Price-Based Resource Allocation in Cognitive Radio Networks via Game Theory with Imperfect Spectrum Sensing

Ali-Mohammad Montazeri and Faezeh Alavi

***Abstract*—In this paper, we consider a price-based resource allocation within an Orthogonal Frequency-Division Multiple Access (OFDMA) based on spectrum sensing in the cognitive networks. Furthermore, there is the primary network which has an opportunity to sell the spectrum to secondary network. We assume the secondary network's interference pricing is used to protect primary network and propose a joint utility maximization of the primary and secondary with a maximum interference constraint at the primary and a transmission power threshold at the secondary transmitter. Accordingly, we devise a novel cost computation strategy which is function of primary and secondary behaviors. To formulate this method, a Stackelberg game and equilibriums are exploited. Numerical results are presented to verify the proposed scheme. The impact of different system parameters is investigated and compared through simulations.**

***Index Terms*—cognitive radio network, spectrum sensing, pricing, game theory.**

I. INTRODUCTION

IN order to account inefficiency in the spectrum usage because of static frequency assignment, dynamic spectrum access and cognitive radio techniques have been proposed [1], [2]. In cognitive networks, secondary users (SUs) which have no spectrum license are allowed to operate in the spectrum assigned to primary users (PUs) provided that a certain level of quality of service is guaranteed for PUs [3]. One of the spectrum access techniques called the sensing-based spectrum sharing [4], where the channel is sensed by the secondary network and based on the result the transmit power is adjusted.

In fact, the potential to learn the surrounding radio environment of CR network is performed by the spectrum sensing (signal detection). The detection of primary users is performed based on the received signal at CR users. This approach includes matched filter, energy and cyclostationary based detectors. Among these methods, the matched filter is optimal, but requires perfect synchronization between the primary transmitter and cognitive device [5]. Due to its simplicity and no requirement on information about the PU, energy detection (ED) is the most popular sensing technique [5], [6].

In addition, when the allocated spectrum is not fully utilized, it is possible to have the spectrum trading mechanism which involves spectrum selling and buying processes. In this mechanism, the primary network has an opportunity to sell the spectrum to the secondary network and obtain the revenue. Hence, one of the challenging issues is the pricing that sets the spectrum price in a competitive environment. In this regard, the primary network offers spectrum to the secondary network, in order to satisfy both the seller and buyers.

This type of problems have been studied by using game theory in cognitive radio networks. Authors in [7]–[10] studied the utility maximization for spectrum sharing cognitive radio networks using different game theory with power control strategies such as evolutionary games, Stackelberg game and repeated Cournot game. In all of them, the authors just consider the underlay methods, it means they only solve the problem with a constraint to protect primary network. In [11], the authors studied price-based resource allocation by using Stackelberg game method for two-tier femtocell networks where macrocell is underlaid femtocells and access the same channels. The interference power constraint from cognitive radio networks is used to control interference in the two-tier spectrum-sharing networks. Unlike [11] which used underlay method and studied two-tier femtocell network over a single frequency band, in this paper we consider the spectrum sensing method and study joint power and subcarrier allocation in cognitive radio network. It is worthwhile to mention that compared to existing approaches in the literature, the pricing issue which considers the spectrum sensing in cognitive radio networks has not been studied yet.

In this paper, we analyze the problem of pricing in a dynamic spectrum access where a primary network offers spectrum access opportunities to a secondary network which operates under sensing-based spectrum sharing. We formulate this as a problem where the resources are allocated in terms of price to gain the highest profit. For a primary service, the cost of the shared spectrum is modeled as a function of the quality of service degradation. For the secondary service, a spectrum demand function is established based on the utility function which depends on the channel quality. To the best of our knowledge, there is no work that propose jointly pricing and resource allocation in CR networks based on spectrum sensing. In addition we have proposed a new cost function based on maximum predefined interference which is the function of the pricing and the secondary behavior.

The rest of paper is organized as follows. Section II presents the system model. Section III formulates the optimization problem. In Section IV, the game model is investigated for

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solving problem. Simulation results are analyzed in Section V and finally Section VI concludes the paper.

