On Ergodic Capacity and Outage Probability for Secondary Users in Point-to-Point and Multiple Access Channels

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***Abstract*—Nowadays, the spectrum is becoming more and more scarce because of not only the demand increase for wireless services and applications but also inflexible and rigid spectrum allocation policies. A promising solution to overcome this challenge is to exploit spectrum holes for secondary users in cognitive radio. Taking advantage of cognitive radio and dynamic spectrum access improves the efficiency of spectrum utilization as it allows secondary users to exploit the available frequency bandwidth. In an opportunistic spectrum access scenario, the secondary users can only utilize the frequency bandwidth when the primary user is inactive. In this paper, we assume that secondary users exploit the available frequency bandwidth using opportunistic spectrum access approach. As a result, one may model the available bandwidth to the secondary users as a random process taking value of 0 or 1. In this paper, we investigate the effects of Rayleigh fading environment on the capacity of AWGN point-to-point channel. Afterwards, we also go beyond point to point channels and analyze Multiple Access Channels. More precisely, we derive the closed form expression of ergodic capacity region of Multiple Access fading Channel with two independent sources. To the best of our knowledge, no prior work has investigated the ergodic capacity region of Multiple Access Channel under Rayleigh fading with the aforementioned approach. Numerical analysis are conducted to validate our theoretical results. Interestingly, numerical results indicate that, against the common sense, capacity of Rayleigh fading environment channels are higher than deterministic ones in Gaussian point-to-point channels.**

***Index Terms*—Capacity, secondary users, cognitive radio, spectrum sharing, Rayleigh fading.**

I. Introduction

Recently, the interest of consumers in wireless devices and applications has increased. As a result, the demand for spectrum has risen rapidly. However, the main reason behind spectrum scarcity is the inflexible spectrum allocation policies. Government agencies regulate the spectrum as a natural source and assign it to licensed users for large geographical regions [1-3]. Under this policy, the primary users have exclusive access to frequency band without any interference in order to protect their systems from distortion. Nevertheless, with many improvements in designing communication systems, this attitude does not seem to be optimal any more and leads to underutilization of spectrum bands [3]. As shown in Figure 1, according to report of the Federal Communications Commission (FCC) published in November 2002, the large amount of spectrum is underutilized [1][2]. As a solution to this problem, exploiting spectrum holes for secondary users (SUs) was proposed. In a dynamic spectrum access scenario, secondary users utilize the spectrum holes. One of the promising solutions for dynamic spectrum access is Cognitive radio (CR) which was first introduced by Mitola in 1998 [4].

Cognitive capability to sense the environmental information and reconfigurability to adapt the transmission parameters are the main features of the cognitive radio. The ultimate goal of the cognitive radio is to detect the spectrum holes and communicate reliably and more efficiently over the channel. Recently, dramatic developments in networking, wireless communications and machine learning made the implementation of cognitive systems feasible. Wide range of studies has been conducted addressing challenges, fundamental limits and implementation of cognitive radios [1][2][5].

Taking advantage of cognitive radio and dynamic spectrum access improves the efficiency of spectrum utilization as it allows secondary users to exploit the available frequency bandwidth. Dynamic spectrum management in CR is achieved by either opportunistic spectrum access or spectrum sharing. In an opportunistic spectrum access scenario, the secondary users wait until the frequency bands or time intervals of primary users (PUs) are available. However, in spectrum sharing systems, the secondary users are allowed to transmit data as long as the interference is tolerable by primary users.

