Contrast Enhancement Based on Eigencanceler Beamforming Applied to Medical Ultrasound Imaging

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Abstract

Using adaptive minimum variance beamforming (MV) results in a significant improvement in image resolution, but it's success in enhancing contrast has not yet been satisfactory. In some researches, eigenspace-based minimum variance beamforming (EIBMV) method has been applied to medical ultrasound imaging system, so that it has improved image contrast while maintaining its resolution. In this paper we apply another eigenspace-based beamforming called eigencanceler (EC) and show it can yield more satisfactory results. However EC needs the noise – interference covariance matrix, whereas the MV and EIBMV use the data covariance matrix. So an altered EC is proposed for the ultrasound imaging. In this method, while canceling the desired signal does not occur, any weight vector is considered to lie in the noise subspace, the subspace orthogonal to the dominant eigenvectors. Otherwise, the weight vector is defined uniformly. Simulation results show the superiority of this method over the MV and EIBMV methods in the contrast aspect. Moreover, the method is more robust against the sound speed errors and can obtain images with better definition of boundaries.

Keywords-beamforming;eigenspace;minimum variance; minimum norm eigencanceler; ultrasaound imaging

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I. Introduction

In standard methods, after applying appropriate delays, an image is formed by weighing and summing the received signal from all channels (DAS Beamforming) [1 – 2]. In recent researches, adaptive beamforming methods have increasingly been utilized in medical ultrasound imaging [3 – 7]. Minimum variance adaptive beamforming (MV) is one of the most utilized adaptive methods which can significantly improve the image resolution [8 – 12].

An important issue for adaptive beamforming ultrasound imaging is to enhance simultaneously the image resolution and contrast. To this end, in [13 – 17] the eigenspace-based minimum variance beamforming (EIBMV) was applied to medical ultrasound images. The EIBMV beamformer utilizes the eigen-structure of the covariance matrix to improve the performance of the MV beamformer in case of image resolution and desired lobes. In EIBMV, the steering vector is projected to the signal subspace which cause the weight vector to lie in the signal subspace. This method is capable of improving the image contrast as well as increasing the robustness of the beamformer against the steering vector errors [13 – 14].

Eigencanceler (EC) is a method which is able to cancel strong interferences. It utilizes the eigenanalysis method like EIBMV, but the difference is that in EC method the weight vectors lie in the noise subspace to highly cancel interferences[18-19].

Since the noise subspace is orthogonal to the interference subspace, any weight vector in the noise subspace has the ability to cancel interferences.

There are different types of EC beamformers, in this paper we just talk about minimum norm EC (MNE). The MNE finds the weight vector in the noise subspace to minimize the norm of the weight vector. This method utilizes the fact that the most power concentrates at the largest eigenvalues, so a few eigenvectors related to that eigenvalues contain all the information about the distribution of the interference and the rest of them are related to the noise subspace which demonstrates the noise covariance matrix [20– 21].It has been shown that the MNE can be formed using dominant eigenvectors only. However, the problem with this method is that it requires noise – interference covariance matrix which is not usually available.

In this paper, we develop an EC based adaptive beamforming method for medical ultrasound imaging. The method termed as thresholded MNE, assumes that the weight vector is in the noise subspace unless it violates the distortionless constraints, where the weights of a DAS beamformer is replaced. The simulation results indicate that the method produces images with better contrast and better definition object boundaries than the MV and EIBMV.

The outline of this manuscript is as follows. The background about MV and EIBMV beamformers are presented in section II. Section III is concerned to introduce the proposed method. The results of the different beamformers applied to the simulated data are shown in section IV and then discussions are included in section V. Finally, some concluding remarks are given in Section VI.

