Task-space Feedback Control of Robot Manipulator with Uncertain Jacobian Matrix, Via Robust Adaptive Fuzzy Sliding Mode Control

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***Abstract*—In most of the researches that have been done in the position control of robot manipulator, the assumption is that robot manipulator kinematic or robot Jacobian matrix turns out from the joint-space to the task-space. Despite the fact that none of the existing physical parameters in the equations of the robot manipulator cannot be calculated with high precision. In addition, when the robot manipulator picks up an object, uncertainties occur in length, direction and contact point of the end-effector with it. So, it follows that the robot manipulator kinematic is also has the uncertainty and for the various operations that the robot manipulator is responsible, its kinematics be changed too, certainly. To overcome these uncertainties, in this paper, a simple adaptive fuzzy sliding mode control has been presented for tracking the position of the robot manipulator end-effector, in the presence of uncertainties in dynamics, kinematics and Jacobian matrix of robot manipulator. In the proposed control, bound of existing uncertainties is set online using an adaptive fuzzy approximator and in the end, controller performance happens in a way that the tracking error of the robot manipulator will converge to zero. In the proposed approximator design, unlike conventional methods, single input-single output fuzzy rules have been used. Thus, in the practical implementation of the proposed control, the need for additional sensors is eliminated and calculations volume of control input decreases too. Mathematical proofs show that the proposed control, is global asymptotic stability. To evaluate the performance of the proposed control, in a few steps, simulations are implemented on a two-link elbow robot manipulator. The simulation results show the favorable performance of the proposed control.**

***Index Terms*—adaptive fuzzy sliding mode, uncertain Jacobian matrix, robot manipulator, task-space, chattering, uncertainties.**

I. INTRODUCTION

Many current proposed robot manipulator controllers work according to the information they get from the robot manipulator joints. In these controllers, actual position of the joint is contrasted with the desired values and the error is defined. In position control to compensate for errors in the joints, control laws are exerted to the actuators. In this way, desired trajectory, control inputs and robot performance are determined in joint-space [1, 2]. But, the final objective is the end-effector position control in the task-space in robot manipulators.

In a rigid and very high quality robot manipulators which have precise dynamic equations and operate within a specific area of task-space, joint-space control method has a good efficiency. However, workplace is unfamiliar in most robot manipulator applications. On the other hand, lots of structured and un-structured uncertainties exist in the dynamics and kinematics of the robot manipulator that challenge the controller's performance. This is why in [3] author believe that control in joint-space won't lead to the favorable control of end-effector position in the task-space in such applications. Insomuch in the control method in task-space, information of the end-effector position is used for control design. For this reason, the position error is distinctive in the task-space and the controller converges this error to zero.

Precise measurement of location and orientation of the end-effector of robot manipulator is necessary in the controller design in task-space. Although is not simple as the joint-space, exact measurement of the variables of the task-space needs for complex sensing techniques such as visual servoing [4-8], lazer [9, 10] and ultrasonic [11]. The inverse kinematics moot point is dislocated with the Jacobian matrix transpose moot point in the control input when the control moot point is directly designed in the task-space [12]. However, to calculate the Jacobian matrix, there must be accessible an accurate knowledge of the values of kinematic parameters of the robot manipulator in these circumstances. In other words, the given presuppositions of kinematics and Jacobian matrix should be fixed to design the controller position in task-space. The designer should also have adequate information of upper bound of present structured and un-structured uncertainties [13].

Till now, different adaptive Jacobian controllers for controlling the robot manipulator have been provided by researchers in task-space [14, 15]. Assuming of being given kinematics and Jacobian matrix has been resolved in these methods. But, in designing these controllers, only parametric uncertainties have been included. While being un-structured uncertainties such as friction, disturbance and un-modeled dynamics can make closed-loop system unstable. To overcome

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these problems, researchers have used nonlinear robust control techniques to control the robot manipulator in task-space [16]. One of the basic problems in designing these types of controllers is the necessary for proper and rational choice of the range of uncertainties. Tracking error mounts and makes closed-loop system unstable, if the values of control input coefficients are selected smaller than the range of uncertainties. And if the values of control input coefficients are selected greater than the range of uncertainties, this causes an increase in the control input amplitude, saturation of the actuators and the occurrence of unpleasant chattering phenomenon in control input signal.

Lately, the theory of variable structure control as a robust control of the robot manipulator has been received a big attention [17-20]. Sliding mode control is one of the most widely used methods of variable structure control. This control method has a lot of benefits such as simple design, robustness against the uncertainties and the assurance of the closed-loop system stability. But, out breaking chattering in control signal is inevitable due to the use of switching in the control law. The undesirable effects of the phenomenon of control input chattering are stimulation of un-modeled dynamics, vibration of mechanical parts and the difficulty of enforcement of control law [21]. Smoothing methods of signal control such as boundary layer methods, fuzzy and adaptive [22-26] have been proposed to dominate control input chattering.

In late years, the adaptive fuzzy sliding mode control (AFSMC) has been proposed for controlling robot manipulators through combining sliding mode control, fuzzy logic theory and adaptive control concepts [27]. The adaptation laws are designed based on Lyapunov stability theory in AFSMC algorithms. AFSMC controllers can be classified into two basic types: indirect AFSMC controllers and direct AFSMC controllers [28]. Indirect AFSMC controller is used to approximate the parameters of the system's dynamics. In [29] a single input-single output (SISO) fuzzy system is suggested to estimate the unknown functions of a nonlinear system. According to the presented approach in [29], authors in [30] designed an indirect AFSMC method accomplished for controlling industrial robot manipulators. To estimate the dynamic equations of the robot manipulator, the multi input-multi output (MIMO) fuzzy systems are employed in [30]. In the following, the number of fuzzy rules are reduced by determining sliding surfaces as the inputs. In [29] a fuzzy system is applied to replace for the discontinuous control term in the proposed methods to avoid chattering phenomenon effects in [30] and [31]. Indirect AFSMC control approaches usually have some shortcomings. Dynamic equations of system under control are used in design of indirect AFSMC controllers. In this case, it is necessary that an approximation of system parameters should be provided by adaptation laws to estimate the upper bounds of uncertainties. Consequently, computational volume of control input is mounted. Furthermore, MIMO fuzzy rules are applied in design of fuzzy approximators in lots of these approaches. Using MIMO fuzzy rules creates an increase in fuzzy rules, consequently, makes their practical implementation difficult. Because if any delay happens in computation of control input, it is not possible to insure the closed-loop system stability.Direct AFSMC controller is employed to truly regulate the parameters of the control law without estimating the system's dynamics. In [32] authors proposed a MIMO fuzzy system to offset for the uncertainties of a robot model. But unfortunately, employing a MIMO fuzzy system needs a great number of fuzzy rules which causes a high computational load. In [33] authors proposed a SISO fuzzy system to adjust the control gain in the control law for a robot manipulator which both reduced the number of fuzzy rules and attenuated chattering. Differ from proposed method in [33], authors in [34] used a PI controller inside a boundary layer to attenuate chattering and the parameters of this PI controller are online adjusted via adaptation laws. In [35] an AFSMC method having an integral-operation switching surface is designed to offset the bound of the approximation error for electrical servo drives. In [29] two schemes of adaptive SMC methods are used so that the fuzzy logic systems approximate the unknown system functions in designing the SMC of nonlinear system. In [36] authors developed a stable AFSMC controller for nonlinear multivariable systems with inaccessible states. In the above mentioned papers, only the parameters of sequel part of fuzzy rules are approximated for diminishing the computational load of control input. However, the structure of proposed approaches is so that for approximating uncertainties bound accurately and diminishing tracking error, it is essential that fuzzy rules be increased. Increase in fuzzy rules causes increase in adaptation laws, in this case still the computational load of control input rises. Based on the matters exposed to discussion, it is essential that lots of sensors to be used in practical implementation of such controllers are for their fuzzy rules structure.

In [37], authors proposed a direct AFSMC method to online tune for both the premise and sequel parts of fuzzy rules. In this paper, a fuzzy controller and a compensation controller are used to give a control law and the bound of the compensation controller is adjusted by adaptation laws. Since given algorithm in [37] designed only for induction servo motor systems, it is not applicable for robot manipulators. In another study, the authors in [38] suggested a direct AFSMC controller by mixing a PI control, sliding mode control and fuzzy logic. This controller can adapt online the parameters of premise and sequel parts of fuzzy rules. Although proposed control method is applicable on robot manipulators, this controlling technique has many adaptation laws which rises the computations' load. In [39] also, a direct AFSMC controller was proposed for controlling the robot manipulator position. In the proposed control method, an adaptive fuzzy approximator is used to approximate the upper bound of uncertainties. Multi-input and single-output fuzzy rules have been applied in the design process of inference engine of this approximator. Therefore, interaction of joints does not affect the desirable performance of controller in suggested method. That's why, robot with proposed control method has a precise tracking capability. But this precise tracking is accompanied with an increase in a number of rules in rule base of fuzzy approximator. So that, it has 120 fuzzy rules in its fuzzy approximator rule base. In other words, structure of proposed control is designed in a manner that through increasing the number of robot joints, the fuzzy rules are increased. Thus, the proposed control has a numerous calculation load and this control method is not applicable in most of industrial robot manipulators.

Researchers have recently utilized the direct AFSMC control to design type-2 AFSMC control [40]. In this method, two adaptive type-2 fuzzy systems have been used to estimate the unknown functions. Ultimate results of simulation show that the proposed control approach has a desirable performance in prevailing the existing uncertainties and it makes zero the position tracking error converge. The reviews represents that the type-2 fuzzy logic extremely increases the calculation's load of the control input although it is very flexible in prevailing the uncertainties in a robot. Based on this description, there are some disadvantages with the practical implementation of the described controller.

