

# Decentralized MRAC for Large Scale Systems with Input and State Delays

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## Abstract

In this paper, the problem of decentralized model reference adaptive control (MRAC) for a class of large scale systems with time varying delay in interconnected term and input and state delays is studied. To compensate the effect of input delay indirectly, a Smith predictor built on. To handle the effects of the time delays in input, the adaptive controller part includes two auxiliary dynamic filters with time varying gains. Under a usual assumption that the interconnections are assumed to be Lipschitz in its variables and uniformly in time with unknown Lipschitz gains, the difficulties from unknown interconnections are dealt. A generalized error is defined and by a suitable Lyapunov function, an adaptive controller is designed to stabilize it. Decentralized adaptive feedback controller can render the generalized error system uniformly ultimately bounded stable is designed. Finally, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed design techniques.

**Keywords:** Input delays, time varying delay, decentralized MRAC, interconnected system.

## 1. Introduction

Large-scale interconnected systems can be found in many diverse fields such as electrical power systems, manufacturing processes, transportation and communication. Decentralized control for the large-scale systems has been a research focus in the past three decades and a lot of achievements have been made, see [1-3], and the references therein. In such systems, often delay exists and due to the effect of it, these systems may possess instability and control performance of these systems is hardly assured. So far, control of delayed interconnected systems attracted a number of researchers over the past years, see for example [2-8] and references therein. According to be large scaling of these systems, priori knowledge of the nonlinearities like interconnection functions and parameters like delays is impossible. Hence adaptive

control is a popular method for controlling of such systems employed in many papers.

In [1] robust adaptive control of large-scale systems without delay was considered by applying dynamic programming. [3] developed a memoryless adaptive output feedback controller for stabilizing a class of large-scale nonlinear systems. An adaptive decentralized neural control laws addressed for a class of pure-feedback interconnected system with unknown time-varying delays in outputs interconnections in [2].

Between adaptive control method, model reference adaptive control (MRAC) has been attracted lot of attention to control of interconnected systems because of its desired closed loop response. Model reference adaptive control for interconnected systems with time delays are considered in [8], but the obtained controllers are dependent on the delays and the interconnections need to be known precisely. Memoryless controller was proposed for large-scale systems with matched nonlinear interconnections in [7] by using MRAC. In [5] by using the adaptive method a memoryless controller was designed to follow a model for nonlinearly interconnected systems with time delays, dead-zone actuators and mismatched time-varying disturbances. In [4] delay-dependent linear state feedback controller was constructed base on the MRAC for a class of large scale systems with time-varying delays and matched interconnections. The final designed controller is very complicated for practical utilizing.

In the all above papers, the actuators delays are not investigated. For systems with input delays, the control problem is very complicated due to the need to predict the input delayed values, especially for large-scale systems. However, to the best of the authors' knowledge, no attempt has been reported to tackle input delayed simultaneously with state and interconnections delays using the model reference method. In [9], globally stable MRAC Smith-predictor like solution for SISO input delayed plants were developed. It was assumed that the process is minimum phase with arbitrary relative degree, though not necessarily stable. The controller structure was similar to the one proposed by [10]. More recently, in [11-13] MRAC controllers were proposed, also with a Smith predictor-like structure. In [12] adaptive control laws were generated by using a high-order tuner and Lyapunov-Krasovski functional was used to adaptive stabilization. The adaptive controller in [13] is based on the Smith predictor and finite spectrum assignment. The problem of input delay with smith predictor was studied in [14], but in the mentioned paper, the

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interconnected term was not considered. In [15] the interconnected term was considered but the input and state delays were not investigated.

In this paper, a new MRAC scheme is developed for a class of interconnected delayed systems with known input and state delays. To handle the effects of the time delays, the adaptive controller part includes two auxiliary dynamic filters with time varying gains. For implicit (indirect) input-delay compensation, a Smith predictor-like filter is used. Decentralized adaptive feedback controller can render the generalized error system uniformly ultimately bounded stable is designed. The paper consists of the following parts. In section 2, problem formulation and assumptions are introduced. The controller design is described in section 3. The stability of system is proof by the introducing a suitable Lyapunov function. In section 4 a numerical example is illustrated and the mentioned theoretical process is implemented. Final section is included the conclusion.