II. Background

A. Minimum Variance Beamforming

Assume an array with M elements that each element receives the signal $x_i(t)$. The received signal from channel *i* for a reflector with ideal steering is given by:

$$x_i(t) = S_d(t) + \sum_{p=1}^{P} S_p(t) + n_i(t) \quad (1)$$

where $S_d(t)$ is the reflected signal which has to be estimated, $S_p(t)$ is the p_{th} interference signal and $n_i(t)$ denotes for noise on channel *i*. After applying delays to each channel to focus at a point in the image, the goal of the adaptive beamformer is to compute the optimal aperture shading before combining the channels. In fact the adaptive beamforming finds a set of optimum sensor weights, w_i , to suppress noise and off – axis signals. The output of the beamformer is defined by:

$$y[n] = \vec{w}^H[n]\vec{x}[n] \quad (2)$$

where $\vec{x}[n] = [x_1[n], x_2[n], ..., x_M[n]]^T$ and $\vec{w}[n] = [w_1^*[n], w_2^*[n], ..., w_M^*[n]]^T$ is the applied weight vectors to the array. The weights of the MV beamformer are determined by minimizing the variance (power) of the beamformer output, $y[n]$, under the constraint that the signal reflecting from the direction of interest is passed without distortion, that is:

$$\min E\left[\|y[n]\|^2\right] = \min \vec{w}^H[n]R[n]\vec{w}[n] \quad (3)$$
$$\text{subject to:} \quad \vec{w}^H[n]\vec{a} = 1$$

where $R[n] = E\left[\vec{x}[n]\vec{x}^H[n]\right]$ is the spatial covariance matrix and $\vec{a}$ denotes the steering vector toward the desired direction. By assuming fixed focus on transmission and dynamic focus on reception, the steering vector simply becomes a vector of ones. The solution to (3) is given by [10]:

$$\vec{w}[n] = \frac{R^{-1}[n]\vec{a}}{\vec{a}^H R^{-1}[n]\vec{a}} \quad (4)$$

If we directly use $R[n]$ to compute the covariance matrix, it is easy to get an ill – conditioned matrix not suitable for the inversion required in (4). The usual way to overcome the problem is the spatial smoothing technique to estimate the covariance matrix. In this technique the array is divided into overlapping subarrays each with length L. Then the covariance matrix is estimated as [7]:

$$\hat{R}[n] = \frac{1}{M-L+1}\sum_{l=0}^{M-L} \vec{x}_l[n]\vec{x}_l^H[n] \quad (5)$$

where $\vec{x}_l[n] = [x_l[n], x_{l+1}[n], ..., x_{l+L-1}[n]]^T$ is the data on *l*'th subarray. By this assumption, the steering vector $\vec{a}$ is $L \times 1$ vector of ones.

To retain speckle statistics similar to DAS beamforming, temporal averaging over $2k+1$ samples is suggested [12]:

$$\hat{R}[n] = \frac{1}{(2K+1)(M-L+1)} \times \sum_{k=-K}^{K}\sum_{l=0}^{M-L} \vec{x}_l[n-k]\vec{x}_l^H[n-k] \quad (6)$$

where K is selected based on the excitation pulse length and $\hat{R}[n]$ is an estimation of the covariance matrix.

Commonly a large number of samples are

required to obtain a stable covariance matrix. A common way to increase robustness of the covariance matrix is to add a constant, ε, into the diagonal of the covariance matrix, replacing $\hat{R}$ with $\hat{R}+\varepsilon\,\mathrm{I}$ (where I represents the unit matrix). This technique is called diagonal loading. The loading factor is usually set to be Δ times the power in the received signals. There are different ways to select Δ, where in this paper it is considered to be $\Delta = 1/100L$:

$$\varepsilon = \Delta tr\{\hat{R}[n]\} \quad (7)$$

After the estimation of the covariance matrix, the $L\times 1$ MV weight is got by equation (4). Then the final output of the MV beamformer is given by:

$$z[n] = \frac{1}{M-L+1}\sum_{l=0}^{M-L} \vec{w}^H[n]\vec{x}_l[n] \quad (8)$$

B. *Eigenspace-Based Minimum Variance Beamforming*

The eigenspace – based minimum variance method (EIBMV) utilizes the eigen structure of the covariance matrix to improve the performance of the MV beamformer which can improve its quality in terms of resolution and reducing sidelobes by simultaneously suppressing off – axis signals and retaining signal of interest.

In the EIBMV method, the covariance matrix is divided into two orthogonal subspaces, the signal subspace and the noise subspace. The EIBMV weight vector is calculated by calculating the MV weight vector and projecting it to the signal subspace, the subspace determined by the dominant eigenvalues of the covariance matrix.