It should be noted that most AFSMC methods that have been so far presented for position control of robot manipulator, were in the joint-space and assuming the accuracy of the Jacobian matrix should be established in all of them. If the robot manipulator to perform the duties is forced to carry a device, being uncertainty in the Jacobian matrix will be inevitable. For this reason, if these controllers are used, accuracy tracking of the end-effector or in other words, carefully tracking in the task-space cannot be guaranteed.

Based on the mentioned items, in this paper, a direct AFSMC controller which has few fuzzy rules is proposed to robust task-space feedback control of robot manipulator. For the sake of reducing computational load of control input, only parameters of sequel part of SISO fuzzy rules are updated in adaptive fuzzy approximator. Hence, this will also lead to decrease in adaptation laws. The proposed control is designed so that in practical implementation of the industrial robot manipulator shouldn't have increase in sensor numbers.

This paper is organized as follows: The joint-space dynamic equations and task-space dynamic equations of a robot manipulator are introduced in sections 2 and 3 respectively. In section 4, in two subsections, the sliding mode controller is designed for robot manipulator in task-space. In the beginning, a sliding mode controller is designed using task-space dynamic equations of the robot manipulator and inverse dynamic approach. Mathematical proof shows that a closed-loop system with this controller has global asymptotic stability. In section 5, a first order TSK fuzzy approximator is designed to eliminate the control input chattering. Despite of the ability that the fuzzy sliding mode control has in restraining the control input chattering, the proposed control has some problems such as failure in approximating the bounds of uncertainties as well as lack of stability proof of the closed-loop system. In section 6, in two subsections, to overcome these problems, a direct robust adaptive fuzzy sliding mode controller is designed. In section 7, a case study on a two-link elbow robot manipulator has been simulated and implemented to demonstrate and compare the efficiency of the proposed controllers in three steps. Finally, section 8 presents the paper's conclusions.

2. JOINT-SPACE DYNAMIC EQUATIONS OF A ROBOT MANIPULATOR

Joint-space dynamic equations of a robot manipulator is a nonlinear, MIMO and second order differential equation which is expressed as follows [41]:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + T_d = u\,, \qquad (1)$$

In expressed relation, $M(q) \in R^{n\times n}$ is the inertia matrix, $V(q,\dot{q}) \in R^{n\times n}$ represents a matrix including sections related to Coriolis and centrifugal forces, $G(q) \in R^n$ stands for the gravitation vector, $T_d \in R^n$ is a vector including disturbances or un-modeled dynamics, $q(t) \in R^n$ is assigned as the vector of joint positions, $\dot{q}(t) \in R^n$ is designated as the vector of joint velocities, $\ddot{q}(t) \in R^n$ is the vector of joint accelerations, and $u \in R^n$ represents the vector of robot manipulator input torque. To simplify equation (1), the following equation is defined:

$$H(q,\dot{q}) = V(q,\dot{q})\dot{q} + G(q) + T_d\,, \qquad (2)$$

With substituting (2) in (1) we obtain:

$$M(q)\ddot{q} + H(q,\dot{q}) = u\ . \qquad (3)$$

Relation (1) has the following specifications:

Specifications 1**:** Inertia matrix $M(q)$ is symmetric and positive-definite.

3. TASK-SPACE DYNAMIC EQUATIONS OF A ROBOT MANIPULATOR

The task-space dynamic equations of a robot manipulator is used to design robust controller in the task-space. For this reason, relation (3) can be simplified as below:

$$\ddot{q} = M^{-1}(q)\big(u - H(q,\dot{q})\big)\,, \qquad (4)$$

In order to achieve the end-effector velocity, the following relation is handled [42]:

$$\dot{X} = J(q)\dot{q}\ , \qquad (5)$$

Wherein, $J(q) \in R^{n\times n}$ is the Jacobian matrix, $\dot{q}(t) \in R^n$ represents the vector of joint velocities, and $\dot{X}(t) \in R^n$ is the velocity vector in the task-space. Differentiating of velocity with respect to time in relation (5), we have:

$$\ddot{X} = J(q)\ddot{q} + \dot{J}(q)\dot{q}\ , \qquad (6)$$

Assumption 1**:** The desired trajectory must be chosen smooth enough, because the being trajectory smooth enough is a condition of existence $\dot{J}(q)$. Suppose that the task-space trajectory is free from singularities, by substituting relation (4) in (6), we have:

$$\ddot{X} = J(q)M^{-1}(q)\big(u - H(q,\dot{q})\big) + \dot{J}(q)\dot{q}\,, \qquad (7)$$

Relation (7) is rewritten as:

$$M(q)J^{-1}(q)\ddot{X} + H(q,\dot{q}) - M(q)J^{-1}(q)\dot{J}(q)\dot{q} = u\ , \qquad (8)$$

$J^{-1}(q)$ is an inverse Jacobian matrix.

Assumption 2**:** It is assumed that the robot is working in a limited task-space such that the Jacobian matrix is full rank.

For transmission of torque-space to force-space, the following relation can be utilized [42]:

$$u = J^T(q)F(t)\,, \qquad (9)$$

In which $J^T(q)$ is Jacobian matrix transpose and $F(t) \in R^n$ is a force vector affecting on the robot end-effector. Relation (9) in (8) is substituted and organized as:

$$J^{-T}(q)M(q)J^{-1}(q)\ddot{X} + J^{-T}(q)H(q,\dot{q}) - J^{-T}(q)M(q)J^{-1}(q)\dot{J}(q)\dot{q} = F(t)\ , \quad (10)$$

Pursuant to the relations (2) and (10), the following relations are described as follows:

$$\begin{cases} M_x(q) = J^{-T}(q)M(q)J^{-1}(q) \\ v_x(q,\dot{q}) = J^{-T}(q)\big(v(q,\dot{q}) - M(q)J^{-1}(q)\dot{J}(q)\dot{q}\big) \\ G_x(q) = J^{-T}(q)G(q) \end{cases}, \quad (11)$$

In expressed relations, similar to the joint-space quantities, $M_x(q) \in R^{n\times n}$ represents the Cartesian mass matrix, $v_x(q,\dot{q}) \in R^{n\times n}$ is a vector of velocity terms in Cartesian space and $G_x(q) \in R^n$ stands for the vector of gravity terms in Cartesian space. $H_x(q,\dot{q})$ is introduced as:

$$H_x(q,\dot{q}) = V_x(q,\dot{q})\dot{q} + G_x(q) + T_{dX}\ , \quad (12)$$

Pursuant to the relations (10) and (12), the task-space dynamic equations of a robot manipulator can be expressed as below:

$$M_x(q)\ddot{X} + H_x(q,\dot{q}) = F(t)\ . \quad (13)$$

In relations (12) and (13), $X(t) \in R^n$ is a proper Cartesian vector representing position and orientation of the robot end-effector [43], $\dot{X}(t) \in R^n$ is the velocity vector of end-effector in Cartesian space, $\ddot{X}(t) \in R^n$ represents the vector of end-effector acceleration in Cartesian space and $T_{dX} \in R^n$ stands for the vector including disturbances or un-modeled dynamics in Cartesian space.

Definition 1**:** Sylvester's law of inertia: If $A \in R^{n\times n}$ is a symmetric square matrix and $C \in R^{n\times n}$ is non-singular matrix, then the number of positive, negative and zero eigenvalues of matrix A and matrix C^TAC are the same, where C^T is the transpose of C [44].

Pursuant to the relation $M_x(q) = J^{-T}(q)D(q)J^{-1}(q)$ and because of the non-singularity of $J^{-1}(q)$ and under consideration the specifications 1 defined in Section 2, utilizing Sylvester's law of inertia, the specifications 2 can be inferred.

Specifications 2**:** Cartesian mass matrix $M_x(q)$ is a positive-definite matrix.

II. 4. DESIGN OF SLIDING MODE CONTROL

A. *4.1. DESIGN OF SLIDING MODE CONTROL (STEP 1)*

In order to design sliding mode control, sliding surface vector is expressed as below [45]:

$$S = \left(\frac{d}{dt} + \lambda\right)^{n-1} e\ , \quad (14)$$

In relation (14), $e = X - X_d$ is the tracking error vector in which $X = [x_1\ x_2\ \dots\ x_n]^T$ represents the vector of end-effector position and $X_d = [x_{1d}\ x_{2d}\ \dots x_{nd}]^T$ stands for the vector of desired trajectory in Cartesian space and $\lambda = diag[\lambda_1, \lambda_2, \dots, \lambda_n]$ represents a diagonal matrix in which $\lambda_1, \lambda_2, \dots, \lambda_n$ are constant and positive coefficients.

In general, in order to design sliding mode controller, the variable $x_r^{(n-1)}$ is expressed as follows:

$$x_r^{(n-1)} = x^{(n-1)} - s\ , \quad (15)$$

Because the industrial robot is introduced through the second order differential equation, relation (15) with $n = 2$ is defined as below:

$$\dot{x}_r = \dot{x} - s\ , \quad (16)$$

With differentiation the relation (16), we have:

$$\ddot{x}_r = \ddot{x} - \dot{s}\ , \quad (17)$$

Point 1**:** Because x, $\dot{x}$, $\ddot{x}$ and S are $n \times 1$ vectors, hence $\dot{x}_r$ and $\ddot{x}_r$ are $n \times 1$ vectors.