## 2. Problem formulation and assumptions

The interconnected system considered in this paper composed by  $N$  subsystems with the  $i$ th subsystem

$$S_i : \dot{x}_i(t) = A_i x_i(t) + A_{di} x_i(t - t_i) + \quad (1)$$

$$B_i u_i(t - h_i) + B_i \sum_{j=1}^N x_{ij}(x_{ij}(t), x_{ij}(t - d_{ij}(t)))$$

where  $x_i \in \mathbb{R}^{n_i}$  and  $u_i \in \mathbb{R}^{m_i}$  represent the state and control vectors of the  $i$ -th subsystem respectively;  $A_i$ ,  $A_{di} \in \mathbb{R}^{n_i \times n_i}$  and  $B_i \in \mathbb{R}^{n_i \times m_i}$  are known constant matrices,  $t_i$  and  $h_i$  are known constant delay,  $x_{ij}(x_{ij}(t), x_{ij}(t - d_{ij}(t)))$  are uncertain interconnections, which indicate the interconnections among the current states and the delayed states of systems  $S_i$  and  $S_j$ , and  $d_{ij}(t)$  are bounded time-varying delays, which are differentiable and satisfy:

$$0 \leq d_{ij}(t) \leq \bar{d}_{ij} < \infty, \quad 0 \leq \dot{d}_{ij}(t) \leq \bar{d}_{ij}^* < 1, \quad (2)$$

where  $\bar{d}_{ij}$  and  $\bar{d}_{ij}^*$  are positive scalars. The initial conditions are given by:

$$x_i(t) = \Lambda_i(t), \quad t \in [t_0 - \bar{t}_i, t_0], \quad i = 1, 2, \dots, N$$

where  $\bar{t}_i = \max\{t_i, \bar{d}_{ij}\}$  and  $\Lambda_i(t)$  are continuous functions.

The objective in this paper is to design a decentralized controller  $u_i(t)$  adaptively to ensure that the system states  $x_i(t)$  follow the state of  $x_{mi}(t)$  given by the local reference model following:

$$\dot{x}_{mi}(t) = A_{mi} x_{mi}(t) + B_{mi} R_i(t) \quad (3)$$

with the known piecewise continuous and bounded reference  $R_i(t)$ .

For system (1) and model reference system (3), the following assumptions hold:

**Assumption 1.** There exist matrices  $K_i$  and positive matrices  $P_i$  satisfying the following inequality:

$$(A_{mi} + B_i K_i)^T P_i + P_i (A_{mi} + B_i K_i) = -Q_i \quad (4)$$

where  $i = 1, 2, \dots, N$  and  $Q_i$  are positive definite matrices.

**Assumption 2.** There exist constant vectors  $z_i \in \mathbb{R}^{n_i}$ ,  $m_i \in \mathbb{R}^{n_i}$  and a nonzero constant scalar  $q_{ri}$  such that the following equations are satisfied:

$$A_i - A_{mi} = B_i z_i, \quad A_{id} + B_i m_i = 0, \quad B_{mi} = B_i q_{ri}$$

**Assumption 3.** There exist unknown positive constants  $a_{ij}$  and  $b_{ij}$  such that:

$$\begin{aligned} \sum_{j=1}^N \|x_{ij}(x_j(t), x_j(t - d_{ij}(t)))\| &\leq \sum_{j=1}^N a_{ij} \|x_j(t)\| \\ &+ \sum_{j=1}^N b_{ij} \|x_j(t - d_{ij}(t))\| \end{aligned} \quad (5)$$

**Assumption 4.** The states of system (3) are bounded.

**Assumption 5.** All subsystems are stable.

**Remark 1.** It should be noted that Assumption 1 is to guarantee that the pair  $(A_{mi}, B_i)$  can be stabilized. If  $(A_{mi}, B_i)$  is completely controllable, Assumption 1 will always hold. Assumptions 1 and 2 are used to determine some important design parameters. Assumption 4 is to assure that the underlying model reference system is bounded stable.

