Robust Stabilization for Fuzzy Network Control Systems with Data Drift

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Abstract

In this study, a robust controller is designed for fuzzy network control systems (NCSs) using the static output feedback. Delay and data packet dropout affect on the stability of network control systems, and therefore, the asymptotic stability condition is established considering delay and data packet dropout. Delay is time-varying while the lower and upper bounds for delay is defined, and the number of data packet dropout is unknown. Data drift is also an important phenomena that may occur when data is transmitted from sensors to the controller and from the controller to actuators. This phenomenon is modeled as a stochastic variable with a probabilistic distribution. For stability analyses, Lyapunov–Krasovskii functions, which depend on the limits of delay and data packet dropout, are used. Results of controller design are derived as Linear Matrix Inequalities (LMIs). A numerical example is adopted to show the effectiveness of the proposed approach.

1. Introduction

In recent years, as well as progress in computer science and communication technology, the application of network systems is growing [1]. Network control systems (NCSs) have been made of sensors, controllers and actuators that are connected via a communication network. The NCSs have many advantages such as low cost for wiring, and the flexibility of operation [2].

When data is transferred from sensors to the controller and from the controller to actuators, this transmission may result in delay and data packet dropout [3]. In the stability analysis, delay and data packet dropout is considered. The value of delay and data packet dropout is time-varying, however, an upper and a lower bound is considered for it.

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The problem of controller design for network control systems are studied in [4-6]. It should be noted that robustness against modeling uncertainty is an essential issue in the theoretical and practical works [7, 8]. In [9], the time varying uncertainty is considered. For the controller design and stability analysis, Lyapunov function method is selected while this function depends on the limits of delays. The result of controller design is derived based on an LMI that depends on the bounds of delay.

Data drift is also an important phenomena, which may occur when data is sent out from sensors or the controller, in the networked control systems. In other words, data drift causes a deviation in the value of data when it is transmitted from sensors to the controller or from the controller to actuators. Accordingly, data drift is a virtual phenomenon that should be considered in the stability analysis and controller design in the network control systems. In [10] fault detection of network control systems considering data drift is addressed. In this reference, data drift phenomena of each sensor have been described by an individual stochastic variable with different probabilistic density functions.

Output and state feedback can be exploited for the control of network systems. For example in [4, 6, 11] the output feedback and in [2, 12] the state feedback is utilized, respectively.

Despite of development in network control systems, the analysis of nonlinear network control systems is still an open field for researchers. One approach to analyze a nonlinear network control system is the use of the Takagi-Sugeno fuzzy models. For the stability analysis of these systems, fuzzy Lyapunov functions are employed. Since the uncertainty is an inseparable part of modeling, a robust fuzzy controller should be designed such that the stability of the system is guaranteed [13, 14].

In this paper, the stability of a nonlinear network control system with time- varying uncertainty is studied. The T-S fuzzy approach is used to model the nonlinear system, and the output feedback is employed to stabilize the system. In the stability analysis, delay, data packet dropout, and data drift are considered. Data drift considered as stochastic variable that satisfies a probabilistic distribution. A suitable fuzzy Lyapunov function is exploited to derive the results as an LMI. This LMI depends on the limits of delay, data packet dropout, data drift expectation, and data drift variance. Simulations illustrate the feasibility of the established Theorem. In other words, the main contribution of this paper is the robust controller design for fuzzy network control systems considering delay, data packet dropout, and data drift.

The rest of this paper is organized as follows. In Section 2 the structure of closed loop fuzzy network control system is explored. The problem of controller design for network control systems is addressed in

Section 3. A numerical example is adopted to show the effectiveness of the proposed approach. Finally, the paper is concluded in Section 5.

Notation: The notations used throughout the paper are fairly standard. I and 0 represent identity matrix and zero matrix; $diag\{.\}$ represents a block diagonal matrix. The notation $P > 0$ means that P is a real symmetric and positive definite. The symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. $E\{.\}$ denotes the expectation.