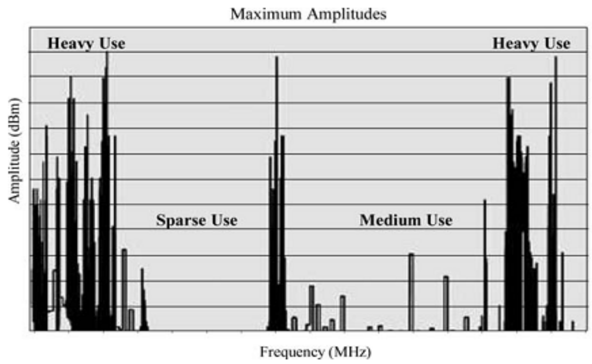

Fig. 1. Spectrum Utilization [1]

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In the context of spectrum sharing systems, early studies obtained the capacity of AWGN channel with constraint on maximum transmitted power [6]. Whereas, Gastpar firstly proposed a new perspective toward defining more relevant constraint [4]. He derived the capacity of different AWGN channels considering received power constraint at a third user's receiver, e.g., primary receiver. Later, Ghasemi and Sousa investigated the ergodic capacity in various fading channels [3] considering either peak or average received-power constraint at a third party's receiver. In [5], Mousavian and Issa derived capacity and power allocation for Rayleigh fading channel under joint average and maximum received power constraints. Common in most of the above mentioned works is that the interference of primary user is not considered in obtaining the capacity of secondary users. Therefore, they obtained the upper bound of the capacity. In particular, when the power level of primary user transmitter is higher than that of secondary user, the interference of primary user on the secondary user's capacity should not be disregarded. Hence, some researches have been carried out aiming to take into account this particular effect [7][8]. Despite of enormous researches in spectrum sharing approach, few studies have been conducted addressing the capacity of secondary users in the opportunistic spectrum access scenario. Ramirez in [9] introduced a new approach toward capacity analysis from secondary users point of view. He modeled the bandwidth fluctuation in an opportunistic spectrum access system as a Poisson random process and he obtained the ergodic capacity of point-to-point AWGN channel in both single carrier modulation and multicarrier modulation.

In this paper, we investigate the ergodic capacity and outage probability of secondary users in an opportunistic spectrum access system where the SUs utilize the PUs bandwidth when it is available. Similar to [9], the channel bandwidth is considered as a random process from the SUs point of view. However, we extend the work in [9] and explore the ergodic capacity of secondary users in a Rayleigh fading environment. Afterwards, we also go beyond point to point channels and analyze Multiple Access Channels which have never been investigated in the literature. More precisely, we derive the closed form expression of ergodic capacity region of Multiple Access fading Channel with two independent sources.

The rest of this paper is organized as follows. Section II introduces system model and basic assumptions. We derived the closed form of secondary users' ergodic capacity and outage probability in point-to-point Gaussian fading channels in Section III. In Section IV, we obtained the ergodic capacity region of Multiple Access fading Channel. Section V presents experimental results and finally, Section VI concludes the paper with a brief summary and discussion.

II. System model

We investigate the ergodic capacity of discrete time point-to-point Rayleigh fading channel and Multiple Access Rayleigh fading channel with perfect channel side information (CSI) available to the receiver. Although the perfect CSI assumption may seem infeasible, this assumption is a crucial footstep toward investigating more realistic models [8].

In an opportunistic spectrum management scenario, secondary users can access to primary users' bandwidth in a randomly fashion. Hence, the capacity is a random process with two independent random variables; fading coefficient and channel bandwidth. We explore the ergodic capacity of channel by expectation over joint probability density function of channel bandwidth and fading coefficient.

Similar to [9], we consider that channel bandwidth available to secondary users changes in a bursty manner. Therefore, we model the channel bandwidth as multiplying W by random process $b(t)$ [9] :

$$W(t) = b(t).W \tag{1}$$

where b(t) is a wide sense stationary random process takes value of 1 or 0 with probability of p and q respectively. We presume that random process b(t) is generated by Poisson points; b(t) takes the value of 1 if the number of points in interval (0,t) is even and similarly takes the value of 0 when the number of points is odd [9]. When $p = q = 1/2$ the mean and autocorrelation function are as follows [9] :

$$E\{b(t)\} = p \tag{2}$$

$$R_{ba}(\tau) = p.(e^{-\alpha|\tau|}cosh(\alpha|\tau|)) \tag{3}$$

where α is the average number of transmission per second.

