Adaptive Observer Design for One-Sided Lipschitz Class of Nonlinear Systems

Mohammad Karkhaneh[1], Mahdi Pourgholi[2*]

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Abstract:
This paper addresses adaptive observer design problem for joint estimation of the states and unknown parameters for a class of nonlinear systems which satisfying one-sided Lipschitz and quadratic inner bounded conditions. It's shown that the stability of the proposed observer is related to finding solutions to a quadratic inequality consists of state and parameter errors. A coordinate transformation is used to reformulate this inequality as a linear matrix inequality (LMI). Sufficient conditions that ensure the existence of adaptive observer are expressed in forms of LMIs, which are easily tractable via standard software algorithms. If the proposed conditions are satisfied, then the state estimation errors are guaranteed to converge to zero asymptotically while, the convergence of the parameters is guaranteed when a persistence of excitation condition is held. The effectiveness of the proposed method is shown by simulation for the joint estimation of states and parameters of a numerical system.

Keywords: Adaptive observer; Nonlinear systems, Nonlinear observers; One-sided Lipschitz; Linear matrix inequality.

1. Introduction

The adaptive observer design problem for nonlinear systems has been an interesting subject for the last three decades. The adaptive observers can estimate the state variables despite of the unknown parameters in the system. In fact, adaptive observers perform the twin tasks of state estimation and parameter identification [1]; An approach to achieve this goal is assuming known parameters and taking some possible observer design for the system and try to find some appropriate adaptation law for the unknown parameters so as to keep the observer convergence in presence of those unknown parameters; this makes the observer a so-called "adaptive observer" [2].

The design of adaptive observer for an LTI system is well analyzed in [3]. In case of nonlinear systems, early results can be found in [4]-[7]; in which the nonlinear systems can be linearized with some change of coordinates and output injections. In [8] Based on existing results on adaptive observer designs, the author proposes a unifying adaptive observer form. Adaptive observers for a class of nonlinear systems satisfying a Lipschitz condition were first developed in [9] and [1], in these works there's no need the considered nonlinear system to be linearizable. Inspired by [1], several authors extended adaptive observer design for Lipschitz systems [10], [11], [12]. In non-adaptive cases where there is no unknown parameter in the system the observer design problem first introduced in [13] and then extended in several works like: [14]-[19] .However, The main shortcoming of all these works is that they can only stabilize the error dynamics for dynamical systems with small Lipschitz constant but, fail to provide a solution when the Lipschitz constant becomes large [20]. To overcome this shortcoming in observer design, in [21] the author used a new tool which it was first introduced in mathematical literature [22], this tool is known as "one-sided Lipschitz" condition, and has widely used in observer design problems [23]-[28].

The one-sided Lipschitz continuity covers a broad family of nonlinear systems which includes the well-known Lipschitz systems as a special case. Moreover, as previously mentioned, observer design techniques based on Lipschitz functions can guarantee stability only for small values of Lipschitz constant which directly translates into small stability regions. However, one-sided Lipschitz constant can be found to be significantly smaller than the classical Lipschitz constant. This makes the one-sided Lipschitz constant much more appropriate for estimating the influence of non-linear terms [22].

Althogh using one-sided Lipschitz condition in observer synthesis is reported in some

1. M.S. Student, E. Eng department, Shahid Beheshti University ,Tehran, Iran .m.karkhaneh@mail.sbu.ac.ir
2. Faculty of Electrical Engineering, Shahid Beheshti University, A.C., Tehran, Iran .m_pourgholi@sbu.ac.ir

researches, but this condition has not been used in adaptive cases, which have some unknown parameters in the system. Therefore, in this paper our objective is to find a systematic approach in adaptive observer synthesis for nonlinear systems where nonlinearities satisfy a one-sided Lipschitz condition. Since our approach is LMI based, less conservative and most efficient results are developed, and these LMIs can be easily solved through standard numerical software packages like YALMIP [29].