2. PROBLEM STATEMENT

Considering the structure of network control systems that is shown in Fig. 1, the model of a network control system is described as follows:

$$x(k+1)=f(x(k))+g(u(k))+m(w(k))$$
$$y(k)=Cx(k) \qquad (1)$$

where $x(k) \in \mathbb{R}^{n\times1}$ is the state vector, $u(k)$ is the control action, $y(k) \in \mathbb{R}^{m\times1}$ is the measured output, $w(k) \in L_2[t_0,\infty)$ is the disturbance input, $f(.), g(.), m(.)$, and $h(.)$ are nonlinear terms.
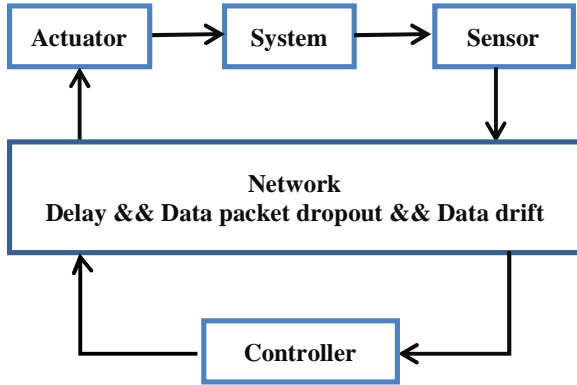

Fig. 1. Structure of network control system

By modeling the nonlinear system with a fuzzy system, the plant is described as:

Plant rule *i*: IF $q_1(k)$ **is** W_i^1 **, . . . ,** $q_n(k)$ **is** W_i^n **,THEN**

$$x(k+1)=\sum_{i=1}^{r} m_i(q)\big(A_{i\Delta}x(k)+B_iu(k)+M_iw(k)\big) \qquad (2)$$
$$y(k)=Cx(k)$$
$$x(k)=J(k),\; k=[-t_M,0],\; i=1,2,...,r$$

where t_M is the upper bound of delay, $J(k)$ is the initial state condition, and $q_1,q_2,...,q_n$ are premise variables. $A_{i\Delta},B_i,M_i,C_i$ are known matrixes with appropriate dimension. Moreover, $m_i(q)$ is defined as:

$$m_i(q(k))=\frac{w_i(q(k))}{\sum_{i=1}^{r}w_i(q(k))}$$

$$w_i(q(k))=\prod_{j=1}^{g}W_i^j(q(k)) \qquad (3)$$

$$m_i(q(k))\geq0, \qquad \sum_{i=1}^{r}m_i(q(k))=1$$

The system uncertainties are described as follows:

$$A_{i\Delta}=A_i+\Delta A_i$$
$$\Delta A_i=L_iF(k)E_i \qquad (4)$$

where L_i and E_i is known matrixes, and $F(k)$ is a time varying matrix that satisfies $F^T(k)F(k)\leq I$, $\forall k>0$.

When data is sent out from sensors to the controller and from the controller to actuators, these transmissions may result in delays that are named t_{sc},t_{ca}, respectively. These delays are time varying that the upper and lower bounds are known. Moreover, in data transmission process, data may not be received to the destination, this phenomenon is named data packet dropout. Delay and data packet dropout affect on system stability, and may cause the instability of the NCSs, therefore, considering delay and data packet dropout is essential in the stability analyses.

Let t_i is the time that a data is received by actuator, $k \in [t_i,t_{i+1})$ and $t_{sa}(k)=t_{sc}(k)+t_{ca}(k)$ and $t(k) @ k - t_i + t_{sc}(k)$, therefore, fuzzy control law considering delay and data packet dropout is given by:

IF $q_1(k)$ **is** W_j^1 **, . . . ,** $q_n(k)$ **is** W_j^n **,THEN**:

$$u(k)=\sum_{j=1}^{r}m_j(q-t(k))K_jCx(k-t(t)) \qquad (5)$$

in which delay and data packet dropout are bounded:

$$\underline{t}_{sa}<t(k)\leq(1+n_p)h+\bar{t}_{sa} \qquad (6)$$

where $\underline{t}_{sa},\bar{t}_{sa}$ is the lower and upper bounds of delay from sensors to actuators, n_p is the maximum number of sequential packet dropouts from sensors to actuators, and h is the sampling period of sensors.