Therefore, using expectation over channel bandwidth and fading coefficient, the ergodic capacity is obtained as:

$$C_{avg} = E_{h,b}([C_{rand}]) \tag{4}$$

where C_{rand} is the random capacity of channel.

III. AWGN Point-to-Point Gaussian fading channel

In a discrete Point-to-Point Gaussian fading channel, the received signal y[n] can be described as:

$$Y[n] = h[n]x[n] + z[n] \tag{5}$$

where x[n] is the signal transmitted by secondary user and h[n] is the channel gain between secondary transmitter and receiver. The capacity of Gaussian fading channel with constraint on the transmitter power can be written as :

$$C = W \log_2(1 + \frac{P_1.|h|^2}{N_0.W})(bps) \tag{6}$$

where W is channel bandwidth and P_1 is the average transmitter power,$|h|^2$ is the channel power gain and N_0 is power spectrum density of noise

According to (1), from the secondary users point of view, we can model the capacity as:

$$C = Wb(t) \log_2(1 + \frac{P_1 b(t).|h|^2}{N_0.Wb(t)})(bps) \tag{7}$$

and the average value of capacity is:

$$C_{avg} = E_{h,b}[Wb(t) \log_2(1 + \frac{P_1.|h|^2}{N_0.W})] \tag{8}$$

because of independency of random variables, we have:

$$\begin{aligned} C_{avg} &= E_h[E_b[Wb(t) \log_2(1 + \frac{P_1.|h|^2}{N_0.W})]] \\ &= E_h[pW \log_2(1 + \frac{P_1.|h|^2}{N_0.W})] \end{aligned} \tag{9}$$

where p is the probability of $b(t) = 1$.

It can be shown that in a Rayleigh fading channel with parameter σ, the channel power gain has an exponential distribution with mean $\lambda = 2\sigma^2$. Therefore, we obtain:

$$\begin{aligned} C_{avg} &= \int_0^{+\infty} pW \log_2(1 + kx)\lambda e^{-\lambda x} dx \\ k &= \frac{P_1}{N_0.W} \end{aligned} \tag{10}$$

By solving (10), we can derive the capacity as (see appendix 1) :

$$C_{avg} = \frac{pWe^{\lambda/k}}{ln2}(-\gamma - \ln(\lambda/k) - \sum_{n=1}^{+\infty} \frac{(-\lambda/k)^n}{n.n!}) \tag{11}$$

where γ is the EulerMascheroni constant.

In addition to ergodic capacity, we are also interested in outage probability as a fundamental parameter for system analysis and implementation. The outage probability is the probability that C_{rand} falls below a minimum threshold. In other word, outage probability is the probability that the receiver cannot decode the transmitted data successfully. According to this definition, we have:

$$P_{out} = P(C_{rand} < R) \tag{12}$$

$$P_{out} = P(C_{rand} < R|b(t) = 1)P(b(t) = 1)+ \tag{13}$$

$$P_{out} = P(C_{rand} < R|b(t) = 1)p \tag{14}$$

$$\begin{aligned} P(C_{rand} < R|b(t) = 1) &= P(|h|^2 < \frac{N_0.W}{P}.(2^{R/W} - 1)) \\ &= \int_0^{\frac{N_0.W}{P}.(2^{R/W}-1)} \lambda.e^{-\lambda x} dx \\ &= 1 - e^{-\lambda.\frac{N_0.W}{P}.(2^{R/W}-1)} \end{aligned} \tag{15}$$

Therefore, we have:

$$P_{out} = p.(1 - e^{-\lambda.\frac{N_0.W}{P}.(2^{R/W}-1)}) \tag{16}$$

IV. AWGN Multiple Access Fading Channel

In this section, we investigate the capacity region of AWGN Multiple Access fading channel with two uncorrelated sources. The received signal is:

$$Y(n) = \sum_{i=1}^{2} h_i(n)X_i(n) + Z(n) \tag{17}$$

where$X_i(n)$ and $h_i(n)$ are transmitted signal and frequency non-selective fading coefficient for the i-th user and $Z(n)$ is Gaussian random variable with zero mean and variance N_0. The fading processes are presumed to be jointly stationary and ergodic [10].