The signal subspace, E_s, contains mainlobe signals and it significantly reduces sidelobe effects. It is constructed form the eigenvectors related to the largest eigenvalues [13], [22]:

$$E_s = [\vec{V}_1, \vec{V}_2, ..., \vec{V}_{NUM}] \quad (9)$$

where $\vec{V}_i = [v_{i1}, v_{i2}, ..., v_{iL}]^T$ denotes the orthonormal eigenvectors related to the eigenvalues and NUM expresses the number of the eigenvectors which determine efficiently the signal subspace. The EIBMV weight vector is calculated by projecting the MV weight vector on the signal subspace:

$$\vec{w}_{EIBMV} = E_s E_s^H \vec{w}_{MV} \quad (10)$$

It has been shown that the EIBMV beamformer has less sensitivity to the steering vector errors than the MV beamformer. This method improves the beamformer quality in terms of contrast, reducing sidelobes and robustness.

Due to the high correlation of the on – axis signals, the mainlobe energy concentrates on the eigenvectors related to the largest eigenvalues. In other words, sidelobe energy distributes on the other eigenvectors. For each concentrated point after estimating the covariance matrix, the sufficient number of the eigenvectors for constructing the signal subspace which can describe mainlobe and reduce sidelobe interferences, are estimated by an adaptive procedure. In otherwords, sufficient number of the eigenvectors to retain mainlobe signals and simultaneously reduce sidelobe interferences is different from one point to other ones. It depends on the ratio of the energy of the mainlobe signals to the energy of the sidelobe signals. The EIBMV approaches to MV beamformer by decreasing the number of the rejected eigenvalues.

III. PROPOSED METHOD

The eigencanceler (EC) is an approach for canceling interferences based on spectrum analysis of covariance matrix. It is designed so that the weight vectors will lie in the noise subspace, the subspace orthogonal to the dominant eigenvectors [19]. The EC exploits those properties of the covariance matrix to construct a weight vector that is very effective in canceling the interferences.There are two types of eigencancelers that one of them which we are using is “Minimum Norm Eigencancer” or “MNE”.

Analysis in eigenspace represents that the most power concentrates at the largest eigenvalues of the noise – interference covariance matrix, so a few eigenvectors related to that eigenvalues contain all the information about the distribution of the interference. Due to this fact, it can be expressed that the eigenvectors related to these eigenvalues contain all the unit vectors which travel in the interference subspace. The rest of the eigenvectors are for noise subspace, the subspace orthogonal to the interference subspace. Any weight vector in the noise subspace has the ability to cancel interferences, so by utilizing the noise covariance matrix, the EC weight vectors can be defined.

Let Q_r denotes the interference covariance matrix and Q_v denotes the noise covariance matrix. Since $Q_r \times Q_v = 0$, any weight vector in the noise subspace, $\vec{w} \in span\,[Q_v]$, has the ability to cancel the interferences.

To minimize the norm of the weight vector while maintaining the linear and eigenvector constraints, MNE is designed as below [21]:

$$\min \vec{w}^H \vec{w}$$
$$\text{subject to:}\quad Q_r^H \vec{w} = 0 \qquad (11)$$
$$\vec{a}^H \vec{w} = 1$$

By solving the optimization problem in (13), the MNE weight vector would be obtained as [21]:

$$\vec{w}_e = Q_v Q_v^H \vec{a}[\vec{a}^H Q_v Q_v^H \vec{a}]^{-1} \qquad (12)$$

By using the fact that $Q_r \times Q_r^H + Q_v \times Q_v^H = I$, equation (14) will become [21]:

$$\vec{w}_e = (I - Q_r Q_r^H)\vec{a}[\vec{a}^H (I - Q_r Q_r^H)\vec{a}]^{-1} \qquad (13)$$

According to (12) and (15), it can be seen that the MNE weight vector will completely lie in the noise subspace.