The rest of the paper is organized as follows. Section 2 is devoted to preliminaries and basic concepts. In section 3, we present an adaptive nonlinear state observer for one-sided Lipchitz systems in the presence of unknown parameters. To verify the efficiency of the proposed approach, the simulation results are presented in the section 4.

2. Preliminaries and basic concepts

In this section, for better understanding the subject we gathered the mathematical Preliminaries.

Definition 1: The non-linear function $\varphi(x,u)$ is said to be locally Lipschitz in a region D including the origin with respect to x, uniformly in u, if there exist a constant $\gamma > 0$ satisfying:

$$\|\varphi(x,u) - \varphi(\hat{x},u)\| \leq \gamma\|x - \hat{x}\| \quad \forall x, \hat{x} \in D \tag{1}$$

where u is any admissible control signal. The smallest constant $\gamma > 0$ satisfying (1) is known as Lipschitz constant. The region D is the operational region or the region of interest. If condition (1)is valid everywhere in R^n, then the function is said to be globally Lipschitz.

Definition 2: The non-linear function $\varphi(x,u)$ is said to be one-sided Lipschitz if there exist $\gamma_1 \in R$ such that $\forall x, \hat{x} \in D$:

$$< \varphi(\mathrm{x,u}) - \varphi(\hat{\mathrm{x}},\mathrm{u}), \mathrm{x} - \hat{\mathrm{x}} > \leq \gamma_1\|\mathrm{x} - \hat{\mathrm{x}}\|^2 \tag{2}$$

where $\gamma_1 \in R$ is called the one-sided Lipschitz constant.

Similarly to Lipschitz property, the one-sided Lipschitz property might be local or global. Note that while the Lipschitz constant must be positive, the one-sided Lipschitz constant can be positive, zero or even negative.

For any Lipschitz function $\varphi(x,u)$ we have:

$$|< \varphi(x,u) - \varphi(\hat{x},u), x - \hat{x} >| \leq \|\varphi(x,u) - \varphi(\hat{x},u)\|\|x - \hat{x}\| \leq \gamma\|\mathrm{x} - \hat{\mathrm{x}}\|^2 \tag{3}$$

Therefore, any Lipschitz function is also one-sided but converse is not true [20]. For continuously differentiable nonlinear functions it is well-known that the smallest possible constant satisfying (1) is the supremum of the norm of Jacobian of the function over the region D, that is:

$$\gamma = \sup\left(\left\|\frac{\partial\varphi}{\partial \mathrm{x}}\right\|\right), \quad \forall \mathrm{x} \in \mathrm{D} \tag{4}$$

Alternatively, the one-sided Lipschitz constant is associated with the logarithmic matrix norm (matrix measure) of the Jacobian. The logarithmic matrix norm of a matrix A is defined as [22]:

$$\mu(\mathrm{A}) = \lim_{\epsilon\to 0}\frac{\|\mathrm{I}+\epsilon\mathrm{A}\|-1}{\epsilon} \tag{5}$$

In (5) the symbol $\|.\|$ represents any matrix norm. Then, we have:

$$\gamma_1 = \sup\left[\mu\left(\frac{\partial\varphi}{\partial \mathrm{x}}\right), \quad \forall \mathrm{x} \in \mathrm{D}\right] \tag{6}$$

If the norm used in (6) is indeed the induced 2-norm (the spectral norm) then it can be shown that $\mu(A) = \lambda_{max}(\frac{A+A^T}{2})$ [30]. On the other hand, from the Fan's theorem we know that for any matrix, $\lambda_{max}\left(\frac{A+A^T}{2}\right) \leq \sigma_{max}(A) = \|A\|$ [31]. Therefore $\gamma_1 \leq \gamma$; i.e. one-sided Lipschitz constant can be found to be much smaller than the Lipschitz constant [22].