In this case, according to [10][11] and equation (4), the ergodic capacity region is set of rate vectors satisfying:

$$R_1 < E_{h_1,b}[Wb(t) \log_2(1 + \frac{P_1.|h_1|^2}{N_0.W})] \tag{18}$$

$$R_2 < E_{h_2,b}[Wb(t) \log_2(1 + \frac{P_2.|h_2|^2}{N_0.W})] \tag{19}$$

$$R_1 + R_2 < E_{h_1,h_2,b}[Wb(t) \log_2(1 + \frac{P_1.|h_1|^2 + P_2.|h_2|^2}{N_0.W})] \tag{20}$$

where $|h_1|^2$ and $|h_2|^2$ have the exponential distribution with parameter $\lambda 1$ and $\lambda 2$, respectively. In order to simplify solving the equation (20), we assumed that $\frac{P_1}{N_0.W} = \frac{P_2}{N_0.W} = 1$.

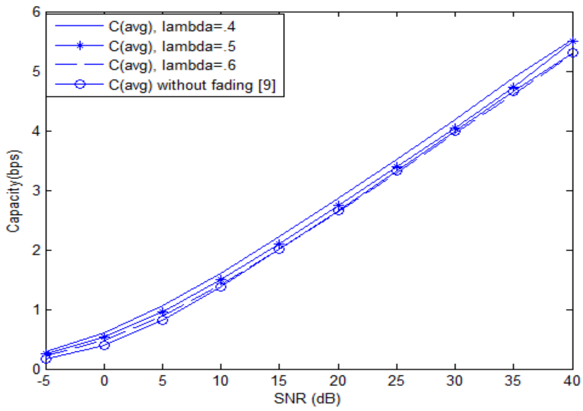

Fig. 2. Ergodic capacity of Gaussian Point-to-Point channel with different Rayleigh fading parameters

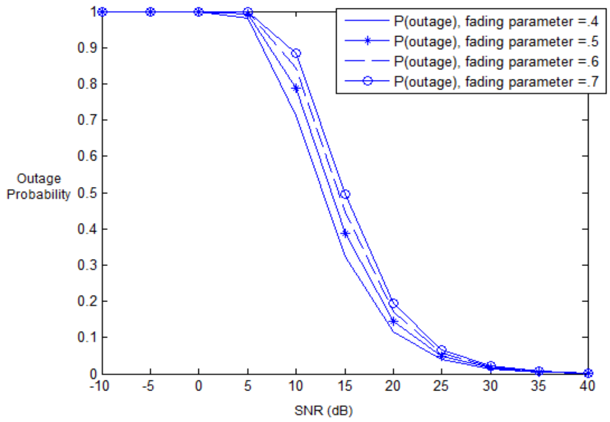

Fig. 3. Outage Probability of Gaussian Point-to-Point channel with different Rayleigh fading parameters

In a similar way to (11), by expectation over random variables, equation (18) and (19) can be written as:

$$R_1 < \frac{pWe^{\lambda_1}}{ln2}(-\gamma - \ln(\lambda_1) - \sum_{n=1}^{+\infty}\frac{(-\lambda_1)^n}{n.n!})$$
$$R_2 < \frac{pWe^{\lambda_2}}{ln2}(-\gamma - \ln(\lambda_2) - \sum_{n=1}^{+\infty}\frac{(-\lambda_2)^n}{n.n!}) \quad (21)$$