II. System Model

In this paper, we consider the down link of a cognitive radio network and a primary network which sells the spectrum to secondary users consisting of one central secondary base station serves the set of its own users which are J secondary users. We assume the secondary and primary networks access the same channels. The total bandwidth of B Hz is shared between primary and secondary networks through orthogonal frequency division multiple access (OFDMA), which is divided into a set of $n = 1, 2, \ldots, N$ subcarriers. The bandwidth of each sub-carrier i.e., $B_c = \frac{B}{N}$, is assumed to be much less than the coherent bandwidth of the wireless channel, so that the channel response in each subcarrier is flat. Secondary user j denotes the scheduled user receiving signal from its BS, where $j = 1, 2, \ldots, J$. It is assumed all nodes are equipped with single antennas. A typical illustration of this network is shown in Fig. 1. The channel power gain of the link between secondary transmitter and receiver of secondary user j on subcarrier n is denoted by $h^j_{ss,n}$, and that between secondary transmitter and the receiver of primary network on subcarrier n is denoted by $h_{sp,n}$. In addition, the channel power gain of primary transmitter and the receiver of secondary user j on subcarrier n is denoted by $h^j_{ps,n}$. It is assumed all the channel gains are independent. The SUs can use the licensed spectrum provided that the interference to the PU does not exceed the predefined threshold level. The thermal noise is considered as an additive Gaussian random variables with zero mean and variance σ^2. Moreover, we assume the noise variance is constant over all subcarriers.

III. Problem Formulation

In this section, the game formulation for the price-based resource allocation scheme is presented. First, the cost function of primary network is considered and then, we present the utility function for secondary network.

It is assumed, pricing the interference from the secondary users is used to protect the primary network [11]. Hence, the revenue of primary network can be obtained as follows.

$$U^{j,n}_{\text{primary}}(\boldsymbol{\mu}, \boldsymbol{p}, \boldsymbol{\rho}) = \rho^j_n \mu_n I^j_n(p^j_n), \tag{1}$$

where $\boldsymbol{\mu}$ is the interference price with $\boldsymbol{\mu} = (\{\mu_1, \mu_2, \ldots, \mu_N\})$, where μ_n denoting the interference price on subcarrier n, $\rho^j_n = \{0, 1\}$ as a binary variable which indicates the sub-carrier allocation, e.g., $\rho^j_n = 0$ if the sub-carrier n is not allocated to secondary user j, otherwise $\rho^j_n = 1$. $I^j_n(p^j_n)$ is the interference power received from secondary user j on subcarrier n, and $\boldsymbol{p}$ is the transmit power for secondary users with $\boldsymbol{p} = (\{p^1_n, p^2_n, \ldots, p^J_n\})^N_{n=1}$, where p^j_n is the transmit power of secondary user j on subcarrier n.

By pricing the interference from secondary network at the primary receiver and consider a maximum interference threshold, the SUs' transmit power can be controlled. This constraint can be written as

$$\sum_{j=1}^{J} \rho^j_n I^j_n(p^j_n) \le Q_n, \tag{2}$$

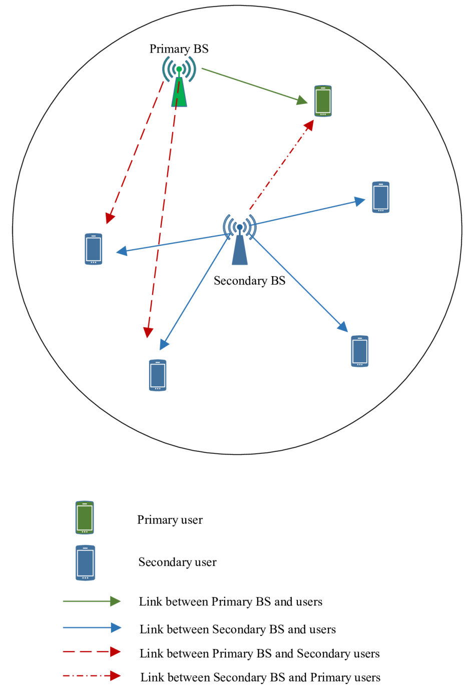

Fig. 1. System Model which Secondary network shares the spectrum with Primary network.

where Q_n is predefined maximum interference at subcarrier n which is acceptable for the primary network.