Definition 3: The non-linear function $\varphi(x,u)$ is called quadratic inner-boundedness in the region $\widetilde{D}$ if $\forall x, \hat{x} \in \widetilde{D}$ there exist $\beta, \rho \in R$ such that [20]:

$$\left(\varphi(x,u) - \varphi(\hat{x},u)\right)^T\left(\varphi(x,u) - \varphi(\hat{x},u)\right) \leq \beta\|x - \hat{x}\|^2 + \rho < x - \hat{x}, \varphi(x,u) - \varphi(\hat{x},u) > \tag{7}$$

By the definition, the Lipschitz function is quadratically inner-bounded with $\rho = 0 \,\&\, \beta > 0$. So the Lipschitz continuity implies quadratic inner-boundedness; However the converse is not true [20]. It should be noted that ρ in (7) can be positive, zero or even negative.

Lemma 1 [32]**:** Every continuous function on a compact set is uniformly continuous (Heine–Cantor theorem).In particular, if a function is continuous on a closed bounded interval of the real line, it is uniformly continuous on that interval.

Lemma 2 [32]**:** Every uniformly continuous function on the bounded set *E* is bounded on *E*.

3. Adaptive observer synthesis

Theorem: consider the following class of continuous time nonlinear dynamical system:

$$\dot{x} = Ax + \varphi(x,u) + bf(x,u)\theta \quad (8)$$

$$y = Cx$$

where: $x \in R^n$, $y \in R^m$, $\theta \in R^p$, $b \in R^{n*s}$, $C \in R^{m*n}$, and $\varphi(x,u): R^n \to R^n$ is one-sided Lipschitz in D with one-sided Lipschitz constant γ_1 and quadratically inner-bounded in $\widetilde{D}$ as defined in (7), and $f(x,u): R^n \to R^{s*p}$ is Lipschitz in D with Lipschitz constant γ_2. The vector of unknown constant parameters θ is bounded as shown below:

$$\|\theta\|_2 \le \gamma_3 \quad (9)$$

If there exist matrices $P = P^T$, H and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that LMI (10) is feasible

$$\begin{bmatrix} \Psi & \nu & (\gamma_2\gamma_3)^{\frac{1}{2}}\|b\|P \\ \nu & -\varepsilon_2 I & 0 \\ (\gamma_2\gamma_3)^{\frac{1}{2}}\|b\|P & 0 & -I \end{bmatrix} < 0 \quad (10)$$

where $\Psi = A^TP + PA - C^TH - H^TC + \gamma_2\gamma_3 I + \varepsilon_1\gamma_1 I + \varepsilon_2\beta I$ and $\nu = P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \rho I}{2}$, and $b^TP = \bar{C}$ which each rows of $\bar{C}$ is a linear combination of rows of C, then the adaptive observer:

$$\dot{\hat{x}} = A\hat{x} + \varphi(\hat{x},u) + bf(\hat{x},u)\hat{\theta} + L(y - C\hat{x}) \quad (11)$$

with adaptive law:

$$\dot{\hat{\theta}} = \frac{f(\hat{x},u)^T\bar{C}\tilde{x}}{\omega}, \omega > 0 \quad (12)$$

is stable and observer gain L in (11), can be computed from $L = P^{-1}H^T$.

Remark 1: The existence of a positive definite matrix P satisfies the condition $b^TP = \bar{C}$ is guaranteed when at least some of the measured outputs are such that the transfer function between these outputs and unknown parameters are dissipative or strictly positive real [1]

Proof of theorem: Let $\tilde{x} = x - \hat{x}$ be the estimation error. Then:

$$\dot{\tilde{x}} = (A - LC)\tilde{x} + \varphi(x,u) - \varphi(\hat{x},u) + bf(x,u)\theta - bf(\hat{x},u)\hat{\theta} \quad (13)$$