By using the relation $R^{-1} = Q_r \Gamma_r Q_r^H + Q_v \Gamma_v Q_v^H$, where Γ_r and Γ_v are the diagonal matrices of the reciprocals of the eigenvalues of the interferences and noise covariance matrices, respectively, we get [21]:

$$\vec{w}_{MV} = (Q_r \Gamma_r Q_r^H + Q_v \Gamma_v Q_v^H) \times \vec{a}\left[\vec{a}^H \left(Q_r \Gamma_r Q_r^H + Q_v \Gamma_v Q_v^H\right)\vec{a}\right]^{-1} \qquad (14)$$

Equation (14) shows that the MV weight vector is a superposition of vectors in the noise subspace Q_v, as well as vectors in the interference subspace Q_r, while the MNE weight vectors lie entirely in the noise subspace.

It is not applicable to use the MNE method directly in ultrasound imaging system, because the noise – interference covariance matrix is not available, we just have signal and noise – interference covariance matrix together. To apply MNE for ultrasound imaging, we proposed a proper thresholding method to detect the presence of the desired signal in the received data.

Imagine that in the desired point, contribution of the desired signal is close to zero. It means that there has not been any reflector, so that in this case, the resultant covariance matrix is exactly the noise – interference covariance matrix. But when the signal distribution is not close to zero and is considerable, applying the MNE beamformer directly will cause cancelation of the desired signal, because in this case desired vector, *w*, almost completely will be orthogonal to the signal subspace. Consequently, the MNE output will approach to zero. Therefore, when the projection of the steering vector in the noise subspace is very small, it will be seen that the signal contribution in the covariance matrix is considerable. So at this desired point, beamforming can be done by DAS, MV or EIBMV method, which we used DAS method. Otherwise, the weight vector will be determined by MNE method. Thus we determined the weight vector as follows:

$$\vec{w} = \begin{cases} 1/L\,, & \| Q_v Q_v^H \vec{a} \| < L\delta \\ \vec{w}_e\,, & otherwise \end{cases} \qquad (15)$$

where, δ is a positive small value close to zero. When δ is assumed too small, the desired signal cancelation maybe occur. On the other hand, when it is large, interference cancelation will not occur properly. In other words, large values of δ makes the MNE act very similar to DAS. Therefore, the performance of MNE beamformer is very dependent to the value of δ. However, its performance would not worse than that of DAS.

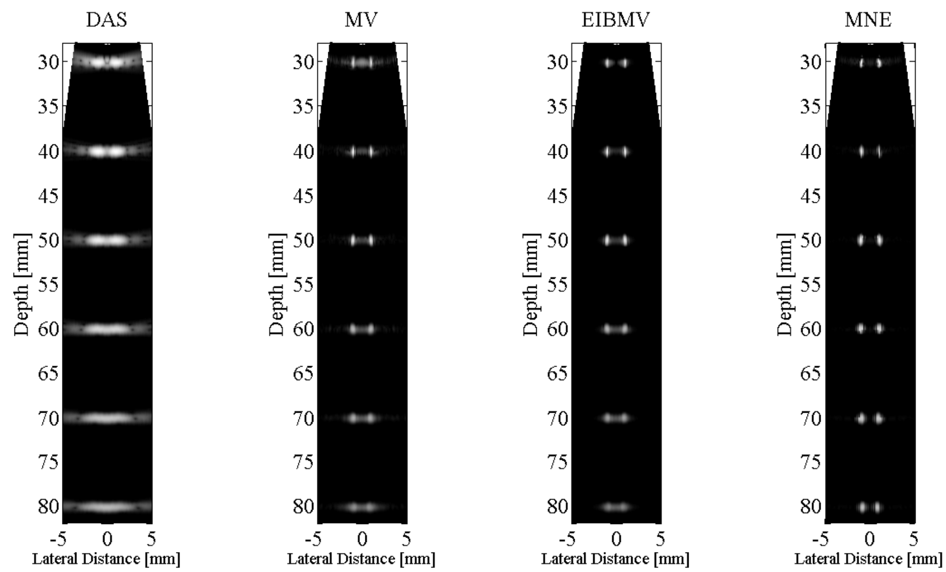
IV. SIMULATIONRESULTS

We have applied the MNE beamformer to some simulated data and compared the results with DAS, MV and EIBMV beamformers. Fixed focus at z=45mm on transmission and dynamic focus on reception was applied. Field II software was used to simulate a linear array with M = 64 elements. The central and sampling frequency of the linear array was set to 3.5 MHz and 80MHz respectively. In the MV method the subarray length is considered $L = M/2$ and temporal averaging was done over $2K+1$ samples in which K was set to 40 at 80

MHz sampling rate.

