control covers can be constructed as  $\mathcal{C}_1 = \mathcal{U}_1(X)$  and  $\mathcal{C}_2 = \mathcal{U}_2(X)$ . Thus, RSUP<sub>1</sub> and RSUP<sub>2</sub> are both feasible reduced supervisors, corresponding to  $P_1$  and  $P_2$ , respectively (Fig. 2). Since states 1 and 2 are not control consistent, states 0, 1 and 2 in SUP cannot be lumped into one state, in order to construct a reduced supervisor. It is obvious that, RSUP<sub>1</sub> is control equivalent to SUP w.r.t. G, under natural projection  $P_1$ , but it is not control equivalent to SUP under  $P_2$ . Also, RSUP<sub>2</sub> is control equivalent to SUP w.r.t. G, under natural projection  $P_2$ , but it is not control equivalent to SUP under  $P_1$  w.r.t. G.

*Example 2:* Let  $\Sigma = \{10,11,12,13\}$  and G, SUP be the plant and the recognizer of supervisor, respectively (Fig. 3). We can find  $\mathcal{C}_1, \mathcal{C}_2$  and  $\mathcal{C}_3$  such that  $K = L_m(\text{SUP})$  is relatively observable w.r.t.  $(\bar{\mathcal{C}}_1, G, P_1)$ , where  $P_1: \Sigma^* \rightarrow \Sigma_1^*$  and  $\Sigma_1 = \{10,12,13\}$  and is relatively observable w.r.t.  $(\bar{\mathcal{C}}_2, G, P_2)$ , where  $P_2: \Sigma^* \rightarrow \Sigma_2^*$  and  $\Sigma_2 = \{11,12,13\}$ . Also, it is relatively observable w.r.t.  $(\bar{\mathcal{C}}_3, G, P_3)$ , where  $P_3: \Sigma^* \rightarrow \Sigma_3^*$  and  $\Sigma_3 = \{10,11,12\}$ . Moreover, we can find  $\mathcal{C}$  such that K is relatively observable w.r.t.  $(\bar{\mathcal{C}}, G, P_0)$ , where  $P_0: \Sigma^* \rightarrow \Sigma_0^*$  and  $\Sigma_0 = \{12\}$ .

We can find the uncertainty set  $\mathcal{U}(X) = \{\{0,1,2,4\}, \{3\}\}$  corresponding to  $P_0$ . Note that RSUP<sub>0</sub> is the partial observation reduced supervisor, corresponding to control cover  $\mathcal{C} = \{\{0,1,2,4\}, \{3\}\}$  (Fig. 4). Since other control covers can be found corresponding to other uncertainty sets relevant to  $P_i, i = 1,2,3$ , the reduced supervisor is not unique. But, other feasible reduced supervisor seems have more number of states. Obviously, we can check that RSUP<sub>0</sub> is control equivalent to SUP under natural projection  $P_0$  w.r.t. G.

*Example 3: Supervisory control of transfer line under partial observation*

Industrial transfer line consists of two machines M<sub>1</sub>, M<sub>2</sub> and a test unit TU, which are linked by buffers B<sub>1</sub> and B<sub>2</sub> (Fig. 5). The capacities of B<sub>1</sub> and B<sub>2</sub> are assumed to be 3 and 1, respectively. If a work piece is accepted by TU, it is released from the system; if rejected, it is returned to B<sub>1</sub> for reprocessing by M<sub>2</sub>. The specification is based on protecting B<sub>1</sub> and B<sub>2</sub> against underflow and overflow [17].

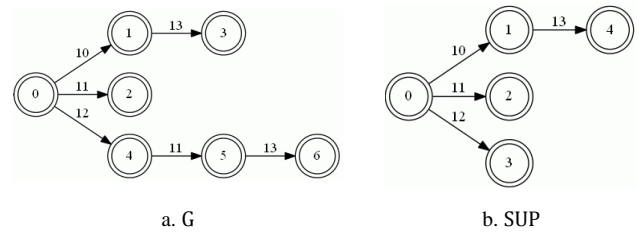


Fig. 3. The plant G and the corresponding supervisor SUP

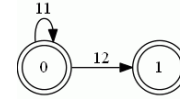
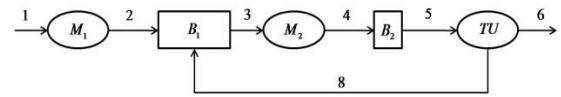
Fig. 4. The partial observation reduced supervisor RSUP<sub>0</sub>