## 3. Model reference adaptive controller design

The problem is to design an adaptive feedback control in order to achieve desired closed loop specifications. The desired specification in this paper is that all signals of the closed loop system remain bounded and that approximate tracking be achieved with small enough asymptotic error. The tracking error and its derivatives is defined as:

$$e_i(t) = x_i(t) - x_{mi}(t) \quad (6)$$

$$\dot{e}_i(t) = A_i x_i(t) + A_{di} x_i(t - t_i) \quad (7)$$

$$\begin{aligned} &+ B_i u_i(t - h_i) + B_i \sum_{j=1}^N x_{ij}(x_{ij}(t), x_{ij}(t - d_{ij}(t))) \\ &- A_{mi} x_{mi}(t) - B_{mi} R_i(t) \end{aligned}$$

From assumption 3, we further obtain the following inequalities with the help of assumption 4:

$$\begin{aligned} &\sum_{j=1}^N \|x_{ij}(x_j(t), x_j(t - d_{ij}(t)))\| \\ &\leq \sum_{j=1}^N a_{ij} \|x_j(t)\| + \sum_{j=1}^N b_{ij} \|x_j(t - d_{ij}(t))\| \\ &= \sum_{j=1}^N a_{ij} \|x_{mj}(t) + e_j(t)\| \\ &+ \sum_{j=1}^N b_{ij} \|x_{mj}(t - d_{ij}(t)) + e_j(t - d_{ij}(t))\| \\ &\leq a_{ij} \|e_j(t)\| + b_{ij} \|e_j(t - d_{ij}(t))\| + d_{ij} \end{aligned} \quad (8)$$

Since

$$\|e_{gi}(t)\| \leq \|e_i(t)\| + \|v_{i1}(t)\| + \|v_{i2}(t)\| \quad (9)$$

Then relationship (8) can be written as:

$$\sum_{j=1}^N \|x_{ij}(x_j(t), x_j(t-d_{ij}(t)))\| \leq a_i \|e_{gi}(t)\| + b_i \|e_{gi}(t-d_{ij}(t))\| + d_i \quad (10)$$

where  $a_i$ ,  $b_i$  and  $d_i$  are unknown positive scalars, and since the states  $x_{mi}$  of reference model system are bounded, there always exist positive scalars such that inequality (10) holds.

Now, we are ready to present our main result in this paper.

**Theorem.** For system (1), the following adaptive feedback controller:

$$u_i = u_{i1} + u_{i2} + u_{i3} \quad (11)$$

where

$$u_{i1} = -z_i x_{mi}(t) + k_i e_{gi}(t) \quad (12)$$

$$+ q_i R_i(t) + m_i x_{mi}(t - t_i)$$

$$u_{i2} = -q_i B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}},$$

$$u_{i3} = \frac{-h_i(t) B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}}}{\left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|}.$$

In which  $q_i(t)$  and  $h_i(t)$  are adaptive parameters with adaptive laws:

$$\begin{aligned} \dot{q}_i &= \frac{1}{2} \Gamma_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 - \frac{1}{2} \Gamma_i s_{i1} q_i, \\ \dot{h}_i &= \frac{1}{2} f_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 - \frac{1}{2} f_i s_{i2} h_i. \end{aligned} \quad (13)$$

where  $\Gamma_i, f_i, s_{i1}$  and  $s_{i2}$  are positive scalars.

Also, with definition generalized as follows:

$$e_{gi}(t) = e_i(t) + v_{i1}(t) + v_{i2}(t) \quad (14)$$

where is the sum of the tracking error  $e_i(t)$  and the internal states  $v_{i1} \in \mathbb{R}^{n_i}$  and  $v_{i2} \in \mathbb{R}^{n_i}$  of the adaptive controller. The controller states  $v_{i1}, v_{i2}$  are generated by the stable dynamic filters  $F_1$  and  $F_2$  with the gains  $\alpha_{i1}(t) \in \mathbb{R}^{n_i}$  and  $\alpha_{i2}(t) \in \mathbb{R}^{2n_i}$ :

$$F_1: \dot{v}_{i1}(t) = A_{mi} v_{i1}(t) \quad (15)$$

$$+ B_{mi} a_{i1}(u_i(t) - u_i(t - h_i))$$

$$F_2: \dot{v}_{i2}(t) = A_{mi} v_{i2}(t) + B_{mi} a_{i2}^T w_i(t) \quad (16)$$

where the input of  $F_2$  is the signal  $w_i(t) = [e_i(t) \ e_i(t - t_i)]^T \in \mathbb{R}^{2n_i}$ .