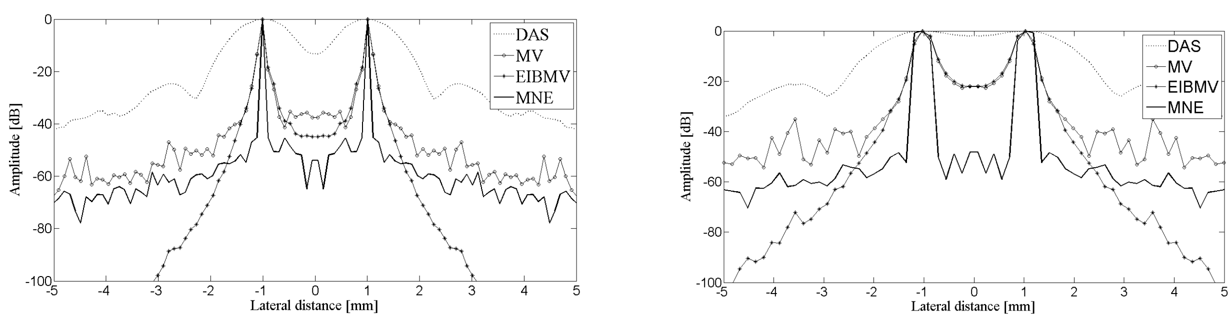
In order to create an appropriate signal subspace for each focused point, the eigenvectors related to 30% of the largest eigenvalues are utilized. In the proposed method, to prevent the desired signal cancelation, $\delta = 10^{-2}$ has been assumed.

Figure 1 shows images obtained with DAS, MV, EIBMV, and MNE beamformers that are displayed over 50dB dynamic range. There are 12 point targets at depth of 30 – 80mm, which separated laterally by 2 mm. From the figure, it is seen that MNE satisfactorily resolved the two points at depth 80 mm, whereas the results of the other beamformers are not satisfactory.



(a) (b) (c) (d)

Figure 1.Simulated point targets using (a) DAS, (b) MV, (c) EIBMV, and (d) MNE beamformer



(a) (b)

Figure 2. Lateral distribution of responses for different beamformers at depth z = 40mm (a) and z = 60mm (b).

Figure 2shows the lateral distribution of the beamformed responses at depth 40mm (a) and 60 mm (b). These figures also indicate better resolving capability of MNE compared to the other beamformers, however the images obtained by EIBMV shows the lowest sidelobes at points laterally far from the reflectors.

To investigate the contrast properties of the beamformers, a cyst phantom in a speckle pattern was simulated, which contains a circular cyst with 5mm radius at depth 45mm. The images reconstructed from these beamformers are shown in figure 3 at 50dB dynamic range. To evaluate the contrast properties, contrast ratio (CR) and contrast–to–noise ratio (CNR) for each beamformer are represented in Table I, where CR is defined as the ratio of the mean value in the background (S_0) to the mean value in the cyst region (S_i), which is expressed in dB as:

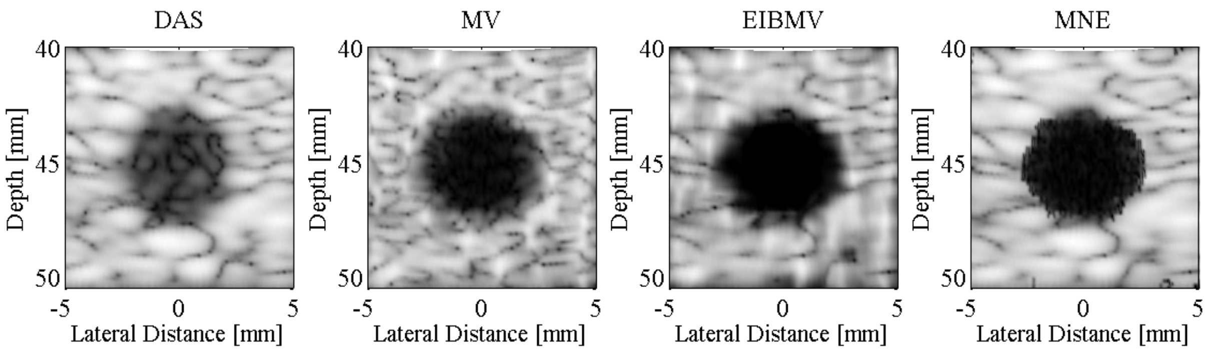
$$CR_{dB} = \left|\langle S_0 \rangle - \langle S_i \rangle\right| \quad (19)$$