Fig. 5. Transfer Line

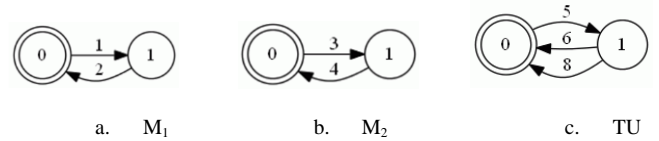
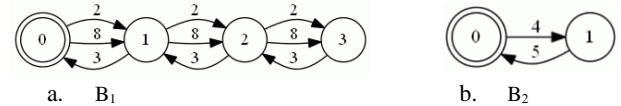
Fig. 6. DES models of M<sub>1</sub>, M<sub>2</sub> and TU

Fig. 7. Specifications of buffers

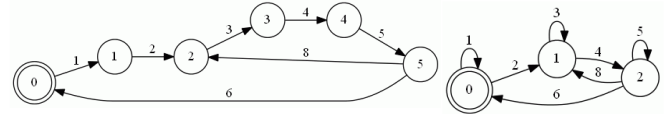


Fig. 8. The supremal relative observable supervisor for transfer line, (SUP)

Fig. 9. SUP<sub>0</sub> and RSUP<sub>0</sub>

All events involved in the DES model are  $\Sigma = \{1,2,3,4,5,6,8\}$ , where controllable events are odd-numbered. State transition diagrams of M<sub>1</sub>, M<sub>2</sub>, TU and specifications of buffers are displayed in Figs. 6, 7, respectively. The recognizer of relative observable supervisor, SUP and the partial observation reduced supervisor, corresponding to  $P_0: \Sigma^* \rightarrow \Sigma_0^*$ ,  $\Sigma_0 = \Sigma - \{1,3,5\}$ , are shown in Figs. 8, 9, respectively. We see that, events 1, 3, 5 appear just as self-loop transitions, each one at one state of the reduced supervisor, RSUP<sub>0</sub> (Fig. 9). Since the recognizer of partial observation supervisor, SUP<sub>0</sub> cannot be further reduced, RSUP<sub>0</sub> and SUP<sub>0</sub> are the same.

*Example 4: Supervisory control of guide way under partial observation*

Consider a guide way with two stations A and B, which are connected by a single one-way track from A to B on a guide way, as shown in Fig. 10. The track consists of 4 sections, with stoplights (\*) and detectors (!) installed at various section junctions [17]. Two vehicles V<sub>1</sub>, V<sub>2</sub> use the guide way simultaneously. V<sub>i</sub>, i = 1, 2 may be in state 0 (at A), state j (while travelling in section j = 1, ..., 4), or state 5 (at B). The

generator of  $V_i, i = 1, 2$  are shown in Fig. 11.

The plant to be controlled is  $G = \text{sync}(V_1, V_2)$ . To prevent collision, control of the stoplights must ensure that  $V_1$  and  $V_2$

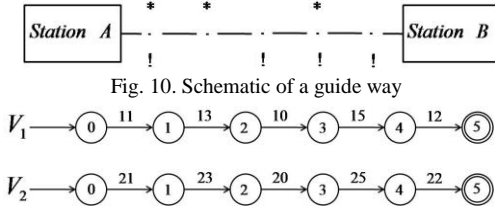


Fig. 10. Schematic of a guide way

Fig. 11. DES model of each vehicle

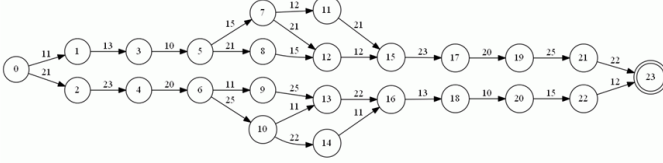


Fig. 12. The relative observable supervisor for the guide way  $\Sigma_{uo} = \{13, 23\}$ , SUP