For designing sliding mode controller, with considering the relations (16) and (17), relation (13) is rewritten as:

$$M_x(q)\ddot{x}_r + M_x(q)\dot{s} + H_x(q,\dot{q}) = F(t)\ , \quad (18)$$

According to the expressed subjects, the control law is suggested as follows:

$$F(t) = \hat{F}(t) - Ksgn(s)\ , \quad (19)$$

Wherein, $K = \mathrm{diag}[k_1, k_2, \dots, k_n]$ represents a positive-definite diagonal matrix and $\mathrm{sgn}(*)$ stands for the sign function. $\hat{F}(t)$ is chosen as below:

$$\hat{F}(t) = \hat{M}_x(q)\ddot{x}_r + \hat{H}_x(q,\dot{q})\ , \quad (20)$$

In relation (20), $\hat{M}_x(q)$ and $\hat{H}_x(q,\dot{q})$ are estimated values of $M_x(q)$ and $H_x(q,\dot{q})$; respectively.

Point 2**:** $M(q)$, $V(q,\dot{q})$, $G(q)$ and $H(q,\dot{q})$ matrixes have a kinematic and dynamic uncertainties. With regard to the relations (11) and (12), it can be concluded that due to the use of Jacobian matrix for calculating values of $M_x(q)$, $V_x(q,\dot{q})$, $G_x(q)$ and $H_x(q,\dot{q})$ matrixes, these matrixes, in addition to the kinematics and dynamics uncertainties, they also have the uncertainties of Jacobian matrix. Therefore, an accurate estimate of the values of $M_x(q)$, $V_x(q,\dot{q})$, $G_x(q)$ and $H_x(q,\dot{q})$ matrixes cannot be provided **.**For this reason, the values of $\hat{M}_x(q)$, $\hat{V}_x(q,\dot{q})$, $\hat{G}_x(q)$ and $\hat{H}_x(q,\dot{q})$ are defined.

By substitution relations (19) and (20) in (18), we have:

$$M_x(q)\ddot{x}_r + M_x(q)\dot{s} + H_x(q,\dot{q}) = \hat{M}_x(q)\ddot{x}_r + \hat{H}_x(q,\dot{q}) - Ksgn(s)\ , \quad (21)$$

Through simplification of relation (21), we obtain:

$$M_x(q)\dot{s} = \left(\hat{M}_x(q) - M_x(q)\right)\ddot{x}_r + \left(\hat{H}_x(q,\dot{q}) - H_x(q,\dot{q})\right) - Ksgn(s)\ , \quad (22)$$

Because of simplicity of the above relations, $\Delta M_x(q) = \hat{M}_x(q) - M_x(q)$, $\Delta H_x(q,\dot{q}) = \hat{H}_x(q,\dot{q}) - H_x(q,\dot{q})$ and $\Delta f = \Delta M_x(q)\ddot{x}_r + \Delta H_x(q,\dot{q})$ are determined and relation (22) is simplified and can be rewritten as follows:

$$M_x(q)\dot{s} = \Delta M_x(q)\ddot{x}_r + \Delta H_x(q,\dot{q}) - Ksgn(s) = \Delta f - Ksgn(s)\ . \quad (23)$$

Point 3**:** $\Delta f \in R^n$ stands for a vector including all structured and un-structured uncertainties.

In the following, to prove closed-loop system stability of relation (22), according to dynamic characteristics of industrial robot manipulator as noted in previous section, Lyapunov function candidate is suggested as below:

$$V(s) = \frac{1}{2}s^T M_x(q)s\ , \quad (24)$$

The first derivative of relation (24) with respect to time is given as:

$$\dot{V}(s) = s^T M_x(q)\dot{s} + \frac{1}{2}s^T \dot{M}_x(q)s\ , \quad (25)$$

The first derivative of all entries of matrix $M_x(q)$ with respect to time is calculated and $\dot{M}_x(q)$ is determined as follows:

$$\dot{M}_x(q) = \begin{bmatrix} \dot{M}_{11} & \cdots & \dot{M}_{1n} \\ \vdots & \ddots & \vdots \\ \dot{M}_{n1} & \cdots & \dot{M}_{nn} \end{bmatrix}, \quad (26)$$

According to relations (23) and (26), relation (25) is redefined, and for better understanding of the mentioned relation, the relations are shown in format of matrix:

$$\dot{V}(s) = [s_1\ s_2\ \dots\ s_n] \times \left(\begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix} - \begin{bmatrix} k_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & k_n \end{bmatrix} \begin{bmatrix} sgn(s_1) \\ sgn(s_2) \\ \vdots \\ sgn(s_n) \end{bmatrix} \right) + \frac{1}{2}[s_1\ s_2\ \dots\ s_n] \begin{bmatrix} \dot{M}_{11} & \cdots & \dot{M}_{1n} \\ \vdots & \ddots & \vdots \\ \dot{M}_{n1} & \cdots & \dot{M}_{nn} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}, \quad (27)$$

Subsequently, after the simplification of relation (27), in four steps, the following relations can be inferred:

$$\dot{V}(s) = [s_1\ s_2\ \dots\ s_n] \times \left(\begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix} - \begin{bmatrix} k_1 sgn(s_1) \\ k_2 sgn(s_2) \\ \vdots \\ k_n sgn(s_n) \end{bmatrix} \right) + \frac{1}{2}\left[\sum_{i=1}^n s_i\dot{M}_{i1}\ \ \sum_{i=1}^n s_i\dot{M}_{i2}\ \ \dots\ \ \sum_{i=1}^n s_i\dot{M}_{in}\right] \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}, \quad (28)$$

$$\dot{V}(s) = [s_1\ s_2\ \dots\ s_n] \times \begin{bmatrix} \Delta f_1 - k_1 sgn(s_1) \\ \Delta f_2 - k_2 sgn(s_2) \\ \vdots \\ \Delta f_n - k_n sgn(s_n) \end{bmatrix} + \frac{1}{2}\left(\sum_{i=1}^n s_i s_1 \dot{M}_{i1} + \sum_{i=1}^n s_i s_2 \dot{M}_{i2} + \cdots + \sum_{i=1}^n s_i s_n \dot{M}_{in}\right)\ , \quad (29)$$

$$\dot{V}(s) = \sum_{i=1}^n (s_i(\Delta f_i - k_i sgn(s_i))) + \frac{1}{2}\sum_{i=1}^n \sum_{j=1}^n s_i s_j \dot{M}_{ij}\ , \quad (30)$$

$$\dot{V}(s) = \sum_{i=1}^n (s_i(\Delta f_i - k_i sgn(s_i))) + \frac{1}{2}\left(\sum_{i=1}^n {s_i}^2 \dot{M}_{ii} + \sum_{i=1}^{n-1}\sum_{j=i+1}^n s_i s_j (\dot{M}_{ij} + \dot{M}_{ji})\right)\ , \quad (31)$$

In relation (31), s_i represents i^{th} entries of sliding surface vector S, Δf_i demonstrates i^{th} entries of vector Δf, K_i shows i^{th} entries of the main diameter of matrix K; furthermore, $\dot{M}_{ij}$ represents entries in i^{th} rows and j^{th} columns of matrix $\dot{M}_x(q)$. In the following, to prove the closed-loop system stability, relation (31) should be less than zero, this means that:

$$\dot{V}(s) = \sum_{i=1}^n (s_i(\Delta f_i - k_i sgn(s_i))) + \frac{1}{2}\left(\sum_{i=1}^n {s_i}^2 \dot{M}_{ii} + \sum_{i=1}^{n-1}\sum_{j=i+1}^n s_i s_j (\dot{M}_{ij} + \dot{M}_{ji})\right) < 0\ , \quad (32)$$

Given that during the process of controlling the robot manipulator, ${s_i}^2$ is always a positive number, therefore, $\dot{M}_{ii}$ must be negative. On the other, the phrase $s_i s_j$ can be a positive or negative value, therefore, $\dot{M}_{ij}+\dot{M}_{ji}$ must be zero. According to the above explanations to satisfy the relation (32) the following three conditions must be met:

$$\dot{M}_{ii} < 0\ , \quad (33)$$

$$\dot{M}_{ij} + \dot{M}_{ji} = 0\ , \quad (34)$$

$$K_i > \|\Delta f_i\|\ . \quad (35)$$

As it stands, only an adjustment and selection of the control parameter K in the relation (35) attributed to the designer and relations (33) and (34) make the control system uses very limited. And the proposed control system is used only to control robot manipulators so that the relations (33) and (34) be established to the derivative of their entries of inertia matrix. So, although with establishing the above relations and by selecting the appropriate control parameter K, the proposed control system is global asymptotic stability, due to constraints created by the relations (33) and (34), the control system will not be widely used. In the next sub-section, to overcome this problem, the control system will be re-designed.