The time varying gains  $a_{i1}(t)$  and  $a_{i2}(t)$  are as:

$$a_{i1} = -q_i^{-1} \quad (17)$$

$$a_{i2} = [-q_i^{-1} n_i \ q_i^{-1} m_i]^T \quad (18)$$

**Definition.** The solution generalized error  $e_{gi}(t, t_0, \mathcal{Y}_i)$  of interconnected system (14) is said to be uniformly

ultimately bounded with respect to the bound  $e_i$  if for each  $\delta_i > 0$  there exists  $T_i = T_i(e_i, d_i) > 0$  independent of  $t_0$  such that  $\|e_{gi}(t, t_0, \mathcal{Y}_i)\| \leq e_i$  for all  $t \geq t_0 + T$  when  $\|x_{i0}\| \leq d_i$ .

**Proof.** Define the following Lyapunov function candidate:

$$\begin{aligned} V_i(e_{gi}, q_i, h_i, t) &= \sum_{i=1}^N \bar{V}_i(e_{gi}, q_i, h_i, t) = \\ &= \sum_{i=1}^N \left\{ V_i(e_{gi}) + \Gamma_i^{-1} (q_i - q_i^*)^2 + f_i^{-1} (h_i - h_i^*)^2 \right. \\ &\quad \left. + q_i \int_{t-d_{ij}}^t \|e_{gi}(x)\|^2 dx \right\} \end{aligned} \quad (19)$$

where

$$V_i(e_{gi}) = e_{gi}^T P_i e_{gi}. \quad (20)$$

$q_i$  are sufficiently small positive scalars,  $q_i^*$  are also positive scalars defined in (25) (below).

Generalized error  $e_{gi}(t)$  is equal to:

$$\begin{aligned} \mathcal{E}_i(t) &= \mathcal{E}_i(t) + \mathcal{E}_{i1}(t) + \mathcal{E}_{i2}(t) \\ &= A_{mi} e_{gi}(t) + B_{mi} a_{i1} [u_i(t) - u_i(t - h_i)] \\ &\quad + B_{mi} a_{i2}^T w_i(t) + B_i z_i x_i - B_i m_i x_i(t - t_i) \\ &\quad - B_{mi} R_i(t) + B_i \sum_{j=1}^N x_{ij}(x_j(t), x_j(t - d_{ij}(t))) \\ &\quad + B_i u_i(t - h_i) \end{aligned} \quad (21)$$

By taking the time derivative of  $V_i(\cdot)$  along the trajectories of the closed loop system, we have

$$\begin{aligned} \frac{\partial \bar{V}_i(e_{gi}, q_i, h_i, t)}{\partial t} &= -e_{gi}^T(t) Q_i e_{gi}(t) \\ &\quad + \frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \sum_{j=1}^N x_{ij}(x_j(t), x_j(t - d_{ij}(t))) \\ &\quad + \frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i (u_{i2}(t) + u_{i3}(t)) \\ &\quad + 2\Gamma_i^{-1} (q_i - q_i^*) \dot{q}_i - 2f_i^{-1} (h_i - h_i^*) \dot{h}_i \\ &\quad + q_i \|e_{gi}(t)\|^2 - q_i (1 - d_{ij}^*(t)) \|e_{gi}(t - d_{ij}(t))\|^2 \\ &= -e_{gi}^T(t) Q_i e_{gi}(t) \\ &\quad + \frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \sum_{j=1}^N x_{ij}(x_j(t), x_j(t - d_{ij}(t))) \\ &\quad + \frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i (u_{i2}(t) + u_{i3}(t)) \\ &\quad + (q_i - q_i^*) \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 - s_{i1} q_i (q_i - q_i^*) \\ &\quad - (h_i - h_i^*) \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 + s_{i2} h_i (h_i - h_i^*) \\ &\quad + q_i \|e_{gi}(t)\|^2 - q_i (1 - d_{ij}^*(t)) \|e_{gi}(t - d_{ij}(t))\|^2 \end{aligned} \quad (22)$$