For spectrum-sensing, the network adopts energy detection. At secondary network's side, because of the nature of wireless communications and the limitations of the spectrum sensing techniques, spectrum sensing is not a perfect function. Consequently, the miss-detected or a false alarm may occur. For considering the effects of spectrum sensing, four different cases should be considered based on the sensing decision which can be present or absent and the status of the primary user which can be active or idle on each channel [12]. As a result, there are four different instantaneous transmission rates of the secondary user, where the first index number (0 for idle and 1 for active) shows the actual status of the primary user and the second index number (0 for absent and 1 for present) shows the decision that is made by the secondary network. Hence, the rate for secondary user j on subcarrier n can be written as

$$r^j_{00,n} = \log_2(1 + \frac{h^j_{ss,n} p^{j0}_n}{\sigma^2}), \tag{3}$$

$$r^j_{01,n} = \log_2(1 + \frac{h^j_{ss,n} p^{j1}_n}{\sigma^2}), \tag{4}$$

$$r^j_{10,n} = \log_2(1 + \frac{h^j_{ss,n} p^{j0}_n}{h^j_{ps,n} p^j_p + \sigma^2}), \tag{5}$$

$$r_{11,n}^{j} = \log_2(1+\frac{h_{ss,n}^{j} p_n^{j1}}{h_{ps,n}^{j} p_p^{j} + \sigma^2}), \tag{6}$$

where $p_{p,n}$ shows the transmit power of the primary network on the n^{th} sub-carrier.

As a result, the average rate of the n^{th} sub-carrier can be formulated as

$$\begin{aligned} \bar{r}_n^j &= \mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j) r_{00,n}^j \\ &+ \mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j r_{01,n}^j \\ &+ \mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j) r_{10,n}^j \\ &+ \mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j r_{11,n}^j, \end{aligned} \tag{7}$$

$\mathscr{P}(H_{0,n}^j)$ indicates the probability that the n^{th} sub-carrier is idle and $\mathscr{P}(H_{1,n}^j)$ indicates the probability that the n^{th} sub-carrier is active. The probability of detection and false alarm for the n^{th} sub-carrier at the secondary detector user $j^{th}$are shown by $\mathscr{P}_{d,n}^j$ and $\mathscr{P}_{fa,n}^j$, respectively.

By defining γ_n as the received signal-to-noise ratio (SNR) from the primary user at the secondary detector on channel n and f_n as the sampling frequency, the probability of detection and false alarm for the n^{th} sub-carrier under the energy detection scheme is given by

$$\mathscr{P}_{d,n} = Q\Big((\frac{\varepsilon_n}{\sigma^2} - \gamma_n - 1)\sqrt{\frac{\tau f_s}{2\gamma_n + 1}}\Big), \tag{8}$$

$$\begin{aligned} \mathscr{P}_{fa,n} &= Q\Big((\frac{\varepsilon_n}{\sigma^2} - 1)\sqrt{\tau f_s}\Big) \\ &= Q\Big(\sqrt{2\gamma_n + 1} Q^{-1}(\mathscr{P}_{d,n}^j) + \sqrt{\tau f_s}\gamma_n\Big), \end{aligned} \tag{9}$$

where τ and ε_n indicate the sensing time and the decision threshold on sub-carrier n, respectively.

To protect the primary network, we consider a target detection probability as $\mathscr{P}_{d,n} = \bar{\mathscr{P}}_d$ for $n = 1, \cdots, N$, then, based on that the decision threshold can be obtained as

$$\varepsilon_n = \sigma^2\Big(\sqrt{\frac{2\gamma_n + 1}{\tau f_s}} Q^{-1}(\bar{\mathscr{P}}_d) + \gamma_n + 1\Big), \tag{10}$$

The interference of sensing-based spectrum access can be written as follows

$$\begin{aligned} I_n^j(p_n^j) &= h_{sp,n} p_n^{j0} \mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j) \\ &+ h_{sp,n} p_n^{j1} \mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j, \end{aligned} \tag{11}$$

The other function which should be considered for secondary network is the cost function, U_{cost}, which is based on the interference that secondary user intends to buy from the primary network under the interference price.