and CNR is defined as the CR divided by thestandard deviation of image intensity in the background region (σ_0):

$$CNR_{dB} = CR / \sigma_0 \quad (20)$$

According to figure 3 and Table I, it is obvious that MNE enhances the contrast in comparison with the other beamformers.

To study more about the reasoning of the enhanced performance of MNE in contrast aspects, the mean intensity of the image at the cyst region and also the background are depicted in Table I. It is seen that the EIBMV reaches to the lowest mean intensity at the cyst region, which can be viewed as a result of low sidelobes of the EIBMV beamformer as shown in figure 2. Also, the table indicates that the mean intensity at background is



(a) (b) (c) (d)

Figure 3. Simulated cyst phantom using (a) DAS, (b) MV, (c) EIBMV, and (d) MNE beamformer.

Table I. Contrast ratio and contrast to noise ratio for different beamformers

Beamformer	Mean intensity in the cyst region (dB)	Mean intensity in the background (dB)	CR	CNR
DAS	-44.2	-23.9	20.3	1.3
MV	-54.3	-27.2	27.1	1.8
EIBMV	-59.2	-30.4	28.8	1.8
MNE	-57.0	-23.8	33.2	2.1

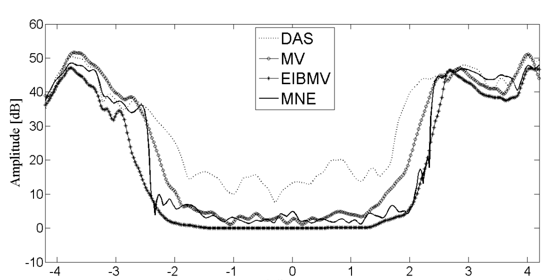


Figure 4. Lateral response of cyst phantom at depth 45 mm for different beamformers

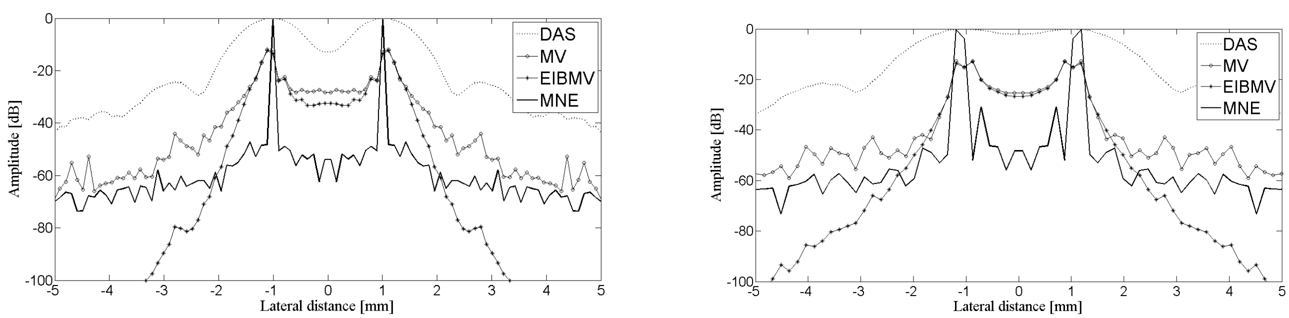
highest for the MNE beamformer, which is a result of minimizing the norm of weight vector as represented in (11). This high background intensity is such that the contrast ratio of MNE is higher than that of EIBMV.

The lateral variations of the intensity of the cyst images at depth 45 mm is depicted in figure 4. This figure shows that a sharp transition from cyst to the background region is occurred in the image obtained by MNE. This means that the boundary of cyst is well defined by MNE, in comparison to the others.