Considering the below relationship

$$\frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i (u_{i2}(t) + u_{i3}(t)) = \quad (23)$$

$$-q_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 - h_i(t) \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

and

$$\frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \sum_{j=1}^N x_{ij} (x_j(t), x_j(t-d_{ij}(t))) \quad (24)$$

$$\leq \bar{a}_i \|e_{gi}(t)\| \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\| +$$

$$\bar{b}_i \|e_{gi}(t-d_{ij}(t))\| \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$+ d_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$\leq \frac{\|\bar{a}_i\|^2}{4e_i} \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 + e_i \|e_{gi}(t)\|^2$$

$$+ \frac{\|\bar{b}_i\|^2}{4q_i(1-d_{ij}^*)} \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2$$

$$+ d_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$+ q_i(1-d_{ij}^*) \|e_{gi}(t-d_{ij}(t))\|^2$$

Let

$$q_i^* = \frac{\|\bar{a}_i\|^2}{4e_i} + \frac{\|\bar{b}_i\|^2}{4q_i(1-d_{ij}^*)} \quad (25)$$

Then, we have

$$\frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \sum_{j=1}^N x_{ij} (x_j(t), x_j(t-d_{ij}(t))) \quad (26)$$

$$\leq q_i^* \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 + d_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$+ q_i(1-d_{ij}^*) \|e_{gi}(t-d_{ij}(t))\|^2 + e_i \|e_{gi}(t)\|^2$$

Considering the following equality:

$$(q_i - q_i^*)(-s_{i1}q_i) + (d_i - h_i)(s_{i2}h_i) \quad (27)$$

$$= -s_{i1} \left( q_i - \frac{1}{2} q_i^* \right)^2 + \frac{1}{4} s_{i1} q_i^{*2}$$

$$- s_{i2} \left( d_i - \frac{1}{2} h_i \right)^2 + \frac{1}{4} s_{i2} h_i^2$$

Substituting (23) to (27) into (22), we further have

$$V_i^*(e, q, t) = \sum_{i=1}^N V_i^*(e, q, t) \quad (28)$$

$$= \sum_{i=1}^N \left( -e_{gi}^T(t) Q_i e_{gi}(t) + (q_i + e_i) \|e_{gi}(t)\|^2 \right)$$

$$+ \sum_{i=1}^N \left( -s_{i1} \left( q_i - \frac{1}{2} q_i^* \right)^2 + \frac{1}{4} s_{i1} q_i^{*2} \right) \\ + \sum_{i=1}^N \left( -s_{i2} \left( d_i - \frac{1}{2} h_i \right)^2 + \frac{1}{4} s_{i2} h_i^2 \right)$$

Because  $Q_i$  is positive definite matrices, parameters  $e_i$  and  $q_i$  can be selected to be small enough to render that the following inequality holds:

$$-I_{\min}(Q_i) + q_i + e_i = -\Pi_i < 0 \quad (29)$$

where  $\Pi_i$  are positive scalars. Furthermore, one has

$$V_i^*(e, q, t) \leq \sum_{i=1}^N \left( -\Pi_i \|e_{gi}\|^2 + \frac{1}{4} s_{i2} d_i^2 + \frac{1}{4} s_{i1} q_i^{*2} \right) \quad (30)$$

Based on Lyapunov stability theory, the proposed decentralized state feedback controller (11)- (18) will guarantee the closed loop error system is uniformly ultimately bounded stable.

**Remark2.** In this section, robust adaptive tracking control is proposed for a class of systems with delayed input and state and without so-called distributed delay blocks. To handle the time delays effects, the auxiliary dynamic filters  $F_1$  and  $F_2$  with variable gains  $a_{i1}(t)$  and  $a_{i2}(t)$  are introduced. The Smith predictor-like filter  $F_1$  is used for indirect elimination of the effect of the dead time  $h_i$ . The filter  $F_2$  with the nonlinear input  $a_{i2}^T(t)w_i(t)$  is introduced to compensate plant parameter uncertainties, perturbations. Note that in our case the Smith predictor has a “classical” structure, with the difference that it is based on the reference model transfer function rather than the plant transfer function, and that it has the input gain  $a_{i1}(t)$ .

**Remark3.** For inequality (30), it is easy to obtain that the stable bounds of error  $e_{gi}$  can be rendered sufficiently small by reducing the values of parameters  $s_{i1}$  and  $s_{i2}$ .