B. 4.2. RE-DESIGN OF SLIDING MODE CONTROL(STEP 2)

Owning to the relation (18), the control law is redefined and can be rewritten as follows:

$$F(t) = \hat{F}(t) - Ksgn(s) - \beta s \ , \tag{36}$$

In relation (36), $\hat{F}(t)$ is considered according to relation (20) and also $\beta = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1n} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \cdots & \beta_{nn} \end{bmatrix}$ is a positive-definite matrix. Substituting relations (36) and (20) in (18), we obtain:

$$M_x(q)\ddot{x}_r + M_x(q)\dot{s} + H_x(q,\dot{q}) = \widehat{M}_x(q)\ddot{x}_r + \widehat{H}_x(q,\dot{q}) - Ksgn(s) - \beta s \ , \tag{37}$$

By simplifying relation (37), with respect to relations (22) and (23), and point 3, the following relation can be concluded:

$$M_x(q)\dot{s} = \Delta M_x(q)\ddot{x}_r + \Delta H_x(q,\dot{q}) - Ksgn(s) = \Delta f - \beta s - Ksgn(s) \ . \tag{38}$$

To prove the closed-loop system stability of relation (38), due to the relations (24) and (26), relation (25) is rewritten as:

$$\dot{V}(s) = [s_1\ s_2\ \dots\ s_n] \times \left(\begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix} - \begin{bmatrix} \beta_{11} & \cdots & \beta_{1n} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \cdots & \beta_{nn} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} - \begin{bmatrix} k_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & k_n \end{bmatrix} \begin{bmatrix} sgn(s_1) \\ sgn(s_2) \\ \vdots \\ sgn(s_n) \end{bmatrix} \right) + \frac{1}{2}[s_1\ s_2\ \dots\ s_n] \begin{bmatrix} \dot{M}_{11} & \cdots & \dot{M}_{1n} \\ \vdots & \ddots & \vdots \\ \dot{M}_{n1} & \cdots & \dot{M}_{nn} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} \ , \tag{39}$$

After the simplification of relation (39), in three steps, the following relations can be inferred:

$$\dot{V}(s) = [s_1\ s_2\ \dots\ s_n] \times \left(\begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{n} s_i\beta_{1i} \\ \sum_{i=1}^{n} s_i\beta_{2i} \\ \vdots \\ \sum_{i=1}^{n} s_i\beta_{ni} \end{bmatrix} - \begin{bmatrix} k_1 sgn(s_1) \\ k_2 sgn(s_2) \\ \vdots \\ k_n sgn(s_n) \end{bmatrix} \right) + \frac{1}{2}\left[\sum_{i=1}^{n} s_i\dot{M}_{i1} \quad \sum_{i=1}^{n} s_i\dot{M}_{i2} \quad \dots \quad \sum_{i=1}^{n} s_i\dot{M}_{in}\right] \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} \ , \tag{40}$$

$$\dot{V}(s) = [s_1\ s_2\ \dots\ s_n] \times \begin{bmatrix} \Delta f_1 - \sum_{i=1}^{n} s_i\beta_{1i} - k_1 sgn(s_1) \\ \Delta f_2 - \sum_{i=1}^{n} s_i\beta_{2i} - k_2 sgn(s_2) \\ \vdots \\ \Delta f_n - \sum_{i=1}^{n} s_i\beta_{ni} - k_n sgn(s_n) \end{bmatrix} + \frac{1}{2}\left(\sum_{i=1}^{n} s_i s_1\dot{M}_{i1} + \sum_{i=1}^{n} s_i s_2\dot{M}_{i2} + \cdots + \sum_{i=1}^{n} s_i s_n\dot{M}_{in}\right) \ , \tag{41}$$

$$\dot{V}(s) = \sum_{i=1}^{n}(s_i(\Delta f_i - k_i sgn(s_i))) - \sum_{i=1}^{n}\sum_{j=1}^{n} s_i s_j\beta_{ij} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} s_i s_j\dot{M}_{ij} \ , \tag{42}$$

In relation (42), β_{ij} represents entries in ith rows and jth columns of matrix β. To prove the stability of closed-loop system, relation (42) should be less than zero, in the sense that:

$$\dot{V}(s) = \sum_{i=1}^{n}(s_i(\Delta f_i - k_i sgn(s_i))) - \sum_{i=1}^{n}\sum_{j=1}^{n} s_i s_j\beta_{ij} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} s_i s_j\dot{M}_{ij} < 0 \ , \tag{43}$$

The relation (43) is satisfied only in the case that:

$$K_i > \|\Delta f_i\| \ , \tag{44}$$

$$\|\beta_{ij}\| > \left\|\frac{\dot{M}_{ij}}{2}\right\| \ . \tag{45}$$

Therefore by choosing appropriate K which satisfies relation (44) and as well as by choosing appropriate β which satisfies relation (45), the closed-loop system will possesses the global asymptotic stability.

Though the closed-loop system with the sliding mode control has a global asymptotic stability in the presence of the all existing uncertainties, the incidence of the undesirable chattering phenomenon in the control input is inevitable due to the use of the sgn(*) function in the control input. Thus, the practical implementation of this controller is difficult. That's why in the next section of the paper, a fuzzy approximator using fuzzy logic is designed to overcome the existing problems. This fuzzy approximator smooths the control input signal and prevents the occurrence of undesirable chattering phenomenon

III. 5. DESIGN OF FUZZY SLIDING MODE CONTROL

First-order fuzzy Takagi- Sugeno- Kang (TSK) system is defined via fuzzy if-then rules which demonstrate the relations between inputs and outputs. In general, first-order fuzzy Takagi- Sugeno- Kang control system rules are expressed as follows:

$$if\ x_1\ is\ A_1^i\ and \dots and\ x_n\ is\ A_n^i\ then\ y^i = a_0^i + a_1^i x_1 + \cdots + a_n^i x_n \ , \tag{46}$$

Wherein $i = 1,2,\dots,M$ and M represents the number of fuzzy rules. y^i's are the output of these M fuzzy rules and $a_0^i, a_1^i, \dots, a_n^i$ are constant coefficients. In order to design sliding mode controller, relation (36) could be defined as [12]:

$$\begin{cases} F_p = \hat{F} + K - \beta s & , \ s < 0 \\ F_n = \hat{F} - K - \beta s & , \ s > 0 \end{cases} , \quad (47)$$

Owing to the relation (47), controller fuzzy rules could be expressed as below:

$$\begin{cases} if\ s\ is\ A_1^1\ and\ F_p\ is\ A_2^1\ and\ F_n\ is\ A_3^1\ \ then \\ y^1 = a_0^1 + a_1^1 s + a_2^1 F_p + a_3^1 F_n \\ \\ if\ s\ is\ A_1^2\ and\ F_p\ is\ A_2^2\ and\ F_n\ is\ A_3^2\ \ then \\ y^2 = a_0^2 + a_1^2 s + a_2^2 F_p + a_3^2 F_n \end{cases} , \quad (48)$$

In the abovementioned relation, $a_0^1 = a_0^2 = a_1^1 = a_2^2 = a_1^2 = a_3^1 = 0$ and $a_2^1 = a_3^2 = 1$ and membership functions will be written as:

$$A_1^1 = \begin{cases} 1 & , \ s \le -0.5 \\ 1 - 2(s + 0.5)^2 & , \ -0.5 \le s \le 0 \\ 2(s - 0.5)^2 & , \ 0 \le s \le 0.5 \\ 0 & , \ s \ge 0.5 \end{cases} , \quad (49)$$

$$A_1^2 = \begin{cases} 0 & , \ s \le -0.5 \\ 2(s + 0.5)^2 & , \ -0.5 \le s \le 0 \\ 1 - 2(s - 0.5)^2 & , \ 0 \le s \le 0.5 \\ 1 & , \ s \ge 0.5 \end{cases} , \quad (50)$$

$$A_2^1 = A_2^2 = 1 \ \ ; \\ lower\ bound\ of\ F \le F_p \le upper\ bound\ of\ F \ , \quad (51)$$

$$A_3^1 = A_3^2 = 1 \ \ ; \\ lower\ bound\ of\ F \le F_n \le upper\ bound\ of\ F \ , \quad (52)$$

Point 4: Designers need to have access to the information of dynamic equations of robot manipulator for designing the robot manipulator controller. Under these conditions, the uncertainties bound of the dynamic equations of robot is specified. Accordingly, because of the favorable performance of robot manipulator, the bound of applied force to end-effector is specified.

If we assume that $x = [s, F_p, F_n]^T$ be input vector of fuzzy system, its output will be calculated according to the combination of fuzzy rules (48) and is defined as below:

$$y = \frac{f^1(x)y^1(x) + f^2(x)y^2(x)}{f^1(x) + f^2(x)} , \quad (53)$$

In the abovementioned relation, $f^1(x)$ and $f^2(x)$ are the firing strengths of the 1th and 2th rules; respectively, which is gained based on the following relation:

$$\begin{cases} f^1(x) = \mu_{A_1^1}(x_1) * \mu_{A_2^1}(x_2) * \mu_{A_3^1}(x_3) \\ f^2(x) = \mu_{A_1^2}(x_1) * \mu_{A_2^2}(x_2) * \mu_{A_3^2}(x_3) \end{cases} . \quad (54)$$

The mark of " $*$ " is the indicative of a t-norm and $\mu_{A_j^i}(x_j)$ indicates the membership degree of the input x_j in the membership function A_j^i from the i[th] rule (for i = 1,2 and j = 1,2,3).

The suggested fuzzy system prevents the sudden changes in the control input and as a result prevents chattering phenomenon. Nevertheless, the presented fuzzy approximator has the following disadvantages:

1. The fuzzy rules of presented approximator is three inputs-one output, so more sensors should be used for practical implementation of the proposed approximator. This increases economic costs of the practical implementation of the controller.
2. In proposed approximator, membership functions in the fuzzy rules should be set based on trial and error to reduce the approximation error. This approach, however, is possible but it is very time consuming.
3. Structure of the proposed fuzzy approximator is in such a way that it cannot approximate the bound of uncertainties. So, the technique of increasing the available coefficient in the control input must be used to overcome the existing uncertainties. As a result, it causes increasing the amplitude of control input and saturation of robot manipulator actuators.
4. The proposed control lacks closed-loop system stability.

In the next section of the paper, an adaptive fuzzy approximator is designed in a way that does not have the above problems.