Let $V = \tilde{x}^TP\tilde{x} + \omega\tilde{\theta}^T\tilde{\theta}$ be the Lyapunov candidate. Then:

$$\begin{aligned}\dot{V} &= \tilde{x}^T[(A-LC)^TP + P(A-LC)]\tilde{x} + 2\tilde{x}^TP[\varphi(x,u) - \varphi(\hat{x},u)] + 2\big[bf(x,u)\theta - bf(\hat{x},u)\hat{\theta}\big]^T P\tilde{x} + 2\omega\tilde{\theta}^T\dot{\tilde{\theta}} \\ &= \tilde{x}^T[(A-LC)^TP + P(A-LC)]\tilde{x} + 2\tilde{x}^TP[\varphi(x,u) - \varphi(\hat{x},u)] + 2[bf(x,u)\theta - bf(\hat{x},u)\theta]^TP\tilde{x} + 2\big[bf(\hat{x},u)\tilde{\theta}\big]^TP\tilde{x} + 2\omega\tilde{\theta}^T\dot{\tilde{\theta}}\end{aligned} \quad (14)$$

Considering Lipschitz condition on f and using (9) we can write (14) as follows: (15)

$$\begin{aligned}\dot{V} &\le \tilde{x}^T[(A-LC)^TP + P(A-LC)]\tilde{x} + 2\tilde{x}^TP[\varphi(x,u) - \varphi(\hat{x},u)] + 2\gamma_2\gamma_3\|b\|\|\tilde{x}\|\|P\tilde{x}\| + 2\big[bf(\hat{x},u)\tilde{\theta}\big]^TP\tilde{x} + 2\omega\tilde{\theta}^T\dot{\tilde{\theta}} \le \\ &\tilde{x}^T[(A-LC)^TP + P(A-LC) + \gamma_2\gamma_3\|b\|^2PP + \gamma_2\gamma_3 I]\tilde{x} + 2\tilde{x}^TP[\varphi(x,u) - \varphi(\hat{x},u)] + 2\big[bf(\hat{x},u)\tilde{\theta}\big]^TP\tilde{x} + 2\omega\tilde{\theta}^T\dot{\tilde{\theta}}\end{aligned}$$

where in the last inequality, $2\gamma_2\gamma_3\|b\|\|\tilde{x}\|\|P\tilde{x}\| \le \tilde{x}^T(\gamma_2\gamma_3\|b\|^2PP + \gamma_2\gamma_3 I)\tilde{x}$ was used.

By setting:

$$2\big[bf(\hat{x},u)\tilde{\theta}\big]^TP\tilde{x} + 2\omega\tilde{\theta}^T\dot{\tilde{\theta}} = 0 \quad (16)$$

We obtain the adaptive law:

$$\dot{\hat{\theta}} = -\dot{\tilde{\theta}} = \frac{f(\hat{x},u)^T b^T P\tilde{x}}{\omega} = \frac{f(\hat{x},u)^T \bar{C}\tilde{x}}{\omega} \quad (17)$$

(15) as inequality (18).

Then we can write

$$\dot{V} \le \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix}^T \begin{bmatrix}(A-LC)^T P + P(A-LC) + \gamma_2\gamma_3\|b\|^2 PP + \gamma_2\gamma_3 I & P\\ P & 0\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \quad (18)$$

From inequality (2), we obtain
$\gamma_1\tilde{x}^T\tilde{x} - \tilde{x}^T\tilde{\varphi} \ge 0$.

Therefore for any $\varepsilon_1 > 0$ we have:

$$\varepsilon_1 \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix}^T \begin{bmatrix}\gamma_1 I & -I/2\\ -I/2 & 0\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \ge 0 \quad (19)$$

Likewise from (7) for any $\varepsilon_2 > 0$:

$$\varepsilon_2 \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix}^T \begin{bmatrix}\beta I & \rho I/2\\ \rho I/2 & -I\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \ge 0 \quad (20)$$

By adding
(19) and (20) to (18) yields:

$$\dot{V} \le \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix}^T \begin{bmatrix}\chi & P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2\rho I}{2}\\ P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2\rho I}{2} & -\varepsilon_2 I\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \quad (21)$$

where
$\chi = (A-LC)^T P + P(A-LC) + \gamma_2\gamma_3\|b\|^2 PP + \gamma_2\gamma_3 I + \varepsilon_1\gamma_1 I + \varepsilon_2\beta I$.