Data drift may deviate the value of data that is sent, and it occurs when data is transmitted from sensors to the controller and from the controller to actuators. If data drift is considered in the close-loop system, the control law is changed as:

$$u(k)=\sum_{j=1}^{r}m_j(q-t(k))\Pi K_jCx(k-t(t)) \qquad (7)$$

where Π is the data drift parameter that is unrelated to the disturbance. It is assumed that the probabilistic density function $F(\Pi)$ of Π lie in the interval $[0, 1]$. The mathematical expectation and variance of Π are $\bar{a}$ and $\bar{b}^2$, respectively.

Substituting the control law (7) in the system, results in the close-loop system as:

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} m_i(q) m_j(q - t(k)) \times$$

$$\left(A_{i\Delta} x(k) + B_i \Pi K_j C x(t - t(k)) + M_i w(k) \right) \tag{8}$$

With attention to system (8), we have:

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} m_i(q) m_j(q - t(k)) \times$$

$$\left(A_i x(k) + B_i (\Pi - \bar{a}) K_j C x(t - t(k)) \right. \tag{9}$$

$$\left. + \bar{a} B_i K_j C x(t - t(k)) + M_i w(k) \right)$$

Lemma 1 [4]: For any positive-definite matrix $R \in \mathbf{i}^{n \times n}$, and considering $t(k) \in [0, t]$, $t(k) \in \mathbf{\dot{c}}_+$, and defining $v(k) = x(k+1) - x(k)$, we have:

$$-t \sum_{i=k-t(k)}^{k-1} v^T(i) R v(i) \le$$

$$-[x(k) - x(k - t(k))]^T R [x(k) - x(k - t(k))]$$

Lemma 2 [13]: Considering real matrixes Ξ_1, Ξ_2, Ξ_3 with appropriate dimensions that satisfy $\Xi_3^T \Xi_3 \le I$, then:

$$\Xi_1 \Xi_3 \Xi_2 + \Xi_2^T \Xi_3^T \Xi_1^T \le e \Xi_1 \Xi_1^T + e^{-1} \Xi_2^T \Xi_2 \qquad \forall e > 0$$

Lemma 3 [15]: thefollowing condition:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} m_i(x(k)) m_j(x(k)) \Gamma_{ij} < 0$$

is equal to:

$$\Gamma_{ii} < 0, \quad i = 1, 2, ..., r$$

$$\frac{2}{L-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \ i \ne j$$

3. MAIN RESULT

In this section, the stabilization condition of fuzzy network control systems is investigated. A robust controller for fuzzy systems considering delay, data packet dropout, and data drift is developed.

THEOREM 1. The closed-loop system (9) is asymptotically stable with upper and lower bounds t_M, t_m for delay and data drift expectation and variance $\bar{a}, \bar{b}^2$, if the following LMI is satisfied for any $P_i > 0, Q_i > 0, R_i > 0, e_i > 0$:

$$\begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} \\ * & \Xi_{22}^{i} \end{bmatrix} < 0 \tag{10}$$

where:

$$\Xi_{11}^{ij} = \begin{bmatrix} y_i & R_i & A_i^T & (A_i - I)^T \\ * & -Q_i - R_i & \bar{a} K_j^T B_i^T & t_M \bar{a} K_j^T B_i^T \\ * & * & P_i - 2I & 0 \\ * & * & * & R_i - 2I \end{bmatrix}$$

$$y_i = -P_i - R_i + (t_M - t_m + 1)Q_i + e E_i^T E_i$$

$$\Xi_{12}^{ij} = \begin{bmatrix} 0 & 0 & 0 \\ \bar{b} C^T K_j^T B_i^T & t_M \bar{b} C^T K_j^T B_i^T & 0 \\ 0 & 0 & P_i L_i \\ 0 & 0 & P_i L_i \end{bmatrix}$$

$$\Xi_{22}^{i} = diag\left(P_i - 2I \quad R_i - 2I \quad -e_i \right)$$

Proof: Assume that $v(k) = x(k+1) - x(k)$, and define a Lyapunov functions as:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k)$$

$$V_1(k) = x^T(k) P x(k),$$

$$V_2(k) = \sum_{i=k-t(k)}^{k-1} x^T(i) Q x(i) \tag{11}$$

$$V_3(k) = \sum_{j=-t_M+1}^{-t_m} \sum_{i=k+j}^{k-1} x^T(i) Q x(i)$$

$$V_4(k) = \sum_{j=-t_M}^{-1} \sum_{i=k+j}^{k-1} v^T(i)(t_M R) v(i)$$

where:

$$P = \sum_{i=1}^{r} m_i(q) P_i \qquad P_i > 0, i = 1, 2, ..., r$$

$$Q = \sum_{i=1}^{r} m_i(q) Q_i \qquad Q_i > 0, i = 1, 2, ..., r \tag{12}$$

$$R = \sum_{i=1}^{r} m_i(q) R_i \qquad R_i > 0, i = 1, 2, ..., r$$

By some computation, one can obtain:

$$E\{\Delta V_1(k)\} = E\{x^T(k+1) P x(k+1) - x^T(k) P x(k)\} \tag{13}$$

and

$$E\{\Delta V_2\} \le x^T(k) Q x(k) - x^T(k - t(k)) Q x(k - t(k))$$

$$+ \sum_{i=k+1-t_M}^{k-t_m} x^T(i) Q x(i) \tag{14}$$

and

$$E\{\Delta V_3(k)\} = (t_M - t_m) x^T(k) Q x(k)$$

$$- \sum_{i=k-t_M+1}^{k-t_m} \left\{ x^T(i) Q x(i) \right\} \tag{15}$$

and

$$E\{\Delta V_4\} \le E\left\{ t_M^2 v^T(k) R v(k) - \sum_{j=k-t_M}^{k-1} v^T(j)(t_M R) v(j) \right\}$$

Now, using Lemma 1, we have:

$$E\{\Delta V_4\} \le t_M^2 E\{v^T(k) R v(k)\} -$$

$$[x(k) - x(k - t(k))]^T R [x(k) - x(k - t(k))] \tag{16}$$

Based on Lyapunov approach, the system (9) is stable if:

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$$E\{\Delta V(k)\}=$$
$$E\{\Delta V_1(k)+\Delta V_2(k)+\Delta V_3(k)+\Delta V_4(k)\}<0 \qquad (17)$$

Therefore, we have:

$$E\{\Delta V(k)\}<E\{x^T(k+1)Px(k+1)-x^T(k)Px(k)$$
$$+x^T(k)Qx(k)-x^T(k-t(k))Q\times x(k-t(k))$$
$$+\sum_{i=k+1-t_M}^{k-t_m}x^T(i)Qx(i)+(t_M-t_m)x^T(k)Qx(k) \qquad (18)$$
$$-\sum_{i=k-t_M+1}^{k-t_m}\{x^T(i)Qx(i)\}-[x(k)-x(k-t(k))]^T$$
$$\times R\,[x(k)-x(k-t(k)]+t_M^2 v^T(k)Rv(k)\}<0$$

Define $x^T(k)=\begin{bmatrix}x^T(k) & x^T(k-t(k))\end{bmatrix}$, then, we can conclude that:

$$\sum_{i=1}^r\sum_{j=1}^r\sum_{i=1}^r\sum_{j=1}^r\sum_{i=1}^r m_i(q)m_j(q-t(k))m_i(q) \qquad (19)$$
$$\times m_j(q-t(k))m_i(q)z^T(k)\Omega_{ij}z(k)\le 0$$

where:

$$\Omega_{ij}=\begin{bmatrix}\Omega_{11}^{ij} & \Omega_{12}^{ij}\\ * & \Omega_{22}^{ij}\end{bmatrix} \qquad (20)$$

$$\Omega_{11}^{ij}=A_{i\Delta}^TP_iA_{i\Delta}-P_i+(1+t_M-t_m)Q_i$$
$$+t_M^2(A_{i\Delta}-I)^TR_i(A_{i\Delta}-I)-R_i$$

$$\Omega_{12}^{ij}=A_{i\Delta}^TP\bar aB_iK_jC+t_M^2(A_{i\Delta}-I)^TR_i\bar aB_iK_jC+R_i$$

$$\Omega_{22}^{ij}=-Q_i-R_i+\bar a\left(B_iK_jC\right)^TP_i\left(B_iK_jC\right)\bar a$$
$$+t_M^2\bar a\left(B_iK_jC\right)^TR_i\left(B_iK_jC\right)\bar a$$
$$+\bar b^2\left(B_iK_jC\right)^TP_i\left(B_iK_jC\right)$$
$$+t_M^2\bar b^2\left(B_iK_jC\right)^TR_i\left(B_iK_jC\right)$$