IV. 6. DESIGN OF ADAPTIVE FUZZY SLIDING MODE CONTROL

A. 6.1. DESIGN OF ADAPTIVE FUZZY SLIDING MODE CONTROL (STEP 1)

Based on the relations (36) and (38), certainly this issue can be inferred that the reason for the incidence of chattering phenomenon in conventional classic sliding mode control rooted in the existence of the constant coefficient K and the Sign function. Nevertheless, suppose that the control gain $Ksgn(s)$ is replaced by a fuzzy gain ρ. Then, the new control input could be defined as follows:

$$F(t) = \hat{F}(t) - \rho - \beta s \ . \quad (55)$$

An adaptive law is designed owning to the warranty that ρ can compensating the disadvantages caused by system uncertainty. It is obvious that via these analyses the value of ρ can be specified by the value of the sliding surface s. Nevertheless, the fuzzy system for ρ should be a SISO system, with s as the input and ρ as the output variable. The rules in the rule base are in the following determined model:

$$if\ s\ is\ A_i^m\ then\ \rho\ is\ B_i^m \ , \quad (56)$$

Wherein A_i^m and B_i^m are fuzzy sets. In this paper, the same type of membership functions, i.e. NB, NM, NS, ZE, PS, PM, PB are chosen for both s and ρ where, N stands for negative, P positive, B big, M medium, S small and ZE zero. With respect to Fig.1, these are all Gaussian membership functions defined by considering to the following relation:

$$\mu_A(x_i) = exp[-\left(\frac{x_i - \gamma}{\zeta}\right)^2] , \quad (57)$$

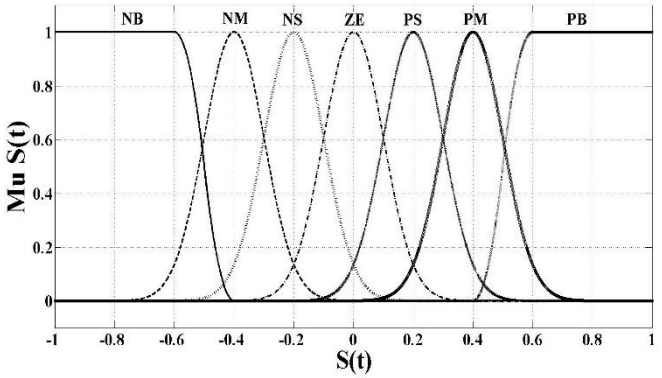


Fig.1. The membership functions of the rule base of the fuzzy inference engine.

Wherein, ''A'' stands for one of the fuzzy sets NB, ... , PB and x_i indicates s or ρ. γ represents the center of ''A'' and ζ is the width of ''A''. Despite the fact that the membership functions for s and ρ have the similar titles, proportionally, the values of the center and the width of the membership function with a similar title for s and ρ are different respectively. The parameters of the membership functions of ρ are updated online insomuch, those of s have predefined quantities. Hence, the controller is an adaptive controller.

Based on the definitions of the input and output membership functions and according to the expressed concepts, the rule base could be defined as follows:

$$\begin{aligned} &if\ s\ is\ NB\ then\ \rho\ is\ NB\\ &if\ s\ is\ NM\ then\ \rho\ is\ NM\\ &if\ s\ is\ NS\ then\ \rho\ is\ NS\\ &if\ s\ is\ ZE\ then\ \rho\ is\ ZE\\ &if\ s\ is\ PS\ then\ \rho\ is\ PS\\ &if\ s\ is\ PM\ then\ \rho\ is\ PM\\ &if\ s\ is\ PB\ then\ \rho\ is\ PB \quad . \end{aligned} \tag{58}$$

Furthermore, according to our consciousness of fuzzy systems and owning to the relation (53), fuzzy gain ρ is defined and can be rewritten as below:

$$\rho = \frac{\sum_{i=1}^{M} \eta^i \mu_{A^i}(s)}{\sum_{i=1}^{M} \mu_{A^i}(s)} = \eta^T \Psi(s)\,, \tag{59}$$

In which M is the amount of the rules. And as well as, $\eta = [\eta^1, \cdots, \eta^i, \cdots, \eta^M]^T$,$\Psi(s) = [\Psi(s)^1, \cdots, \Psi(s)^i, \cdots, \Psi(s)^M]^T$ and $\Psi(s)^i$ defined as:

$$\Psi(s)^i = \prod_{j=1}^{n} \mu_{A_j^i}(s) / \sum_{i=1}^{M} \prod_{j=1}^{n} \mu_{A_j^i}(s)\,. \tag{60}$$

In relation (59) η is selected as a parameter to be updated and that's why its name is parameter vector. In addition, $\Psi(s)$ is defined as the function basis vector and can be considered as the weight of the parameter vector. Eventually, to sum up the main points of presented concepts, the suggested control input is represented as:

$$\begin{cases} F(t) = \hat{F}(t) - \rho - \beta s \\ \hat{F}(t) = \hat{M}_x(q)\ddot{x}_r + \hat{H}_x(q,\dot{q}) \\ \rho = \eta^T \Psi(s) \end{cases} . \tag{61}$$

An approximator with single input-single output fuzzy rules has been used in designing the proposed control. So, the need for additional sensors is resolved in practical implementation of the proposed control. On the other hand, the presented adaptive fuzzy approximator, at the time of the process of robot manipulator control approximates the bound of existing uncertainties online and prevents an increase in the control input amplitude. However, adjusting the membership functions of this approximator must be continued based on trial and error and the proposed control still lacks the closed-loop system stability. In the next subsection of the paper, the posed problems are solved by reviewing the method of designing the adaptive fuzzy approximator.

B. 6.2. RE-DESIGN OF ADAPTIVE FUZZY SLIDING MODE CONTROL (STEP 2)

In this sub-section, to overcome the problems that were mentioned in sub-section 6.1, according to relations (36), (38) and (55), the new control input could be rewritten as:

$$F(t) = \hat{F}(t) - \Gamma - \rho - \beta s \ . \tag{62}$$

In relation (62), Γ represents a positive constant. The following candidate Lyapunov function is suggested in order to design the adaptive fuzzy controller:

$$V(s) = \tfrac{1}{2} s^T M_x(q) s\,, \tag{63}$$

In relation (56), $V(s)$ is introduced as an indicator of the amount of energy of s. The stability of system is guaranteed via choosing a control law in a way that $\dot{V}(s) < 0$ and $\dot{V}(s) = 0$ only when $s = 0$. A fuzzy gain ρ is used in the adaptive fuzzy sliding mode control to avoid adverse effects of the system uncertainty and reduction of the energy of s. The first derivative of the relation (63) with respect to time is given as:

$$\dot{V}(s) = s^T M_x(q)\dot{s} + \tfrac{1}{2} s^T \dot{M}_x(q) s\,, \tag{64}$$

According to relations (37) to (42), relation (64) is rewritten as below:

$$\begin{aligned} &\dot{V}(s) = \textstyle\sum_{i=1}^{n} (s_i(\Delta f_i - \rho_i - \Gamma_i)) - \\ &\textstyle\sum_{i=1}^{n}\sum_{j=1}^{n} s_i s_j \beta_{ij} + \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} s_i s_j \dot{M}_{ij} \quad , \end{aligned} \tag{65}$$

With respect to relation (65), it can be inferred that $\dot{V}(s) < 0$ only in the case that:

$$\begin{cases} \rho_i < \Delta f_i - \Gamma_i & , \quad s_i < 0 \\ \rho_i > \Delta f_i - \Gamma_i & , \quad s_i > 0 \end{cases} , \tag{66}$$

$$\|\beta_{ij}\| > \left\|\tfrac{\dot{M}_{ij}}{2}\right\| \ . \tag{67}$$

If $\|s_i\|$ is too small, a smaller value of ρ_i can further guarantee the stability of system. And as the same way, if $\|s_i\|$ is too large then, a larger value of ρ_i can further guarantee the stability of closed-loop system with adaptive fuzzy sliding mode control. Eventually, if $s_i = 0$ then, the value of ρ_i could be chosen to be equal to zero.

Based on the details, This theme is analogous to the concept of utilizing the function Sat $(*)$. The difference is that the

control gain is different along with the sliding surface all times. Owning to the descriptions of the input and output membership functions and base on the relations (56) and (57), the rule bases could be defined as below:

$$\begin{array}{l} if\ s\ is\ NB\ then\ \rho\ is\ NB \\ if\ s\ is\ NS\ then\ \rho\ is\ NS \\ if\ s\ is\ ZE\ then\ \rho\ is\ ZE \\ if\ s\ is\ PS\ then\ \rho\ is\ PS \\ if\ s\ is\ PB\ then\ \rho\ is\ PB\ \ . \end{array} \quad (68)$$

Next, according to the relations (53), (59) and (60), define η^* so $\rho = \eta^{*T}\Psi(s)$ is the optimal compensation for Δf. Based on the Wang's theorem [46], there is $\alpha > 0$ which satisfies the below mentioned inequality:

$$\Delta f - \rho = \Delta f - \eta^{*T}\Psi(s) < \alpha\ , \quad (69)$$

According to the noted inequality, α is approximation error and it can be as small as possible. Afterwards, define $\tilde{\eta}$ as a parameter:

$$\tilde{\eta} = \eta - \eta^*\ , \quad (70)$$

Based on the relations (59) and (70) it is inferred that:

$$\rho = \tilde{\eta}^T\Psi(s) + \eta^{*T}\Psi(s)\ . \quad (71)$$

After analyzing all the details of the adaptive fuzzy controller design, the candidate Lyapunov function is modified and the relation (63) is redefined and can be rewritten as below:

$$V(s) = \frac{1}{2}s^T M_x(q)s + \frac{1}{2\delta}\tilde{\eta}^T\tilde{\eta}\ , \quad (72)$$