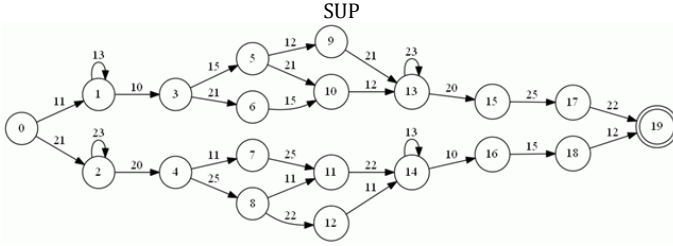


Fig. 13. The feasible supervisor for the guide way  $\Sigma_{uo} = \{13, 23\}$ , SUP0

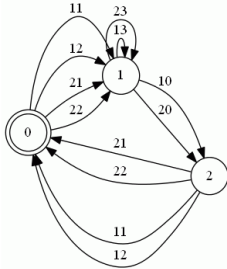


Fig. 14. The partial observation reduced supervisor for the guide way, RSUP0

never travel on the same section of track simultaneously. Namely,  $V_i, i = 1, 2$  are mutual exclusion of the state pairs  $(i, i)$ ,  $i = 1, \dots, 4$ . Controllable events are odd-numbered and the unobservable events 13, 23 are considered to synthesize the supremal relative observable supervisor, i.e.  $P_0: \Sigma^* \rightarrow \Sigma_0^*$ ,  $\Sigma_0 = \Sigma - \{13, 23\}$ . The supremal relative observable supervisor, SUP is shown in Fig. 12, and its corresponding partial observation supervisor SUP0 is shown in Fig. 13. The reduced supervisor, in which unobservable events 13, 23 are self-looped at state 1, is shown in Fig. 14.

Moreover, events 15, 25 are self-looped at all states of the reduced supervisor (hence, they are not shown). Thus, the supervisor is normal w.r.t.  $(L_m(G), P_N)$ , where  $P_N: \Sigma^* \rightarrow \Sigma_N^*$ ,  $\Sigma_N = \Sigma - \{15, 25\}$ . It can be checked that  $P_0(\text{RSUP}_0)$  and  $P_0(\text{SUP})$  are isomorph. Moreover, if the supervisor does not observe events 13, 23, they cannot be disabled at states 0, 2 in RSUP0. It means that, they appear as self-loop transitions at

states 0, 2. But the state size of the reduced supervisor does not change.

*Example 5: Supervisory control of AGV under partial observation*

A work cell consists of two machines  $M_1, M_2$  and an automated guided vehicle AGV as shown in Fig. 15. AGV can be loaded with a work piece either from  $M_1$  (event 10) or from  $M_2$  (event 22), which it transfers respectively to  $M_2$  (event 21) or to an output conveyor (event 30) [17]. Let  $\text{CELL} = \text{sync}(M_1, M_2, \text{AGV})$ . We can see CELL is blocking in state 9, i.e. the sequence of events reaches to a state from which no further transitions are possible (Fig. 16). To prevent blocking, we define  $\text{SPEC} = \text{trim}(\text{CELL})$ , as an appropriate specification (Fig. 17). The supremal relative observable supervisor, SUP is shown in Fig. 18, and its corresponding partial observation supervisor SUP0 is shown in Fig. 19. In Fig. 19, states 0, 3 and states 1, 2 are control consistent, respectively. Thus, the partial observation based reduced supervisor, RSUP0 is as shown in Fig. 20. Assume  $P_0: \Sigma^* \rightarrow \Sigma_0^*$ ,  $\Sigma_0 = \Sigma - \{11\}$ , we can easily check that Corollary 1 is satisfied for SUP and RSUP0.

## V. CONCLUSIONS

This paper addresses an extension to supervisor reduction procedure, proposed in [9], by considering partial observation; namely not all events are observable. We reduced a feasible partial observation supervisor instead of the original one. In the resulting reduced supervisor, only observable events can cause state changes. We finally clarified the extended theory by some examples.