If $\Omega_{ij}<0$ the condition (17) is satisfied, then using Schur complement for $\Omega_{ij}<0$, we have:

$$\begin{bmatrix}\Pi_{11}^{ij} & \Pi_{12}^{ij}\\ * & \Pi_{22}^i\end{bmatrix}<0 \qquad (21)$$

where:

$$\Pi_{11}^{ij}=\begin{bmatrix}-P_i-R_i+(1+t_M-t_m)Q_i & R_i & A_{i\Delta}^T\\ * & -Q_i-R_i & \bar aC^TK_j^TB_i^T\\ * & * & -P_i^{-1}\end{bmatrix}$$

$$\Pi_{12}^{ij}=\begin{bmatrix}(A_{i\Delta}-I)^T & 0 & 0\\ t_M\bar aC^TK_j^TB_i^T & \bar bC^TK_j^TB_i^T & t_M\bar bC^TK_j^TB_i^T\\ 0 & 0 & 0\end{bmatrix}$$

$$\Pi_{22}^i=diag\left(-R_i^{-1} \quad -P_i^{-1} \quad -R_i^{-1}\right)$$

Now, considering equation $(P-I)^TP^{-1}(P-I)>0$, $(P-I)^TP^{-1}(P-I)>0$ and using Lemma 2 for uncertain terms, the LMI (10) is obtained. The proof of Theorem 1 is complete. ∎

Remark1. For solving LMI (10), Theorem 3 can be used to obtain variable matrix.

4. SIMULATION RESULTS

In order to evaluate the performance of proposed method a fuzzy network control system with two rules is considered:

$$x(k+1)=\sum_{i=1}^r m_i(q)\left(A_{i\Delta}x(k)+B_iu(k)+M_iw(k)\right)$$
$$y(k)=Cx(k)$$

$$A_1=\begin{bmatrix}1.2 & -0.2\\ 0.1 & -0.4\end{bmatrix}, A_2=\begin{bmatrix}1.2 & -0.2\\ 0.2 & 0\end{bmatrix}$$

$$B1=\begin{bmatrix}1.3\\ 0.7\end{bmatrix}, B2=\begin{bmatrix}1.5\\ 1\end{bmatrix}, M1=\begin{bmatrix}0.5\\ 0.7\end{bmatrix}; M2=\begin{bmatrix}0.6\\ 0.5\end{bmatrix},$$

C=[10,-2];

The membership function defines as:

$$m_1(x_1(k))=\frac{1}{1+e^{-x_1(k)}}, m_2(x_1(k))=1-m_1(x_1(k))$$

For uncertain terms assume that:

$$L_1=L_2=\begin{bmatrix}0.2\\ 0.2\end{bmatrix}, E1=E2=[0.1,0.2], F(k)=\sin(\frac{p\mathrm{k}}{6})$$

The upper and lower bands of delay are 1, 0, respectively, and the initial condition is $x^T(0)=\begin{bmatrix}10 & 5\end{bmatrix}^T$. In this paper two conditions for data drift are considered, in condition 1, it is assumed thatdata drift occurs with the following probability:

$$F(\Pi)=\begin{cases}0.1 & \Pi=0.9\\ 0.8 & \Pi=1.0\\ 0.1 & \Pi=1.1\end{cases}$$

It is clear that the $\bar a=1$and $\bar b^2=0.002$.

In condition 2, it is assumed thatdata drift occurs with the following probability:

$$F(\Pi)=\begin{cases}0.1 & \Pi=0.8\\ 0.85 & \Pi=1.0\\ 0.05 & \Pi=1.2\end{cases}$$

Therefore, $\bar a=0.99$and $\bar b^2=0.0059$. Using Theorem 1, the controller is obtained.

Based on condition 1 the controller parameters are calculated as $K_1=$ -0.0272, $K_2=$ -0.0272 , and based on condition 2 controller parameters are $K_1=$ -0.0322 and $K_2=$ -0.0300 .