Wherein, δ represents a constant parameter that is greater than zero. The first derivative of the relation (72) with respect to time is given as:

$$\dot{V}(s) = s^T M_x(q)\dot{s} + \frac{1}{2}s^T\dot{M}_x(q)s + \frac{1}{2\delta}\left(\dot{\tilde{\eta}}^T\tilde{\eta} + \tilde{\eta}^T\dot{\tilde{\eta}}\right), \quad (73)$$

Due to the relation (65), relation (73) is simplified as follows:

$$\dot{V}(s) = s^T[\Delta f - \rho - \Gamma] + s^T s(\frac{\dot{M}}{2} - \beta) + \frac{1}{\delta}\tilde{\eta}^T\dot{\eta}\ , \quad (74)$$

With substituting relation (71) in relation (74), we obtained:

$$\dot{V}(s) = s^T\left[\Delta f - \tilde{\eta}^T\Psi(s) - \eta^{*T}\Psi(s) - \Gamma\right] + s^T s(\frac{\dot{M}}{2} - \beta) + \frac{1}{\delta}\tilde{\eta}^T\dot{\eta}\ , \quad (75)$$

After that, with rearranging the relation (75), we have:

$$\dot{V}(s) = s^T\left[\Delta f - \eta^{*T}\Psi(s) - \Gamma\right] + s^T s(\frac{\dot{M}}{2} - \beta) + \tilde{\eta}^T\left[\frac{1}{\delta}\dot{\eta} - s^T\Psi(s)\right]\ , \quad (76)$$

The adaptive rule with respect to relation (76) could be chosen as:

$$\dot{\eta} = \delta s^T\Psi(s)\ , \quad (77)$$

Due to the structure of the selected adaptive rule, relation (76) is reorganized as below:

$$\dot{V}(s) = s^T\left[\Delta f - \eta^{*T}\Psi(s) - \Gamma\right] + s^T s(\frac{\dot{M}}{2} - \beta)\ , \quad (78)$$

Based on the relations (69) and (78), the following inequality could be inferred:

$$\dot{V}(s) < \|s^T\|(\alpha - \Gamma) + \|s^T\|\|s\|(\frac{\dot{M}}{2} - \beta), \quad (79)$$

With respect to relation (67), relation (79) illustrates that via properly selecting the coefficients Γ and β, $\dot{V}(s) < 0$ is satisfied. According to satisfying the relation (79), the closed-loop system with adaptive fuzzy sliding mode control is globally asymptotically stable in presence of all structured and unstructured uncertainties. Eventually, to sum up the presented concepts, the suggested control input is defined as:

$$\begin{cases} F(t) = \hat{F}(t) - \Gamma - \rho - \beta s \\ \hat{F}(t) = \hat{M}_x(q)\ddot{x}_r + \hat{H}_x(q,\dot{q}) \\ \rho = \eta^T\Psi(s) \\ \dot{\eta} = \delta s^T\Psi(s)\ . \end{cases} \quad (80)$$

V. 7. A CASE STUDY ON THE TWO-LINK ELBOW ROBOT MANIPULATOR

In this section, the robust controllers which have been designed and scrutinized in this paper are conducted on the two-link elbow robot manipulator of Fig.2.

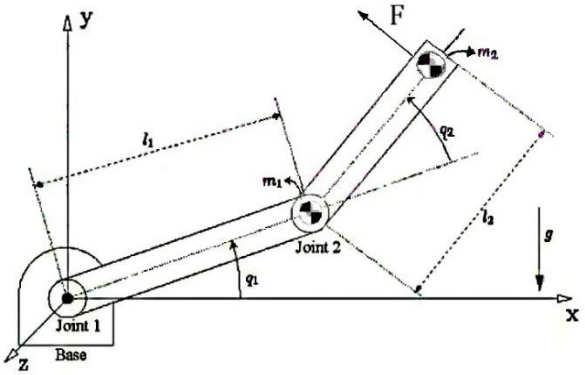


Fig.2. Two-link elbow robot manipulator.

The dynamic equations of this robot are presented as below [41]:

$$M_x(q)\ddot{X} + V_x(q,\dot{q})\dot{q} + G_x(q) + T_{dX} = F(t)\ , \quad (81)$$

In which:

$$M_x(q) = \begin{bmatrix} m_1 + \frac{m_1}{(sinq_2)^2} & 0 \\ 0 & m_2 \end{bmatrix}, \quad (82)$$

$$V_x(q,\dot{q}) = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \quad (83)$$

$$V_{11} = -(m_2L_1(cosq_2) + m_2L_2)\dot{q}_1 - 2m_2L_2 + m_2L_1(cosq_2) + m_1L_1\frac{cosq_2}{(sinq_2)^2})\dot{q}_2\ , \quad (84)$$

$$V_{12} = -m_2L_2\dot{q}_2\ , \quad (85)$$

$$V_{21} = m_2 L_1 (sinq_2)\dot{q}_1 + m_2 L_1 (sinq_2)\dot{q}_2 \ , \quad (86)$$

$$V_{22} = 0 \ , \quad (87)$$

$$G_x(q) = \begin{bmatrix} m_1 g \frac{cosq_1}{sinq_2} + m_2 g (sinq_1)(sinq_2) \\ m_2 g (cosq_1)(cosq_2) \end{bmatrix} , \quad (88)$$

$$T_{dX} = \begin{bmatrix} T_{d_x} \\ T_{d_y} \end{bmatrix} . \quad (89)$$

Point 5: In each robot link, the mass distribution is intended as point particle and the center of mass of each link is considered to be determined at the end of the link.

In abovementioned relations, L_1 is the length of the first link, L_2 represents the length of the second link, m_1 is the mass of the first link, m_2 stands for the mass of the second link, g is the gravity, T_{dx} represents the disturbance or un-modeled dynamic and F is the force applied on the end-effector of robot. The quantities for the robot which are applied in this simulation have been shown in table 1.

Point 6: The quantities $\hat{L}_1$, $\hat{m}_1$, $\hat{L}_2$ and $\hat{m}_2$ are the estimations from the actual quantities of L_1, m_1, L_2 and m_2 which have been used in calculation of $\hat{F}$.

TABLE 1
PARAMETERS OF TWO-LINK ELBOW ROBOT MANIPULATOR

$m_1 = 5kg$	$\hat{m}_1 = 4.5kg$
$L_1 = 1m$	$\hat{L}_1 = 1.1m$
$m_2 = 4kg$	$\hat{m}_2 = 3.5kg$
$L_2 = 0.8m$	$\hat{L}_2 = 0.9kg$
$T_{d_x} = T_{d_y} = 5\sin(t)$	$g = 9.8\,{}^{m}/_{s^2}$

The quantities of controlling parameters in controller (36) which have been applied in this simulation are shown in table 2.

Point 7**:** The quantities k_1 and k_2 are calculated according to relation (44) and also quantities β_{11}, β_{12}, $\beta_{21}$and β_{22} are calculated with respect to relation (45).

TABLE 2
PARAMETERS OF CONTROLLER (36) IN TWO-LINK ELBOW ROBOT MANIPULATOR

k_1=170	k_2=150
λ_1=35	λ_2=70
$\beta_{11} = 80$	$\beta_{12} = 0$
$\beta_{21} = 0$	$\beta_{22} = 80$

The Jacobian matrix of robot manipulator is determined as follows:

$$J(q) = \begin{bmatrix} L_1 sinq_2 & 0 \\ L_1 cosq_2 + L_2 & L_2 \end{bmatrix} . \quad (90)$$

To study the desirable performance of the suggested control, the three-step simulations are carried out.

A. 7.1. STEP 1 OF SIMULATION

In step 1 of simulation, sliding mode control of relation (37) is applied for two-link elbow robot manipulator. Fig.3 shows the desired and actual trajectories in Cartesian space for end-effector.

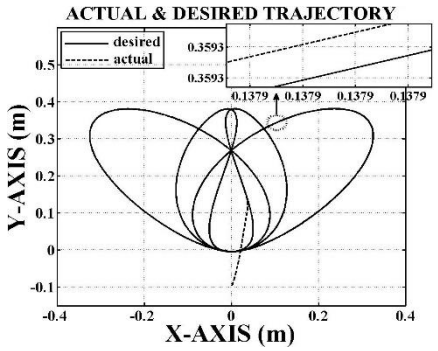


Fig.3. The desired and actual trajectories in Cartesian space for end-effector.

After execution the simulation, tracking errors of the end-effector position in Cartesian space for X and Y axes are illustrated in Fig.4.

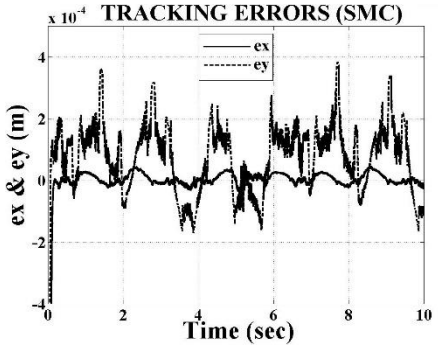
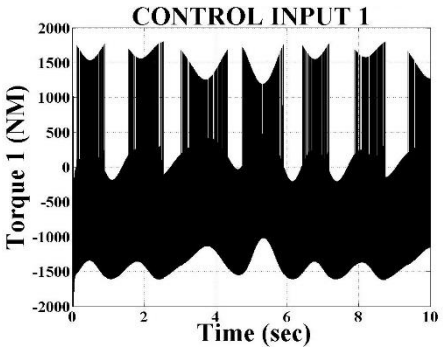
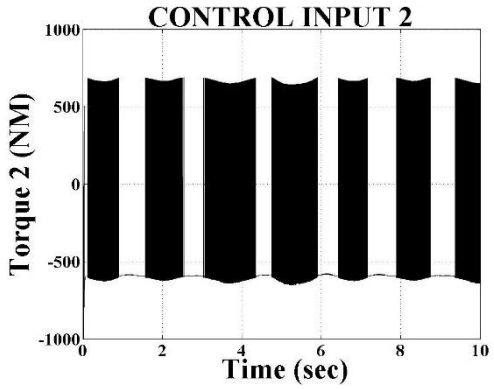


Fig.4. Tracking error of the end-effector position.