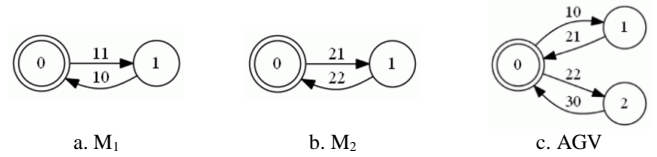


Fig. 15. DES model of each machine and AGV

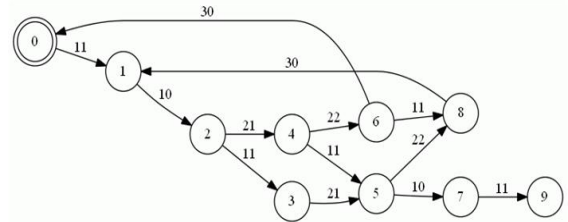


Fig. 16. DES model of CELL

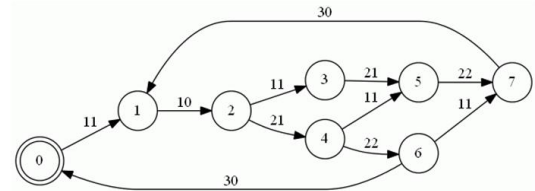


Fig. 17. DES model of specification, SPEC

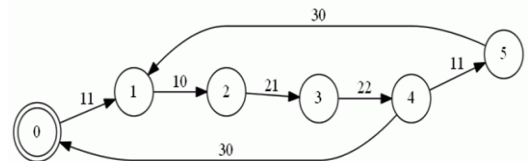
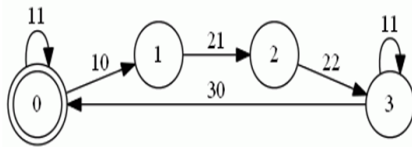
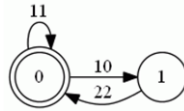


Fig. 18. The relative observable supervisor of AGV  $\Sigma_{uo} = \{11\}$ , SUP



Fig. 19. The feasible supervisor of AGV  $\Sigma_{uo} = \{11\}$ , SUPOFig. 20. The partial observation reduced supervisor for AGV, RSUP<sub>0</sub>

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## Appendix

In this appendix, a quick review of TCT commands is presented.  $\text{DES}=\text{sync}(\text{DES1}, \text{DES2}, \dots, \text{DESk})$  is the synchronous product of  $\text{DES1}, \text{DES2}, \dots, \text{DESk}$ .

$\text{DES3}=\text{supcon}(\text{DES1}, \text{DES2})$  for a controlled generator  $\text{DES1}$ , forms a trim recognizer for the supremal controllable sublanguage of the marked (“legal”) language generated by  $\text{DES2}$  to create  $\text{DES3}$ . This structure provides a proper supervisor for  $\text{DES1}$ .

$\text{DES3}=\text{supconrobs}(\text{DES1}, \text{DES2}, [\text{NULL}/\text{IMAGE}/\text{IMAGE\_DE S}])$  is a trim DES which represents the supremal controllable and relatively observable sublanguage of the legal language represented by  $\text{DES2}$ , with respect to the plant  $\text{DES1}$  and natural projection specified by the listed Null or Image events, or the latter’s allevents representation.

$\text{DAT3}=\text{condat}(\text{DES1}, \text{DES2})$  returns control data  $\text{DAT3}$  for the supervisor  $\text{DES2}$  of the controlled system  $\text{DES1}$ . If  $\text{DES2}$  represents a controllable language (with respect to  $\text{DES1}$ ), as when  $\text{DES2}$  has been previously computed with  $\text{supcon}$ , then  $\text{condat}$  will display the events that are disabled at each state of  $\text{DES2}$ . In general,  $\text{condat}$  can be used to test whether a given language  $\text{DES2}$  is controllable: just check that the disabled events tabled by  $\text{condat}$  are themselves controllable (have odd-numbered labels).

$\text{DES3}=\text{supreduce}(\text{DES1}, \text{DES2}, \text{DAT2})$  is a reduced supervisor for plant  $\text{DES1}$  which is control-equivalent to  $\text{DES2}$ , where  $\text{DES2}$  and control data  $\text{DAT2}$  were previously computed using  $\text{supcon}$  and  $\text{condat}$ . Also returned is an estimated lower bound  $\text{slb}$  for the state size of a strictly state-minimal reduced supervisor.  $\text{DES3}$  is strictly minimal if its reported state size happens to equal the  $\text{slb}$ .

$\text{DES2}=\text{project}(\text{DES1}, \text{NULL}/\text{IMAGE EVENTS})$  is a generator of the projected closed and marked languages of  $\text{DES1}$ , under the natural projection specified by the listed Null or Image events.

$\text{True/False}=\text{isomorph}(\text{DES1}, \text{DES2})$  tests whether  $\text{DES1}$  and  $\text{DES2}$  are identical up to renumbering of states; if so, their state correspondence is displayed.