#### 4. Numerical example

To illustrate the utilization of our approach, in this section, we consider the following numerical example. Here, a large scale time-delay system with interconnected terms is composed of two subsystems described by

$$\begin{aligned}
 \mathbf{x}_1(t) &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t-h_1) \\
 &+ \begin{bmatrix} 0 & 0 \\ -0.6 & -0.8 \end{bmatrix} x_1(t-t_1) \\
 &+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} (d_{11} x_{11}(t) + d_{12} x_{22}(t-d_1(t))) \\
 \mathbf{x}_2(t) &= \begin{bmatrix} 0 & 1 \\ -2.5 & -3.5 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2(t-h_2) \\
 &+ \begin{bmatrix} 0 & 0 \\ -0.6 & -0.8 \end{bmatrix} x_2(t-t_2) \\
 &+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} (d_{21} x_{21}(t) + d_{22} x_{12}(t-d_2(t)))
 \end{aligned} \quad (31)$$

To build the adaptive controller we choose the reference model:

$$\begin{aligned}
 \mathbf{x}_{m1}(t) &= \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} x_{m1}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R_1(t) \\
 \mathbf{x}_{m2}(t) &= \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} x_{m2}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} R_2(t)
 \end{aligned} \quad (32)$$

with  $R_1(t) = 100 \sin \frac{P}{8} t$ ,  $R_2(t) = 100 \cos \frac{P}{8} t$ .

Therefore, we obtain the following controller from theorem 1:

$$\begin{aligned}
 u_1 &= -(2x_{m11} + 2x_{m12}) + (0.6x_{m11}(t-t_1) \\
 &+ 0.8x_{m12}(t-t_1)) - (e_{g11} + 10e_{g12}) + R_1(t) \\
 &- 2q_1 B_1^T P_1 e_{g1} - 2h_1 \frac{B_1^T P_1 e_{g1}}{\|B_1^T P_1 e_{g1}\|}
 \end{aligned} \quad (33)$$

$$\begin{aligned}
 u_2 &= -(2.5x_{m21} + 2.5x_{m22}) + (0.6x_{m21}(t-t_2) \\
 &+ 0.8x_{m22}(t-t_2)) - (e_{g21} + 10e_{g22}) + R_2(t) \\
 &- 2q_2 B_2^T P_2 e_{g2} - 2h_2 \frac{B_2^T P_2 e_{g2}}{\|B_2^T P_2 e_{g2}\|}
 \end{aligned} \quad (34)$$

and the adaptive laws

$$\begin{aligned}
 \dot{q}_i &= \|2B_i^T P_i e_{gi}\|^2 - 0.01q_i, \\
 \dot{h}_i &= \|2B_i^T P_i e_{gi}\| - 0.01h_i, \quad i = 1, 2.
 \end{aligned} \quad (35)$$

We chose

$$\begin{aligned}
 t_1 = t_2 = 1, \quad h_1 = h_2 = 1.5, \quad d_{11} = d_{21} = 1, \\
 d_{22} = d_{12} = 0.5, \quad d_1(t) = d_2(t) = 0.5(1 + \cos t) \\
 Q_i = 100I.
 \end{aligned}$$

and the initial conditions are

$$\begin{aligned}
 x_1(0) &= \begin{bmatrix} 8 \\ 4 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} -4 \\ -8 \end{bmatrix}, \\
 x_{m1}(0) &= \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad x_{m2}(0) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.
 \end{aligned}$$

The state responses  $x_i(t)$  and  $x_{mi}(t)$  are shown in Figures 1-4. From figures it can be seen that the closed-loop system is uniformly ultimately bounded stable. Also the control signals and the generalized errors  $e_{gi}(t)$  are shown in Figures 5 and 6 respectively.

## 5. Conclusion

In this paper, model reference adaptive control problem for a class of large-scale time-delay systems with both input delay and state delay and interconnection is investigated. The decentralized feedback controllers and corresponding adaptive laws are designed. Our approach relies on a decomposition of the adaptive control design by introducing the “generalized error”, and two auxiliary linear filters with time varying gains. The first filter is the “classical” Smith predictor, but based on the reference model transfer function rather than on the plant transfer function. The second filter has the same dynamics as the reference model but driven by the signal  $a_{i2}^T w_i(t)$ . The effect of such decomposition is to pull the input delay out of the design procedure. That leads to control laws with the following distinctive features: (i) they do not incorporate distributed-delay elements and (ii) they provide robustness with respect to plant parameter uncertainties. The “price” paid for this controller configuration is its restrictive applicability to stable plants, to plants stabilizable with memory less state feedback or to plants stabilizable with so called “delayed state feedback”. Based on Lyapunov stability theory, we prove the resulting closed-loop error system is uniformly ultimately bounded stable. A numerical example is given to verify the feasibility and validity of the main results. Therefore, the results obtained are expected to apply to a large class of

