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«Research Note»

Downlink Radio Resource Allocation in Cellular CDMA/TDMA Networks: A Novel Heuristic Algorithm

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Abstract- It is shown in [1] that the optimal downlink radio resource allocation for non-realtime traffic in cellular CDMA/TDMA networks can be mapped to a Multi-dimensional Multiple-choice Knapsack Problem (MMKP) which is NP-hard. In this correspondence we propose a heuristic algorithm with polynomial time complexity for this problem. Numerical results indicate significant computational performance improvement in comparison to existing heuristic algorithms for MMKP.

Keywords: Base-Station Assignment, Cellular Networks, Heuristic Algorithms, Packet Scheduling.

1- Introduction

The MMKP is a combination of the Multi Dimensional Knapsack Problem (MDKP) and Multiple-Choice Knapsack Problem (MCKP) which both are NP-hard [2]. It was shown in [3] that finding the exact solution of an MMKP problem is also NP-hard and is significantly more difficult than that of MDKP and MCKP [3]. In [1] we show that the optimal downlink radio resource allocation for non- realtime traffic in cellular CDMA/ TDMA networks is a Multi-dimensional Multiple-choice Knapsack Problem (MMKP).

An alternative to finding the exact solution is using heuristic algorithms which obtain an approximation of the exact solution with much lower computational complexity. To the best of our knowledge, [3] and [4] are the only works which propose polynomial time heuristic algorithms for MMKP.

In [3] a heuristic polynomial time algorithm is proposed based on Lagrange Multipliers method. In [4] two polynomial time heuristics have been presented in which an iterative procedure is utilized to modify a feasible solution until converging to the optimal solution.

In this correspondence we develop a new heuristic algorithm based on Lagrange Multiplier method in which, by exploiting the unique structure of the optimal downlink radio resource allocation problem, the computational complexity is significantly reduced compared to the ones in [3] and [4].

The numerical results indicate that in practical conditions the proposed method reaches the solution with higher probability and lower computational complexity.

2- System Model

The detailed system model is presented in [1] here we briefly restate it for easy reference. The system is time-slotted with each slot containing M frames of T_f seconds. The total transmit power of a BS is allocated to a single user within each frame while the rest of the users are kept inactive. The length of the time-slot is chosen so that the channel variations are negligible within the slot. Fixed L-bit packets are transmitted in an integer number of frames. At time t, there are $N(t)$ packets waiting in the system to be served. The set of B base-stations (BSs) in the network is indicated by BS = $\{BS_1,\ldots, BS_B\}$.

An active set $A_i(t)$, corresponding to each packet i, is defined as the set of BSs which can be assigned to the destination user of packet i, $d(i)$, as the server:

$$A_i(t) = \{j | j \in \text{BS}, \gamma_{d(i),j}(t) \geq \gamma_{min}\}. \quad (1)$$

In (1), $\gamma_{d(i),j}(t)$ is the bit-energy-to-interference-plus-noise spectral density, E_c/I_0, level of the pilot channel of BS_j received by $d(i)$, and γ_{min} is the minimum required E_c/I_0. $NA_i \leq N_A$ indicates the number of BSs in $A_i(t)$, where N_A is a system parameter.

Let $m_{ij}(t)$ be the number of required frames for transmission of packet i by BS_j at time t, $m_{ij}(t) = \lceil L/r_{ij}(t)T_f \rceil$, where $\lceil . \rceil$ gives the upper nearest integer since an integer number of frames should be allocated to each packet, $r_{ij}(t)$ is the bit-rate of the channel between BS_j and $d(i)$ at time t.

For a packet i, we associate a utility-function, $u_{ij}(t)$, that indicates the "profit" earned by the network as a result of transmitting packet i from BS_j at time t. Utility function, $u_{ij}(t)$, serves as an optimization objective for packet transmission and is a function of the allocated network resources to that packet as well as the quality of service experienced by that packet. The earned profit modeled by the utility function provides a priority metric for each packet served by a given BS; the higher the value of a utility function, the higher the priority of transmitting the corresponding packet. For a survey on the utility functions see [5].

We define the total network utility at time t as the summation of the utilities of the packets served at time t. The objective is to maximize the total network utility:

***Problem** O*:

$$\max_{\mathcal{B}_b(t)} \sum_{i=1}^{N(t)} \sum_{j=0}^{B} u_{ij}(t) b_{ij}(t) \quad (2)$$

$$\text{s.t.} \sum_{i=1}^{N(t)} m_{ij}(t) b_{ij}(t) \leq M, \quad j = 1, \ldots, B, \quad (3)$$

$$\sum_{j=0}^{B} b_{ij}(t) = 1, \quad i = 1, \ldots, N(t), \quad (4)$$

$$b_{ij}(t) \in \{0, 1\}. \quad (5)$$

In Problem O, $b_{ij}(t)$ is the assignment indicator at time t; $b_{ij}(t)= 1$ if packet i is transmitted by BS_j, and $b_{ij}(t)= 0$, otherwise; we also define

$\mathrm{B}_{b(t)}:=\{b_{ij}(t) | i = 1, \ldots, N(t), j=0,\ldots, B\}$.