By letting $H = L^T P$ in (21)and applying Schur's complement [33], for all $\tilde{x} \neq 0$ we conclude that $\dot{V}(t) \le 0$ provided that LMI (22)is feasible.

$$\begin{bmatrix}\Psi & \nu & (\gamma_2\gamma_3)^{\frac{1}{2}}\|b\|P\\ \nu & -\varepsilon_2 I & 0\\ (\gamma_2\gamma_3)^{\frac{1}{2}}\|b\|P & 0 & -I\end{bmatrix} < 0 \quad (22)$$

where
$\Psi = A^T P + PA - C^T H - H^T C + \gamma_2\gamma_3 I + \varepsilon_1\gamma_1 I + \varepsilon_2\beta I$ and $\nu = P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2\rho I}{2}$.

Remark 2: since φ is continuous and $x \in L_\infty$, therefore, from lemma 1 we can conclude that φ is uniformly continuous, and from lemma 2 we conclude that $\varphi \in L_\infty$.

Proof of Convergence of $\tilde{x}$: assume there exists a $\xi > 0$ such that (23) is satisfied.

$$\begin{bmatrix}\chi & P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2\rho I}{2}\\ P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2\rho I}{2} & -\varepsilon_2 I\end{bmatrix} \le -\xi I \quad (23)$$

Then:

$$\dot{V} \le -\xi \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix}^T \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \quad (24)$$

By integrating:

$$\xi \int_0^t \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix}^T \begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \le V(0) - V(t) \quad (25)$$

Since $V(t) \in L_\infty$ and $V(0)$ is finite, therefore $\begin{bmatrix}\tilde{x}\\ \tilde{\varphi}\end{bmatrix} \in L_2$ and hence $\tilde{x} \in L_2$. Besides, from remark 2 and knowing that $f(x,u)$ is Lipschitz, from (13)we can see that $\dot{\tilde{x}} \in L_\infty$. Therefore, by Barbalat's lemma [34], $\tilde{x} \to 0$.

Proof of Convergence of $\tilde{\theta}$:

$$\int_0^\infty \dot{\tilde{x}}\, dt = \lim_{t\to\infty} \tilde{x}(t) - \tilde{x}(0) = -\tilde{x}(0) \quad (26)$$

Since $\tilde{x}(0)$ is finite then (26) is finite. Besides, f is Lipschitz continuous and from remark 2 we know that φ is uniformly continuous, so we can see that $\dot{\tilde{x}}$ in (13) is uniformly continuous. Hence, by Barbalat's lemma, $\dot{\tilde{x}} \to 0$. This implies that $bf(x,u)\theta - bf(\hat{x},u)\hat{\theta} \to 0$.

Remark 3: In terms of parameter convergence, we could only show that $bf(x,u)\theta - bf(\hat{x},u)\hat{\theta} \to 0$. Since $\hat{x} \to x$ we can write that
$bf(x,u)(\theta - \hat{\theta}) \to 0$. This leads to the following persistency of excitation condition:
If $\exists \alpha_0,\ \alpha_1, \delta > 0$ such that:

$$\alpha_0 I \ge \int_{t_0}^{t_0+\delta} bf(x(\tau),u(\tau))f(x(\tau),u(\tau))^T b^T\, d\tau \quad (27)$$

$\alpha_1 I \quad for\ all\ t_0$

Then $\theta - \hat{\theta} \to 0$.

4. Illustrative Example

In this section, we illustrate the effectiveness of the proposed adaptive observer design method through an example.

Example: suppose that the equations of motion of a moving object are given in as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -x_1({x_1}^2 + {x_2}^2) \\ -x_2({x_1}^2 + {x_2}^2) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \theta \tag{28}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

In [20], it's proved that the system is globally one-sided Lipschitz with one-sided Lipschitz constant $\gamma_1 = 0$, while about Lipschitz continuity, we can only say that the system (28) is locally Lipschitz, and on any set $D = \{x \in R^2 : \|x\| \le r\}$ the Lipschitz constant is $\gamma = 3r^2$, i.e. the Lipschitz constant rapidly increases with the increase of r.