To derive a mathematical closed form of equation (20):

$$R_1 + R_2 < E_{h_1,h_2}[pW\log_2(1 + \frac{P_1.|h_1|^2 + P_2.|h_2|^2}{N_0.W})] \quad (22)$$

where $\frac{P_1}{N_0.W} = \frac{P_2}{N_0.W} = 1$. Therefore:

$$R_1 + R_2 < \int_0^{h_1}\int_0^{h_2} pW\log_2(1 + x_1 + x_2)\lambda_1 e^{-\lambda_1 x_1} \lambda_2 e^{-\lambda_2 x_2} dx_1 dx_2 \quad (23)$$

Solving the equation above, we have (see Appendix 2):

$$R_1 + R_2 < e^{\lambda_2}E_1(\lambda_2)(1 + \frac{\lambda_2}{\lambda_2 - \lambda_1}) - \frac{\lambda_2 e^{\lambda 2}}{\lambda_2 - \lambda_1}(+2E_1(\lambda_2 - \lambda_1) + e^{-(\lambda_2 - \lambda_1)}(+2\gamma + 2\ln(\lambda_1) + E_1(\lambda_1))) \quad (24)$$

V. Numerical Results

Here we present the numerical results to validate our theoretical claims. These results are obtained through Monte Carlo simulations. Figure (2) illustrates the ergodic capacity of Gaussian Point-to-Point channel with different Rayleigh fading parameters. We also compared our result with the deterministic case in [9]. It is obvious that capacity decreases as the fading parameter increases. As shown in (2), a capacity gain is achieved in Rayleigh fading environment compared to deterministic case in [9]. In this simulation, the probability of $b(t) = 1$ is fixed and equals 0.4.

In the case of AWGN Gaussian point-to-point channel, the outage probability with different fading coefficients is presented in Figure (3). As expected, the outage probability tends to rise as the fading parameter increases.

In order to have a better understanding of equation (24), we plotted this equation for fixed value of λ_1 and different values of λ_2 in Figure (4). As it is shown, the sum-rate rises as the difference between λ_1 and λ_2 increases.

VI. Conclusion

Due to the increase of wireless applications and more importantly inflexible spectrum management policy, the spectrum has become scarce over the years. Cognitive Radio and dynamic spectrum sharing were introduced as promising solutions; the efficiency of spectrum utilization increases by allowing secondary users to access

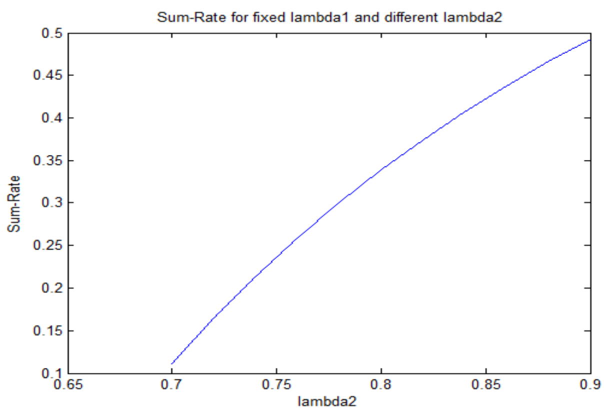

Fig. 4. Sum-Rate for fixed λ_1 and different values of λ_2

primary users' bandwidth whenever they are inactive. Building upon this argument, we investigated the capacity of secondary users in Gaussian Point-to-Point channel and capacity region of Gaussian Multiple Access channel with Rayleigh fading. The available bandwidth of secondary users was modeled as a random process and the ergodic capacity was derived in closed form expression by expectation over random variables. In the case of Gaussian Point-to-Point channels, the simulation results indicated that a gain is achieved if the channels are varying due to the fading.