A practical imaging system should be robust against sound velocity error resulting from medium inhomogeneities. Assuming the sound speed is known with a 5% overestimation, different beamforming methods are applied to obtain the images of the point targets. Figure 5 illustrates the lateral responses of the beamformers at depths 40 mm and 60 mm. It is obvious from the figure that the performance of MNE is not affected seriously by the sound speed error: its resolving capability almost retains and its amplitude estimation of the reflectors is the same as DAS, whereas MV-based methods are degraded both in resolving capability and amplitude estimation aspect.

points of a cyst. Besides, MNE minimizes the norm of the weighting vector which results in a smoother and higher image intensity at speckle regions, compared to the images obtained by the MV-based beamformers. This property along with the better boundary definition are responsible for the better contrast properties of the MNE.

It seems combining the MNE and EIBMV beamformers in a proper manner can produce an enhanced image. A simple way of combining is to choose the minimum value of the MNE output and EIBMV output. In this regards, the obtained image is expected to have a good definition of walls, a smooth and high intensity of speckles and dark cysts.



(a) (b)

Figure 5. Lateral distribution of responses for different beamformers at the presence of 5% over estimation of sound speed error at depth z = 40mm (a) and z = 60mm (b).

V. Discussions

MNE and EIBMV are two beamforming methods that utilize eigenspace decomposition of the covariance matrix. In the MNE method, the weight vector lies in the noise subspace, while in EIBMV it lies in the signal subspace. Consequently, MNE completely suppresses the interference signals, which results in a better distinction a strong reflector from the around medium, as can be seen from figures 1 and 2. This feature is also responsible for the better definition of cyst boundary in MNE beamformer, as is shown in figure 4. In contrary, EIBMV completely cancels out the noise signal and hence, achieves better performance when the signal and interferences are very weak. This leads to a very small output at points far from a strong reflector and also, at inner

VI. Conclusions

In this paper we have investigated an eigencanceler based (EC) beamformer for ultrasound imaging system and proposed a thresholded MNE. This new beamformer calculates the projection of steering vector on the noise subspace and when the projection is not smaller than a threshold, the data are combined through a weight vector parallel to the projection. Otherwise, a DAS beamforming is applied.

The simulation results have shown that the proposed method can perform a better resolving capability compared to MV and EIBMV methods and also provides a significant contrast enhancement which can be viewed as a result of a high intensity at speckle regions and a sharp decreased response at the vicinity of a reflector.

One problem in many adaptive beamforming methods is their sensitivity to steering vector errors. Due to the fact that the MNE method like EIBMV uses the eigen-decomposition of the covariance matrix to create the desired weight vector, it is expected its robustness would be acceptable. Besides, the proposed MNE method applies DAS in the presence of the desired signal. These two features gives a good robustness to MNE, as the simulation results verifies this.

The main problem encountered in the proposed method is to determine an appropriate threshold value of δ. Since this parameter has a direct effect on the performance of the beamformer, it is better to be defined by an adaptive procedure which is considered in our future researches.

References

[1] S.Holm, J.F. Synnevag, and A. Austeng, “Capon Beamforming For Active Ultrasound Imaging Systems,” *Proc. IEEE 13th DSP Workshop,* Jan. 2009.

[2] B. D. Van Veen and K. M. Buckley, "Beamforming: a versatile approach to spatial filtering," *ASSP Magazine, IEEE,* vol. 5, pp. 4-24, 1988.

[3] J.F Synnevag, A. Austeng, and S. Holm, “Benefit of minimum variance beamforming in medical ultrasound imaging,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Conrol,* vol. 56, no. 9, pp. 1868-1879,Sep.2009.

[4] B. Mohammadzadeh, and A. Mahloojifar, “Contrast Enhancement and Robustness Improvement of Adaptive Ultrasound Imaging Using Forward-Backward Minimum Variance Beamforming,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control,* vol. 48, no. 4,April 2011.

[5] C.C Nilsen, and I. Hafizovic, “Beamspace Adaptive Beamforming for Ultrasound Imaging,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 56, no. 10,October 2009.

[6] C.C. Gaudes, E.M.G. mez, and T.S. Paules, “Robust array beamforming with sidelobe control using support vector machines,”*IEEE Trans. Signal Process.*, vol. 55,no. 2, pp. 574-584, Feb.2007.