After defining these functions, the utility for secondary user n can be defined as

$$U_{\text{secondary}}^{j,n}(\boldsymbol{\mu}, \boldsymbol{p}, \boldsymbol{\rho}) = \rho_n^j\big[\bar{r}_n^j - U_{\text{cost}}(\boldsymbol{\mu}, \boldsymbol{p})\big], \tag{12}$$

For the cost functions, we can design different approaches based on maximum interference that the primary network can tolerate. Moreover, based on status of the primary network which can be active or idle, we propose the different strategies based on the average interference as

$$U_{\text{cost}}^{j,n} = \begin{cases} \text{Active primary:} \begin{cases} \mu_n^1 I_n^j, & \text{if} \quad 0 < I_n^j \leq Q \\ \infty, & \text{if} \quad I_n^j > Q \end{cases} \\ \text{Idle primary:} \qquad \mu_n^0 \end{cases} \tag{13}$$

where μ_n^1 and μ_n^0 are denoting the interference price for active and idle statuses at sub-carrier n, respectively. Therefore, $U_{n,\text{secondary}}^j(\boldsymbol{\mu}, \boldsymbol{p}, \boldsymbol{\rho})$ can be obtained as follows:

$$\begin{aligned} &U_{\text{secondary}}^{j,n}(\boldsymbol{\mu}, \boldsymbol{p}, \boldsymbol{\rho}) = \\ &\rho_n^j\big[\mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j)(r_{00,n}^j - \mu_n^0) \\ &+ \mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j(r_{01,n}^j - \mu_n^0) \\ &+ \mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)(r_{10,n}^j - \mu_n^1 h_{sp,n} p_n^{j0}) \\ &+ \mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j(r_{11,n}^j - \mu_n^1 h_{sp,n} p_n^{j1})\big], \end{aligned} \tag{14}$$

Furthermore, an average transmit power constraint should be considered in order to keep the transmit power budget of the secondary network. Hence, power allocation strategy is needed at the secondary network to maximize its utility. The average transmission power of secondary network is given by

$$\begin{aligned} \bar{P} &= \sum_{j=1}^{J}\sum_{n=1}^{N} \rho_n^j\big[\mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j)(p_n^{j0}) \\ &+ \mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j(p_n^{j1}) \\ &+ \mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)(p_n^{j0}) \\ &+ \mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j(p_n^{j1})\big]. \end{aligned} \tag{15}$$

The maximum allowable transmission power that can be used at the secondary transmitter is denoted by P^{max}. Mathematically, this problem can be formulated as

$$\begin{aligned} \max_{\boldsymbol{p}, \boldsymbol{\rho}} \quad & \sum_{j=1}^{J}\sum_{n=1}^{N} U_{\text{secondary}}^{j,n}(\boldsymbol{\mu}, \boldsymbol{p}, \boldsymbol{\rho}) \\ s.t. \quad & \bar{P} \leq P^{max}. \end{aligned} \tag{16}$$

Primary network's objective is to maximize its revenue obtained from selling the spectrum to secondary users which can be expressed as

$$\begin{aligned} \max_{\boldsymbol{\mu}} \quad & \sum_{j=1}^{J}\sum_{n=1}^{N} U_{\text{primary}}^{j,n}(\boldsymbol{\mu}, \boldsymbol{p}, \boldsymbol{\rho}) \\ s.t. \quad & \sum_{j=1}^{J} \rho_n^j I_n^j(p_n^j) \leq Q_n, \quad \forall n. \end{aligned} \tag{17}$$

where U_{primary} is obtained as:

$$U^{j,n}_{\text{primary}}(\boldsymbol{\mu},\boldsymbol{p},\boldsymbol{\rho}) = \rho^j_n\big[\mu^0_n\mathscr{P}(H^j_{0,n}) +\mu^1_n\big(\mathscr{P}(H^j_{1,n})(1-\mathscr{P}^j_{d,n})h_{sp,n}p^{j0}_n +\mathscr{P}(H^j_{1,n})\mathscr{P}^j_{d,n}h_{sp,n}p^{j1}_n\big)\big]. \quad (18)$$

We should solve this two problems together. Therefore, we jointly investigate to maximize the revenue of the primary network and the individual utilities of secondary users for the proposed price.