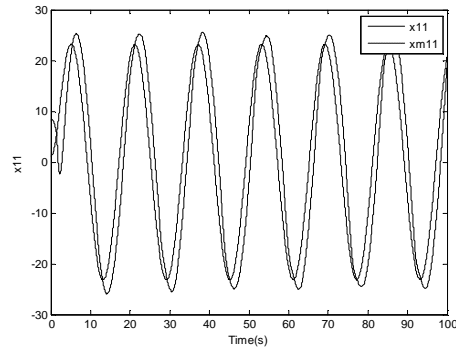


Fig. 1. The response of the state variable  $x_{11}$  and  $x_{m11}$ .

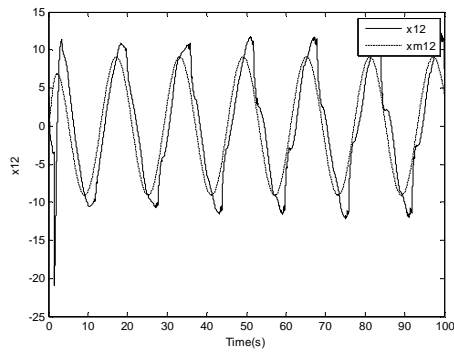


Fig. 2. The response of the state variable  $x_{12}$  and  $x_{m12}$ .

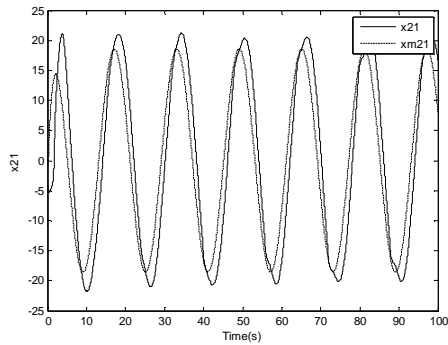


Fig. 3. The response of the state variable  $x_{21}$  and  $x_{m21}$ .

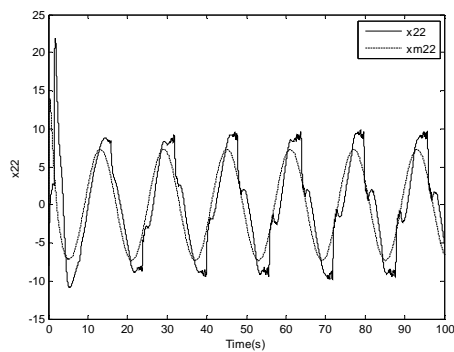


Fig. 4. The response of the state variable  $x_{22}$  and  $x_{m22}$ .

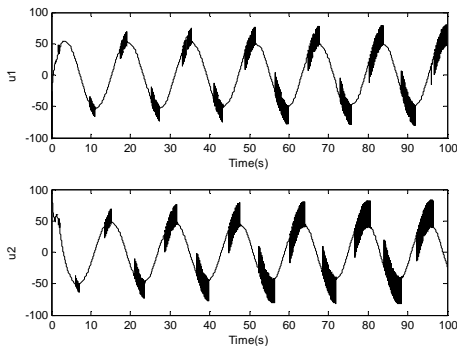


Fig. 5. Adaptive control signals  $u_1$  and  $u_2$ .

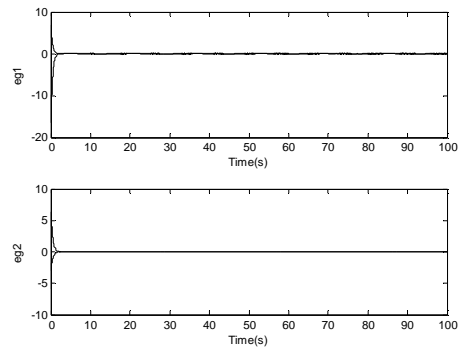


Fig. 6. The generalized errors  $e_{g1}$  and  $e_{g2}$  converge to zero.

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