According to Fig.3 and 4, it is obvious that the precise tracking on X and Y axes have been occurred, so that the maximum tracking error of the end-effector position is 513×10^{-7} meters for X axis and 383×10^{-6} meters for Y axis. Exerted control input to the joints 1 and 2 are shown in Fig.5.



(a)



(b)

Fig.5. Exerted control inputs to joints 1 and 2, in SMC control, (a) The exerted control input to joint 1, and (b) The exerted control input to joint 2.

As can be seen in Fig.5, the chattering domain of exerted control inputs to joints 1 and 2 are 1814 to 3406 Newton meters and 6 to 1249 Newton meters, respectively. This chattering can lead to the activation of the nonlinear dynamic modes of the two-link elbow robot manipulator and finally causes instability in the control system and damage to the physical structure of the robot manipulator. In step 2 of simulation, to overcome the adverse chattering phenomenon in control inputs, fuzzy sliding mode control input is simulated for two-link elbow robot manipulator.

B. 7.2. STEP 2 OF SIMULATION

After applying fuzzy sliding mode control input and execution of simulation, the tracking error of the end-effector position on X and Y axes have been demonstrated in Fig.6.

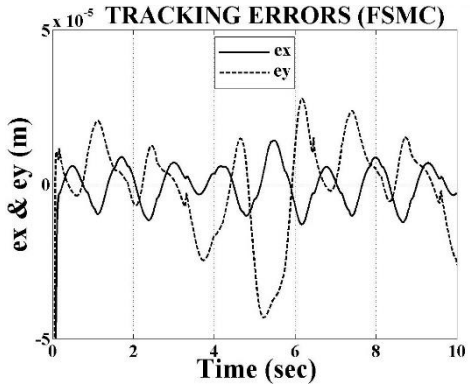
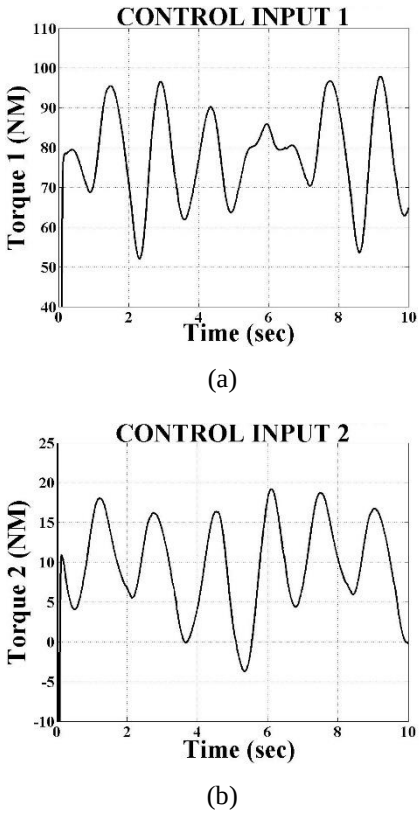


Fig.6. Tracking error of the end-effector position on X and Y axes, in FSMC control.

By comparing Fig.4 and 6, it is obvious that a more precise tracking in comparison with the previous step of simulation is conducted on X and Y axes, such that the maximum tracking error on X and Y axes are 14×10^{-6} meters and 43×10^{-6} meters, respectively. In Fig.7, the applied control inputs to the joints 1 and 2 are displayed.



(a)

(b)

Fig.7. Exerted control inputs to joints 1 and 2, in FSMC control, (a) The exerted control input to joint 1, and (b) The exerted control input to joint 2.

According to Fig.7, it is observed that the control inputs have no chattering. In addition, the maximum control inputs 1 and 2 are 97.87 Newton meters and 19.2 Newton meters, respectively.

The simulation results confirm the desired performance of the fuzzy sliding mode control in position control of robot manipulator. But, as stated in section 5 of the paper, this controller has flaws that increases the economic costs of its practical implementation. On the other, the proposed method lacks stability of closed-loop system and the presented fuzzy approximator does not have the ability to approximate the bound of existing uncertainties. For this reason, the next step of simulation allocates to evaluate the performance of the proposed controllers in section 6 of the paper.

C. 7.3. STEP 3 OF SIMULATION

In this step of simulation, according to the designed adaptive fuzzy sliding mode controllers of section 6, the control inputs of equations (61) and (80), are applied for two-link elbow robot manipulator.

1) 7.3.1. STEP 3-1 OF SIMULATION

In this step, the control input of equation (61) is applied for two-link elbow robot manipulator. After applying adaptive fuzzy sliding mode control input and execution of simulation, the tracking error of the end-effector position on X and Y axes have been indicated in Fig.8.

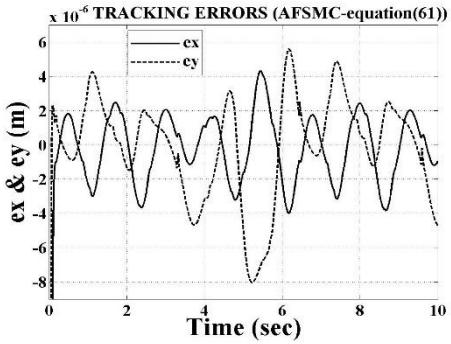
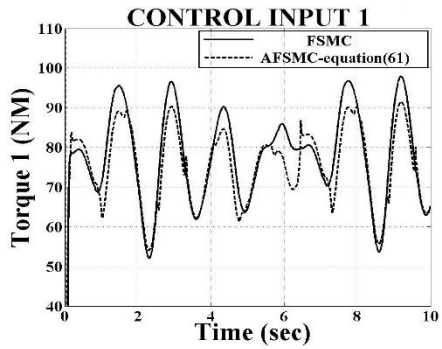
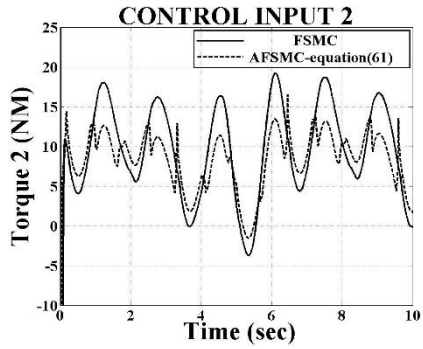


Fig.8. Tracking error of the end-effector position on X and Y axes, in AFSMC control (equation (61)).

By comparing Fig.8 with Fig.4 and 6, the considerable reduction of the tracking error in this step of the simulations is remarkable. The maximum tracking error on X axis is $4.31 \times 10^{-6}\ Rad$ and on Y axis, it is $8.03 \times 10^{-6}\ Rad$. Fig.9 shows exerted control inputs in adaptive fuzzy sliding mode control (equation (61)) compared with fuzzy sliding mode control for joints 1 and 2.



(a)



(b)

Fig.9. Exerted control inputs to joints 1 and 2, in AFSMC control (equation (61)) compared with FSMC control, (a) The exerted control input to joint 1, and (b) The exerted control input to joint 2.

According to Fig.9, it is observed that the adaptive fuzzy sliding mode control inputs 1 and 2 have no chattering and have smaller amplitude compared with fuzzy sliding mode control, in the most time of simulation. The diagram of the variations in fuzzy gains ρ_x and ρ_y versus time is shown in Fig.10.

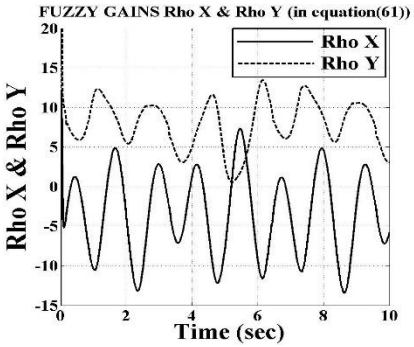


Fig.10. The fuzzy gains ρ_x and ρ_y , in AFSMC control (equation (61)).

According to Fig.10, smooth approximation of the fuzzy gains ρ_x and ρ_y is observable. This suitable approximation shows that the adaptive fuzzy system functions satisfactorily and has specified the bounds of the existing uncertainties.
The simulation results show the favorable performance of the proposed control. By comparing the performance of the fuzzy sliding mode controller and adaptive fuzzy sliding mode controller (61), it follows that in the same working conditions, precision tracking in adaptive fuzzy sliding mode controller is more and control input amplitude of this controller is smaller. In addition, adaptive fuzzy approximator has properly approximated bound of existing uncertainties. However, adaptive fuzzy sliding mode control (61) lacks closed-loop system stability. That's why in the next step of the simulation, the function of adaptive fuzzy control (80) will be examined that in its designing any attempt has been made to resolve this problem.

2) 7.3.2. STEP 3-2 OF SIMULATION

In this step, the control input of equation (80) is applied for two-link elbow robot manipulator. In this step of the simulation, the number of fuzzy rules of adaptive fuzzy approximator (80) was reduced up to 5 numbers, to reduce calculations volume of control input and the membership functions of the Fig.11 was used for designing adaptive fuzzy approximator.