[7] J.F. Synnevag, A. Austeng, and S. Holm, “Adaptive beamforming applied to medical ultrasound imaging,” *IEEE Trans. Ultrason. Ferroelectr.Freq.Control,* vol. 54 , NO. 8, pp. 1606-1613, Aug. 2007.

[8] Sh.L Wang, Ch.H Chang, H.Ch Yang, Y.H Chou and P.Ch Li, “Performance Evaluation of Coherence-Based Adaptive Imaging Using Clinical Breast Data,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control,* vol. 54, NO. 8, pp. 1669-1679, Augt 2007.

[9] Sh.L Wang, and P.Ch Li, “High Frame Rate Adaptive Imaging Using Coherence Factor Weighting and the MVDR Method,” *IEEE International Ultrasonics Symposium Proceedings* ,2008,pp. 1175-1178.

[10] B. M. Asl and A. Mahloojifar, “Minimum variance beamforming combined with adaptive coherence weighting applied to medical ultrasound imaging,” IEEE Trans. Ultrason. Ferroelectr. Freq. Control, vol, 56, no. 9, pp.1923-1931, 2009.

[11] C.-I. C. Nilsen and S. Holm, “Wiener beamforming and the coherence factor in ultrasound imaging,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control,* vol.57, no. 6,pp. 1329-1346, 2010.

[12] J C. C. Nilsen and S. Holm, "Wiener beamforming and the coherence factor in ultrasound imaging," *IEEE Trans. Ultrason., Ferroelectr. Freq. Control,* vol. 57, pp. 1329-1346, 2010.

[13] B. M. Asl and A. Mahloojifar, “Eigenspace-based minimum variance beamforming applied to medical ultrasound imaging,” *IEEE Trans. Ultrason., Ferroelectr. Freq. Control,* vol. 57, no. 11, pp. 2381–2390, Nov. 2010.

[14] S. Mehdizadeh, A. Austeng, T. F. Johansen, and S. Holm, "Eigenspace Based Minimum Variance Beamforming Applied to Ultrasound Imaging of Acoustically Hard Tissues," *IEEE Trans. Med. Imag.,* vol. 31, pp. 1912-1921, 2012.

[15] X. Zeng, C. Chen, and Y. Wang, "Eigenspace-based minimum variance beamformer combined with Wiener postfilter for medical ultrasound imaging," *Ultrasonics,* vol. 52, no. 8, pp. 996-1004, 2012.

[16] X. Zeng, Y Wang, J. Yu, and Y.i Guo., " Beam-domain eigenspace-based minimum variance beamformer for medical ultrasound imaging," *IEEE Trans. Ultrason., Ferroelectr. Freq. Control,* vol. 60, no. 12, pp. 2670-2676, 2013.

[17] S. Mehdizadeh, S. Muller, T. F. Johansen, and S. Holm, "Application of eigen-spaced beamformers for imaging lamina of vertebral arch," *in*

Ultrasonics Symposium (IUS), 2012 IEEE International, pp. 2238-2241, 2012.

[18] M. Haimovich and Y. Bar-Nass “An Eigenanalysis Interference Canceler,” *IEEE Trans. Signal Process.*, vol. 39, No. 1, pp. 1-9, 1991

[19] A. Haimovich, "The eigencanceler: adaptive radar by eigenanalysis methods," Aerospace and Electronic *Systems, IEEE Transactions on*, vol. 32, pp. 532-542, 1996.

[20] W. J. L. Queiroz and M. S. Alencar, "Eigencanceler study applied to local scatter channel model," in *Microwave and Optoelectronics Conference, 2003. IMOC 2003. Proceedings of the 2003 SBMO/IEEE MTT-S International*, vol. 1, pp. 43-47, 2003.

[21] A. Haimovich, "The eigencanceler: a new space-time interference canceler," in *Radar Conference, 1994., Record of the 1994 IEEE National*, vol. 4, pp. 194-199, 1999.

[22] Y. Jung-Lang and C. C. Yeh, “Generalized eigenspace-basedbeamformers,” *IEEE Trans. Signal Process.*, vol. 43, pp. 2453-2461, 1995.