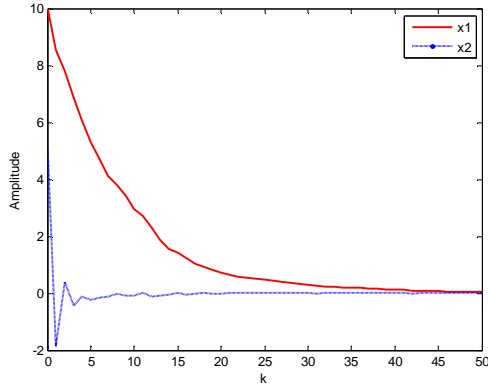The state of closed-loop system based on condition 1, is shown in Fig. 2, and the control law is shown in Fig. 3.

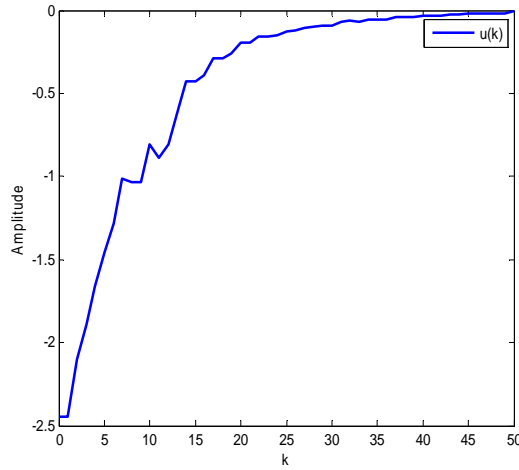**Fig. 2.** State of the close-loop system based on condition 1

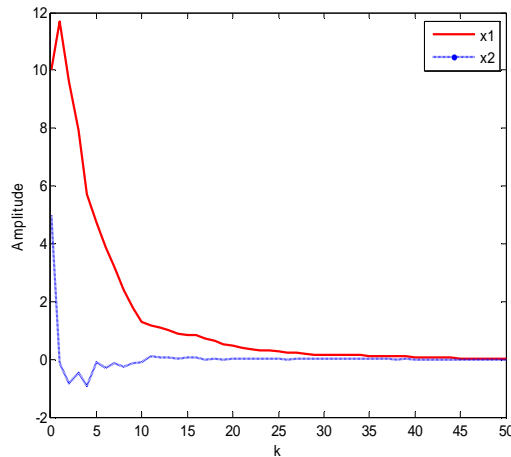**Fig. 3.** Control law based on condition 1

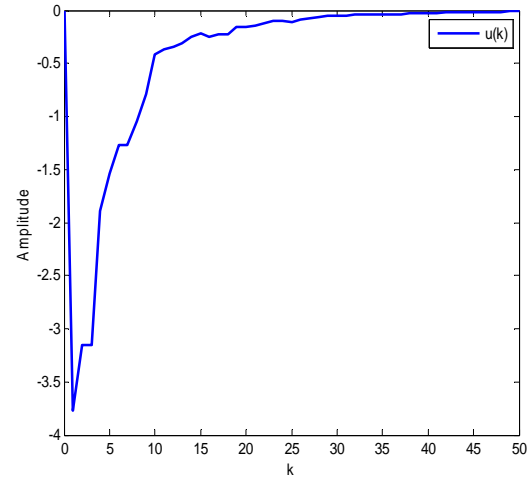**Fig. 4.** State of the close-loop system based on condition 2

Fig. 5. Control law based on condition 1

Moreover, the state of closed-loop system and control law based on condition 2 are shown in Fig. 4 and Fig. 5, respectively. Obviously, the system is stable and control law is acceptable. Therefore, the proposed method is able to handle delay, data packet dropout, and data drift simultaneously.

5. CONCLUSIONS

In this paper,the problem of controller design for nonlinear network control systems, which is modeled by fuzzy systems, is investigated. Since the modeling uncertainty is unavoidable, the stability is guaranteed using a robust controller. In the stability analysis, delay, and data packet dropout is considered. Moreover, data drift, which might change the value of data that is transmitted through the network, is also considered in the design procedure. For the stability analysis, the fuzzy Lyapunov function is used, and the result is developed based on an LMI that depends on the limits of delay, data packet dropout, and expectation and variance of data drift. Simulation results verify the good performance of the proposed approach.

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