A null BS, BS_0, with M virtual frames is introduced in the formulation for which $u_{i0}(t) = 0$ for all packets. If $b_{i0}(t) =1$, packet i is not scheduled for transmission at time t, therefore, $\tau_i(t+1)=\tau(t)+M.T_f$, where $\tau_i(t)$ is the packet experienced delay until time t. The inequality in (3) shows the downlink resource constraint for the BSs in BS. Furthermore, (4) indicates that each packet is transmitted by only one BS.

It is shown in [1] that by adding a NULL variable corresponding to a packet which is not scheduled to be transmitted, Problem *O* can be mapped to an MMKP which, for each time slot *t*, results in a joint packet scheduling and BS assignment: it gives the packets which are scheduled for transmission, and for each packet, it also gives the corresponding BS that transmits the packet to the destination user. Since Problem *O* is solved for each time-slot, the time index *t* is dropped for brevity.

Since MMKP is NP-hard [2], the computational complexity of finding the exact solution is not reasonable in practice. Here, we first define MMKP and subsequently we propose a novel polynomial time heuristic algorithm for Problem *O*.

MMKP [2]: Consider a knapsack with *B* distinct resources represented by $(M_1, M_2, \ldots, M_B)$. There are *N* groups, each with K_i items. Each item *j* of a group *i* has a particular value, u_{ij}, and requires *B* distinct resources represented by the vector $(m_{ij1}, m_{ij2}, \ldots, m_{ijB})$. The MMKP objective is to select one item from each group to maximize the total value of the collected items subject to *B* resource constraints.

3-The Heuristic Algorithm

We use the Lagrange Multiplier approach as a basis for the approximation of the optimization solution. Lagrange Multipliers method is based on finding the solution of the unconstrained optimization problem

$$\max\Big\{\sum_{i=1}^{N}\sum_{j=1}^{N_{\mathcal{A}}+1}\big(u_{ij}-\lambda_j m_{ij}\big)b_{ij}\Big\}, \qquad (6)$$

which is shown in [6] to be the solution of the constraint optimization Problem *O*, where λ_j is Lagrange multiplier and $b_{ij}^{*}=1$ if $u_{ij}-\lambda_j m_{ij}>0$, and 0 otherwise. This might result in more than one solution; among them the one which satisfies (4) is the optimal solution. As a matter of fact, if λ_j, $j=1, \ldots, B$, are known, the solution of Problem *O* can be obtained easily. If these multipliers are computed so that $M-\sum_{i=1}^{N} m_{ij}b_{ij}^{*} \geq 0$, then the solution satisfies (3), thus is feasible; the solution becomes optimal if

$$\sum_{j=0}^{B}\lambda_j\big(M-\sum_{i=1}^{N}m_{ij}b_{ij}^{*}\big)=0. \qquad (7)$$

The proposed algorithm has two main procedures which are executed consequently: the *Main Body* and the *Solution Improvement*.

Main Body
s1.1. Resetting the Lagrange multipliers:
    $\lambda_j \leftarrow 0$, for all j
s1.2. Normalized required resources and NULL assignment:
    $m_{ij} \leftarrow m_{ij}/M$, $b_{ij} \leftarrow 0$ for all i, j
s1.3. for each packet the BS, with the maximum achieved utility is assigned:
    $a(i) \leftarrow \arg\max_j(u_{ij})$, $b_{ia(i)} \leftarrow 1$, for all i
s1.4. Compute the resource constraint violations:
    $\pi_j \leftarrow \sum_{i=1}^{N} m_{ij}b_{ij}$ for all j

while $\pi_j > 1$, for any j
    s1.5. Determine the most offending constraint violation: $j^* \leftarrow \arg\max_j\{\pi_j\}$
    s1.6. Find the packet to be changed its assignment:
        for $\{i|a(i)=j^*\}$
            for $j=0:N_{\mathcal{A}_i}$
            $\Delta_{ij} \leftarrow \frac{\big(u_{ij^*}-u_{ij}-\lambda_{j^*}(m_{ij^*}-m_{ij})\big)}{(m_{ij^*}-m_{ij})}$
            end
        end
    $I^*J^* \leftarrow \arg\min_{ij}\{\Delta_{ij}\}$, for all j, $i|a(i)=j^*$
    s1.7. Re-evaluate the assigned: BSs:
        $\lambda_{j^*} \leftarrow \lambda_{j^*} + \Delta_{I^*J^*}$
        $b_{I^*a(I^*)} \leftarrow 0$, $a(I^*) \leftarrow J^*$, $b_{I^*J^*} \leftarrow 1$
        $\pi_{j^*} \leftarrow \pi_{j^*} - m_{I^*j^*}$
        $\pi_{J^*} \leftarrow \pi_{J^*} + m_{I^*J^*}$
end