VII. Appendix 1

In this section, we aim to solve the equation (10) and obtain the ergodic capacity of secondary users in Gaussian Point-to-Point channel.

$$\begin{aligned} C_{avg} &= \int_0^{+\infty} pW \log_2(1+kx)\lambda e^{-\lambda x}dx \\ &= \int_0^{+\infty} \frac{p.W}{ln2}\ln(1+kx)\lambda e^{-\lambda x}dx \\ k &= \frac{P_1}{N_0.W} \end{aligned} \tag{25}$$

By substituting $1+kx$ with u we have:

$$C_{avg} = \frac{p.W.e^{\lambda/k}}{ln2}\int_1^{+\infty}\ln(u)\frac{\lambda}{k}e^{-\frac{\lambda}{k}u}du \tag{26}$$

Now we proceed using integration by parts:

$$C_{avg} = \frac{p.W.e^{\lambda/k}}{ln2}(-\ln(u)e^{-\frac{\lambda}{k}u}|_1^{+\infty} + \int_1^{+\infty}\frac{e^{-\frac{\lambda}{k}u}}{u}du) \tag{27}$$

As we know, the second term is called Exponential Integral and is defined as:

$$\begin{aligned} Ei(x) &= -\int_{-x}^{+\infty}\frac{e^{-t}}{t}dt \\ &= \gamma + \ln|x| + \sum_{k=1}^{+\infty}\frac{x^k}{k.k!} \end{aligned} \tag{28}$$

$$E_1(x) = -Ei(-x) \tag{29}$$

where γ is the Euler Mascheroni constant.Therefore, we have:

$$C_{avg} = \frac{pWe^{\lambda/k}}{ln2}E_1(\lambda/k); \tag{30}$$

VIII. Appendix 2

In this section, we aim to solve the equation (23).

$$\begin{aligned} R_1+R_2 < &\int_0^{+\infty}\int_0^{+\infty}[pW\log_2(1+x_1+x_2)\lambda_1 e^{-\lambda_1 x_1} \\ &\lambda_2 e^{-\lambda_2 x_2}]dx_1dx_2 \\ &= \frac{pW}{\ln(2)}\int_0^{+\infty}\int_0^{+\infty}[\ln(1+x_1+x_2)\lambda_1 e^{-\lambda_1 x_1} \\ &\lambda_2 e^{-\lambda_2 x_2}]dx_1dx_2 \end{aligned} \tag{31}$$

Without loss of generality, we assume that $\lambda_2 > \lambda_1$. By substituting $1+x_1+x_2 = u$ in the integral:

$$\begin{aligned} &\int_0^{+\infty}\int_0^{+\infty}\ln(1+x_1+x_2)\lambda_1 e^{-\lambda_1 x_1}\lambda_2 e^{-\lambda_2 x_2}dx_1dx_2 \\ &= \int_0^{+\infty}\lambda_2 e^{-\lambda_2 x_2}\int_{1+x_2}^{+\infty}\ln(u)\lambda_1 e^{-\lambda_1(u-1-x_2)}dudx_2 \\ &= \int_0^{+\infty}\lambda_2 e^{x_2(\lambda_1-\lambda_2)}e^{\lambda_1}\int_{1+x_2}^{+\infty}\ln(u)\lambda_1 e^{-\lambda_1 u}dudx_2 \end{aligned} \tag{32}$$

The inner integral, similarly to the previous section, is solved as:

$$\begin{aligned} &\int_{1+x_2}^{+\infty}\ln(u)\lambda_1 e^{-\lambda_1 u}du \\ &= \ln(1+x_2)e^{-\lambda_1(1+x_2)} - \ln(\lambda_1(1+x_2)) \\ &-\sum_{n=1}^{+\infty}\frac{(-\lambda_1)^n(1+x_2)^n}{n.n!} - \gamma \end{aligned} \tag{33}$$