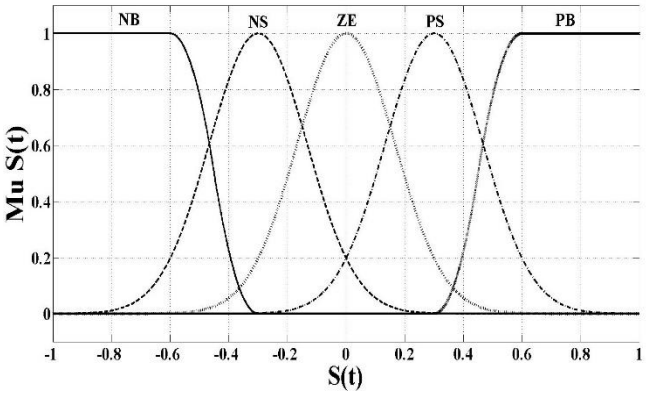


Fig.11. The membership functions of the rule base of the fuzzy inference engine.

After applying adaptive fuzzy sliding mode control input and execution of simulation, the tracking error of the end-effector position on X and Y axes have been indicated in Fig.12.

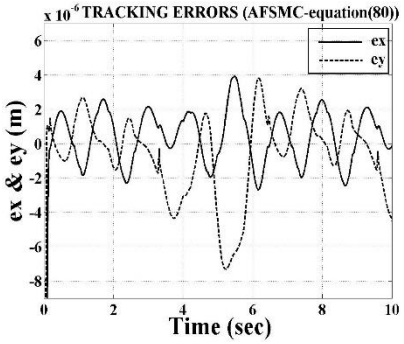
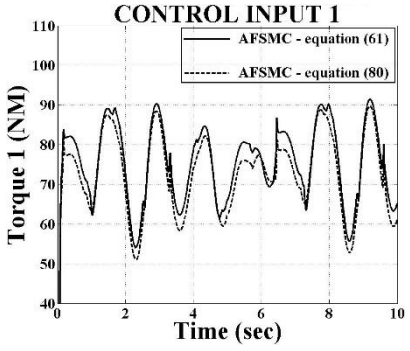
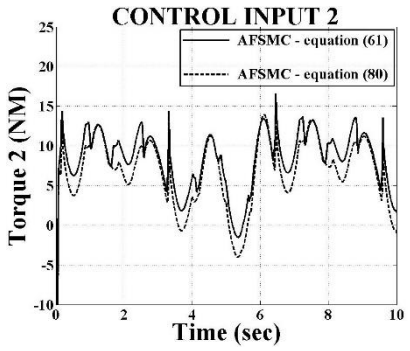


Fig.12. Tracking error of the end-effector position on X and Y axes, in AFSMC control (equation (80)).

By comparing Fig.12 with Fig.8, the negligible reduction of the tracking error in this step of the simulations is visible. The maximum tracking error on X axis is $3.92 \times 10^{-6}\ Rad$ and on Y axis, it is $7.31 \times 10^{-6}\ Rad$. Fig.13 shows exerted control inputs in adaptive fuzzy sliding mode control (equation (80)) compared with adaptive fuzzy sliding mode control (equation (61)) for joints 1 and 2.



(a)



(b)

Fig.13. Exerted control inputs to joints 1 and 2, in AFSMC control (equation (80)) compared with AFSMC control (equation (61)), (a) The exerted control input to joint 1, and (b) The exerted control input to joint 2.

According to Fig.13, it is observed that the adaptive fuzzy sliding mode control inputs 1 and 2 (equation (80)) have no chattering and have smaller amplitude compared with adaptive fuzzy sliding mode control (equation (61)), in the most time of simulation. The diagram of the variations in fuzzy gains ρ_x and ρ_y versus time is shown in Fig.14.

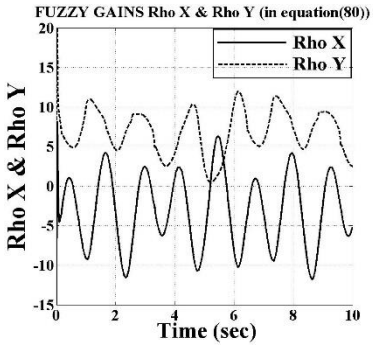


Fig.14. The fuzzy gains ρ_x and ρ_y , in AFSMC control (equation (80)).

According to Fig.14, smooth approximation of the fuzzy gains ρ_x and ρ_y is observable. This suitable approximation shows that the adaptive fuzzy system functions satisfactorily and has specified the bounds of the existing uncertainties.

3) 7.3.3. STEP 3-3 OF SIMULATION

The suggested AFSMC control in [38] has been considered in recent years by robotic researchers. For performance evaluation of the proposed controller, the mentioned referenced controller is implemented on a two-link elbow robot manipulator and the simulation results are compared with the previous step of simulation. After using mentioned controller in [38] and execution of simulation, the tracking error of the end-effector position on X and Y axes have been shown in Fig.15.

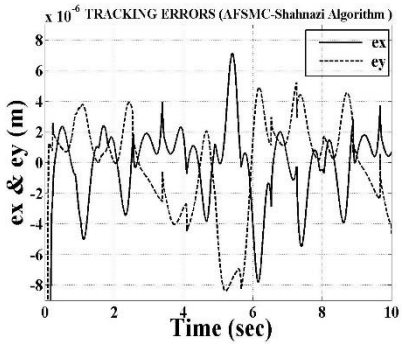
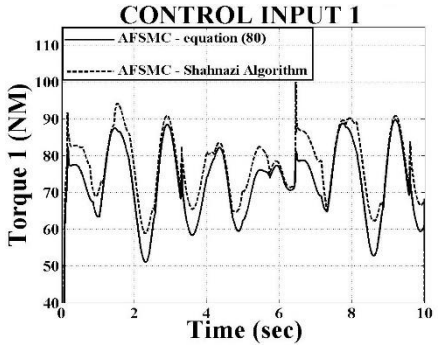
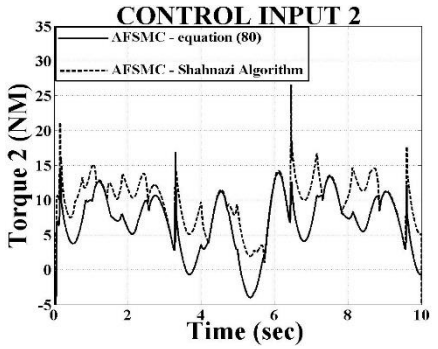


Fig.15. Tracking error of the end-effector position on X and Y axes, in AFSMC control (shahnazi algorithm).

By comparing Fig.15 with Fig.12, the modest increases of the tracking error in this step of the simulations is visible, that's negligible. The maximum tracking error on X axis is $7.82 \times 10^{-6}\ Rad$ and on Y axis, it is $8.39 \times 10^{-6}\ Rad$. Fig.16 displays exerted control inputs in proposed AFSMC compared with AFSMC in [38], for joints 1 and 2.



(a)



(b)

Fig.16. Exerted control inputs to joints 1 and 2, in AFSMC control (equation (80)) compared with AFSMC control (shahnazi algorithm), (a) The exerted control input to joint 1, and (b) The exerted control input to joint 2.

Based on the Fig.16, it is obvious that the shahnazi's AFSMC control inputs 1 and 2 have bigger amplitude compared with proposed AFSMC, in the most time of simulation. Hence, the proper functioning of the suggested controller can be inferred. In total, Along with above mentioned simulation results, by investigating the design approach of abovementioned controllers the following remarks can be pointed out:

1. In Shahnazi's AFSMC control, the combination of PI control, sliding mode control and adaptive fuzzy control were utilized. In the suggested adaptive fuzzy approximator, the parameters of premise and consequence parts of fuzzy rules are updated online. Accordingly, the number of adaptation laws for approximating control input coefficients is increased enormously. As a result, the increase of adaptation laws leads to the increase of computational load of control input.
2. In multi-link robot manipulators, utilization of Shahnazi's AFSMC control will be encountered with high economic cost because of using multiple adaptation laws.
3. In Shahnazi's AFSMC control, to better approximation of uncertainties bound and provide more precision in the tracking of robot manipulator position, designer should increase the fuzzy rules of adaptive fuzzy approximator. The increase of fuzzy rules leads to the increase of adaptation laws which makes the computations and the design process more complex. In the other words, addition of one linguistic variable to premise and consequence parts of fuzzy rules cause the addition of 4 more adaptation laws, consequently.

VI. 8. CONCLUSIONS

In this paper, adaptive fuzzy sliding mode control was presented for position control of robot manipulator in task-space and in the presence of dynamic, kinematic and Jacobian matrix uncertainties. To do this, at first, sliding mode control was designed for position control of robot manipulator in task-space. Mathematical proof shows that closed-loop system in the presence of existing uncertainties has the global asymptotic stability. But because of chattering in the control input, practical implementation of the proposed control is not possible. Then, to resolve this problem, a fuzzy system was designed and added to the controller. Although fuzzy sliding mode control input lacks chattering, the proposed fuzzy approximator has flaws that makes it difficult the possibility of practical implementation. In the following, by changing the structure of the designing fuzzy approximator, an adaptive fuzzy approximator presented in such a way that approximates bound of existing uncertainties and has very low calculations volume. In the design of adaptive fuzzy approximator, tips were considered that don't have fuzzy approximator's problems. But due to lack of the closed-loop system stability, the structure of adaptive fuzzy approximator was changed in such a way that adaptive fuzzy sliding mode control, makes closed-loop system in the presence of dynamic, kinematic and Jacobian matrix uncertainties has global asymptotic stability. To demonstrate the operation of proposed controller, simulations in several steps were implemented on two-link elbow robot manipulator. The simulation results confirm desired performance of the proposed control and the adaptive fuzzy approximator.

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