Figure 1 The Main Body Algorithm

1) ***Main Body*** (see Fig. 1): The algorithm starts by setting the Lagrange multipliers to zero in (s1.1) and normalizing the required time-slots in (s1.2). Then in (s1.3) each packet i is assigned to a BS $a(i)$ (i.e., $b_{ia(i)} \leftarrow 1$ in A_i, where $a(i)$= arg max u_{ij}. Note that so far only one BS is assigned to each user, therefore, the constraints in (4) and (7) are satisfied. However, some of the constraints in (3) might be violated; if so, the initial BS assignments are adjusted in the *while loop* until (3) is held. To adjust the initial BS assignments, the most offending BS constraint violation j^*, determined in (s1.5), is iteratively improved in the rest of the Main Body: in (s1.6) we consider the packets whose assigned BSs are j^*.

For each BS j in the active-set of these packets, we then compute the increase of the Lagrange multiplier λ_{j*}, denoted by Δa_{ij}, resulting from exchanging the previously assigned BSj^* by another BS in A_i. Among those users whose j^* is in their active set, we choose the user I^* and the corresponding BS J^* in its active set, so that the corresponding exchange causes the least increase of multiplier λ_{j*}, i.e., minΔ_{ij}. In (s1.7) the exchange is done and the corresponding parameters are updated accordingly. This new assignment minimizes the gap between the optimal solution characterized by (7) and the previous assignment. The *while loop* in the Main Body is repeated for each packet until a BS has been assigned to that packet and (3) is satisfied (i.e., $\pi_j \leq 1$).