The system (28) is quadratically inner-bounded in $\widetilde{D} = \{x \in R^2 : \|x\| \le r\}$ with [20]:

$$r = \min\left(\sqrt{-\frac{\rho}{4}}, \sqrt[4]{\beta + \frac{\rho^2}{4}}\right)$$

Then we have $\rho < 0$, $\beta + \frac{\rho^2}{4} > 0$. Hence, by manipulating ρ and β, the region $\widetilde{D}$ can be made arbitrarily large. Note that as the system is globally one-sided Lipschitz ($D = R^2$), our solution is valid in $D \cap \widetilde{D} = \widetilde{D}$. Now by letting $\rho = -100, \beta = -98.5$ from LMI (22), we obtain:

$$P = \begin{bmatrix} 15.5694 & 0 \\ 0 & 15.7879 \end{bmatrix},$$

$$H = [0.0397 \quad 3.6129]$$

and hence we obtain L as:

$$L = P^{-1} H^T = [0.0025 \quad 0.2288]$$

To illustrate the effectiveness of the proposed method, Fig1 shows the trajectory tracking of the adaptive observer (11) for x_1 and x_2. Also the estimation of unknown parameter θ by adaptive law (12) with $\theta = 0.5$ is presented in Fig2.

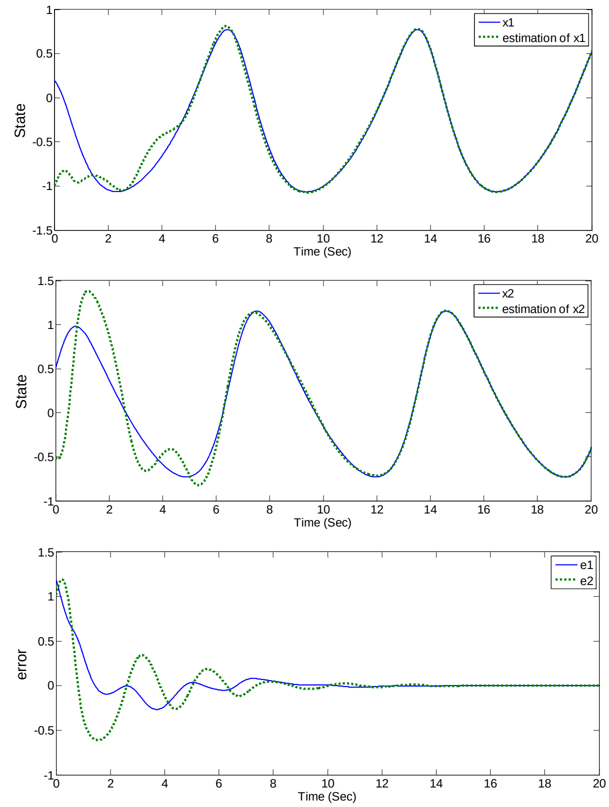

Fig 1. Actual and estimated values of the states

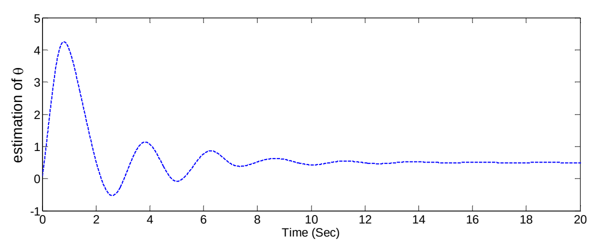

Fig 2. Estimation of system parameter

5. Conclusion

In this paper, we proposed an approach to design nonlinear observers for systems which satisfy one-sided Lipschitz and quadratic inner bounded conditions, in the presence of unknown parameters. Based on Lyapunov technique, necessary conditions for stabilizing the adaptive observer are derived. These conditions converted into LMIs, by solving the proposed LMI through standard software packages the observer gain will obtain. Simulation results are presented to show the effectiveness of the proposed method.

6. References

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