Substituting $1+x_2$ with u, we proceed to solve the equation (32):

$$\begin{aligned} &\int_0^{+\infty}\lambda_2 e^{x_2(\lambda_1-\lambda_2)}e_1^{\lambda}\int_{1+x_2}^{+\infty}\ln(u)\lambda_1 e^{-\lambda_1 u}dudx_2 \\ &= \int_1^{+\infty}\lambda_2 e^{\lambda_2}e^{-u(\lambda_2-\lambda_1)}(\ln(u)e^{-\lambda_1(u)} - \ln(\lambda_1(u)) \\ &-\sum_{n=1}^{+\infty}\frac{(-\lambda_1)^n u^n}{n.n!} - \gamma) \end{aligned} \tag{34}$$

Furthermore, we intend to solve each integral.

$$\begin{aligned} I_1 &= \int_1^{+\infty}\lambda_2 e^{\lambda_2}e^{-u(\lambda_2-\lambda_1)}\ln(u)e^{-\lambda_1 u}du \\ &= \int_1^{+\infty}\lambda_2 e^{\lambda_2}e^{-u\lambda_2}\ln(u)du \\ &= e^{\lambda_2}(-\gamma - \ln(\lambda_2) - \sum_{n=1}^{+\infty}\frac{(-\lambda_2)^n}{n.n!}) \\ &= e^{\lambda_2}E_1(\lambda_2) \end{aligned} \tag{35}$$

$$
\begin{aligned}
I_2 =& \int_1^{+\infty} \lambda_2 e^{\lambda_2} e^{-u(\lambda_2-\lambda_1)} \ln(\lambda_1 u) du \\
=& \lambda_2 e^{\lambda_2} (-\frac{e^{-u(\lambda_2-\lambda_1)}}{\lambda_2-\lambda_1} \ln \lambda_1 u|_1^{+\infty} + \int_1^{+\infty} \frac{e^{-u(\lambda_2-\lambda_1)}}{(\lambda_2-\lambda_1)u} du) \\
=& \frac{\lambda_2 e^{\lambda 2}}{\lambda_2-\lambda_1} (-\gamma - \ln(\lambda_2-\lambda_1) - \sum_{n=1}^{+\infty} \frac{-(\lambda_2-\lambda_1)^n}{n.n!} \\
& + e^{-(\lambda_2-\lambda_1)} \ln(\lambda_1)) \\
=& \frac{\lambda_2 e^{\lambda 2}}{\lambda_2-\lambda_1} (E_1(\lambda_2-\lambda_1) + e^{-(\lambda_2-\lambda_1)} \ln(\lambda_1))
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
I_3 =& \int_1^{+\infty} \lambda_2 e^{\lambda_2} e^{-u(\lambda_2-\lambda_1)} \sum_{n=1}^{+\infty} \frac{(-\lambda_1)^n u^n}{n.n!} du \\
=& \frac{\lambda_2 e^{\lambda 2}}{\lambda_2-\lambda_1} (e^{-(\lambda_2-\lambda_1)} (\gamma + E_1(\lambda_1) + \ln(\lambda_1)) \\
& + E_1(\lambda_2-\lambda_1) - E_1(\lambda_2))
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
I_4 =& \int_1^{+\infty} \lambda_2 e^{\lambda_2} e^{-u(\lambda_2-\lambda_1)} \gamma \\
=& \frac{\gamma \lambda_2 e^{\lambda 2}}{\lambda_2-\lambda_1} e^{-(\lambda_2-\lambda_1)}
\end{aligned} \tag{38}
$$

Now, we substitute equations (35, 36, 37, 38) in equatiosn (34) and (31).

$$
\begin{aligned}
R_1 + R_2 <& e^{\lambda_2} E_1(\lambda_2)(1 + \frac{\lambda_2}{\lambda_2-\lambda_1}) - \frac{\lambda_2 e^{\lambda 2}}{\lambda_2-\lambda_1} (+2E_1(\lambda_2-\lambda_1) \\
& + e^{-(\lambda_2-\lambda_1)} (+2\gamma + 2\ln(\lambda_1) + E_1(\lambda_1)))
\end{aligned} \tag{39}
$$

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