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Solution Improvement
while more assignments can be exchanged
    s2.1.  Compute total utility increases:
    for i = 1 : N
        for j = 0 : N_{A_i}
            Evaluate μ_ij:
            if u_ij − u_ia(i) > 0 && π_j − m_ia(i) + m_ij ≤ 1
                then μ_ij ← u_ij − u_ia(i)
            else μ_ij ← 0
        end
    end
    s2.2.  Find the best exchangeable
    assignment:
        I*J* ← arg max_ij {μ_ij}, for all i, j
    s2.3.  Exchange the selected
    assignment:
        π_a(I*) ← π_a(I*) − m_I*J*
        π_J* ← π_J* + m_I*J*
        b_I*a(I*) ← 0, b_I*J* ← 1
end
```

Figure 2 The Solution Improvement Algorithm

After finding a feasible solution through the Main Body, there may still some available unused resources in the BSs. The remaining resources are utilized to adjust the feasible solution in the *Solution Improvement* algorithm.

2) *Solution Improvement* (see Fig. 2): In this algorithm, each BS j in the active-set of every packet i is checked against the currently assigned BS of that packet, i.e., $a(i)$. In (s2.1) we obtain the amount of increase in the achieved utility, denoted by μ_{ij}, caused by the new assignment of BS j, while making sure that the resource constraints are not violated. This condition is verified using the *if-else* statement in (s2.1). Then, in (s2.2) user I^* is selected among all users, so that replacing its previously assigned BS by a new BS J^* causes the largest increase in the total achieved utility, i.e., max μ_{ij}. In (s2.3) the exchange is done and the corresponding parameters are updated accordingly. The *while loop* is repeated until no more exchanges are possible. The output of the Solution Improvement algorithm is the solutions of Problem O.

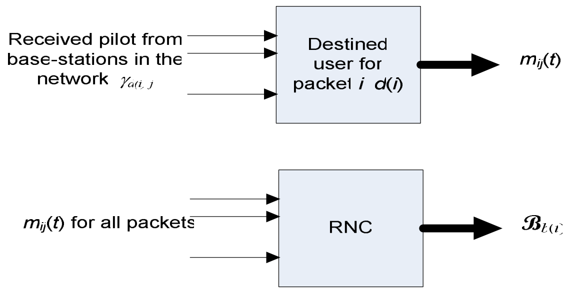

Figure 3 Implementation Block Diagram

4-Computational Complexity

The schematic of the system implementation is depicted in Fig. 3. The proposed optimization is performed in a Radio Network Controller (RNC) which controls a number of BSs in the network. The inputs of the proposed algorithm are $u_{ij}(t)$ and $m_{ij}(t)$, and the output is the assignment matrix, $B_{b(t)}$. For each packet i, the corresponding BSs with acceptable pilot signal strength in the destined user are then considered as the active-set of packet i, A_i (see (1)). The corresponding channel bit-rates $r_{ij}(t)$ are measured by the users and sent to the RNC via BSs. Packet experienced delay, $\tau_i(t)$, is also available in the RNC. Therefore, $u_{ij}(t)$ and $m_{ij}(t)$ are obtainable. The following proposition indicates that the proposed algorithm has a polynomial-time computational complexity.

Proposition 1: The proposed heuristic algorithm has a maximum computational complexity of $O(N^2 N_A^3)$.

Proof: In the Main Body, (s1.1) has the complexity order of $O(B)$, and (s1.2)-(s1.4) have the complexity order of $O(NN_A)$. In the *while loop*, (s1.5) and (s1.7) have the complexity order of $O(B)$ and $O(1)$, respectively. In (s1.6) for each of the N users, there are at most N_A non-selected BSs in the corresponding active-set, thus for each user the maximum complexity order is $O(N_A)$. There is one iteration for each BS in the active-set of each user resulting in a total complexity order of $O(NN_A^2)$ for (s1.6). In every iteration of (s1.6), one assigned BS is removed from one user, thus, in the worst case the *while loop* in the *Main Body* is executed NN_A times. Noting that in a cellular network usually $B \ll NNA$, the overall complexity order for the execution of the while loop of the Main Body is $O(N^2NA^3)$.

Table 1
COMPUTATIONAL COMPLEXITY

Heuristic Algorithm Complexity	**Complexity**
Branch and Bound [2]	$O(2^{NNA})$
Heuristic in [3]	$O(BN^2N^2A)$
Heuristic in [4]	$O(BN^2N^2{}_A)$
Proposed Heuristic	$O(N^2N^3{}_A)$

Table 2
VALUE OF δ FOR B= 10, N_A = 3 AND DIFFERENT NUMBER OF USERS.

NUMBER OF USERS	**[3]**	**[4]**	**PROPOSED HEURISTIC**
5	0.0023	0.0027	0.0015
10	0.0026	0.0029	0.0017
15	0.0031	0.0030	0.0020
20	0.0037	0.0033	0.0024

In the *Solution Improvement*, the complexity order of (s2.2) and (s2.3) are $O(NN_A)$ and $O(1)$, respectively. In (s2.1) for each of the N users, at most N_A non-selected BSs are in the corresponding active-set which the total utility increase should be computed for them. Each computation has a complexity of $O(N_A)$. There is one iteration for each BS in the active-set of each user, resulting in a complexity order of $O(NN_A^2)$ for (s2.1). Since for each user there can be, at most, NA BSs in its active-set which could have higher utility than the assigned BS, the outer *while loop* of the solution improvement algorithm is at most executed NN_A times. This gives an overall complexity of $O(N^2 N_A^3)$ for the Solution Improvement. Thus, the overall computational complexity is $O(N^2 N_A^3)$.

5- Numerical Results

For comparison we implement the algorithms in [3] and [4] as well as our proposed algorithm. As the benchmark for comparison, we also implement Branch and Bound (BB) algorithm which gives the exact solutions [2]. In Table 1 the computational complexity of the algorithms in [3], [4], and BB, as well as the proposed heuristic in this paper, are presented. As it can be seen, the computational complexity of the proposed algorithm in this paper is lower than those of the other heuristics in [3], [4]. Noting the fact that in cellular networks number of BSs, B, is usually larger than that of number of BSs in the active set, N_A; in RNC which controls B BSs, the order of complexity for our proposed algorithm is much less than those of in [3] and [4].

We consider a network of 10 BSs controlled by an RNC; N_A is assumed to be 3. The air interface is based on UMTS with the same parameters as in [1]. We obtain the total achieved utility U_O which is defined in (2) based on the exact solution obtained by the BB algorithm. Then for the algorithms in Table 1 and for 10,000 independently generated snap-shots of simulations, we obtain $\Delta = \Sigma| U_O - \underline{U}_O |$ for each algorithm, where $\underline{U}_O$ is the achieved utility using a heuristic algorithm. The value of δ indicates the accuracy of the algorithm in obtaining the exact optimal solution. Table 2 presents values of δ for different number of users. As it can be seen the proposed algorithm in this paper performs more efficiently than the other algorithms; note that better accuracy is achieved along with lower complexity.

6- Conclusions

We propose a heuristic algorithm with polynomial time complexity for optimal downlink radio resource allocation. Numerical results indicate significant computational performance improvement.

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8- References

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