To this end, we define new variable s^j_n which is equal to $s^j_n = \rho^j_n p^j_n$. Hence, the problem variables are changed from $(\boldsymbol{\mu},\boldsymbol{p},\boldsymbol{\rho})$ to $(\boldsymbol{\mu},\boldsymbol{s},\boldsymbol{\rho})$. By using this variable, the proposed functions is updated with following new functons:

$$r^j_{00,n} = \log_2(1+\frac{h^j_{ss,n}s^{j0}_n}{\sigma^2\rho^j_n}), \quad (19)$$

$$r^j_{01,n} = \log_2(1+\frac{h^j_{ss,n}s^{j1}_n}{\sigma^2\rho^j_n}), \quad (20)$$

$$r^j_{10,n} = \log_2(1+\frac{h^j_{ss,n}s^{j0}_n}{\rho^j_n(h^j_{ps,n}p^j_p+\sigma^2)}), \quad (21)$$

$$r^j_{11,n} = \log_2(1+\frac{h^j_{ss,n}s^{j1}_n}{\rho^j_n(h^j_{ps,n}p^j_p+\sigma^2)}), \quad (22)$$

$$I^j_n(p^j_n) = \frac{h_{sp,n}s^{j0}_n}{\rho^j_n}\mathscr{P}(H^j_{1,n})(1-\mathscr{P}^j_{d,n}) + \frac{h_{sp,n}s^{j1}_n}{\rho^j_n}\mathscr{P}(H^j_{1,n})\mathscr{P}^j_{d,n}, \quad (23)$$

$$\bar{P} = \sum_{j=1}^{J}\sum_{n=1}^{N}\mathscr{P}(H^j_{0,n})(1-\mathscr{P}^j_{fa,n})(s^{j0}_n) +\mathscr{P}(H^j_{0,n})\mathscr{P}^j_{fa,n}(s^{j1}_n) +\mathscr{P}(H^j_{1,n})(1-\mathscr{P}^j_{d,n})(s^{j0}_n) +\mathscr{P}(H^j_{1,n})\mathscr{P}^j_{d,n}(s^{j1}_n), \quad (24)$$

Furthermore the revenue and utility functions are changed to:

$$U^{j,n}_{\text{primary}}(\boldsymbol{\mu},\boldsymbol{s},\boldsymbol{\rho}) = \rho^j_n\big(\mu^0_n\mathscr{P}(H^j_{0,n})\big) +\mu^1_n\big(\mathscr{P}(H^j_{1,n})(1-\mathscr{P}^j_{d,n})h_{sp,n}s^{j0}_n +\mathscr{P}(H^j_{1,n})\mathscr{P}^j_{d,n}h_{sp,n}s^{j1}_n\big), \quad (25)$$

and

$$U^{j,n}_{\text{secondary}}(\boldsymbol{\mu},\boldsymbol{s},\boldsymbol{\rho}) = \rho^j_n\big[\mathscr{P}(H^j_{0,n})(1-\mathscr{P}^j_{fa,n})(r^j_{00,n}-\mu^0_n) +\mathscr{P}(H^j_{0,n})\mathscr{P}^j_{fa,n}(r^j_{01,n}-\mu^0_n) +\mathscr{P}(H^j_{1,n})(1-\mathscr{P}^j_{d,n})(r^j_{10,n}-\mu^1_n h_{sp,n}\frac{s^{j0}_n}{\rho^j_n}) +\mathscr{P}(H^j_{1,n})\mathscr{P}^j_{d,n}(r^j_{11,n}-\mu^1_n h_{sp,n}\frac{s^{j1}_n}{\rho^j_n})\big], \quad (26)$$

To solve the problem, a constitute process should be taken. A set of prices based on the received interference power from each secondary user is poses by the primary network. Then, the resource allocation of secondary users is updated to maximize their utilities based on the assigned interference prices [13].

IV. Game Model

A game contains three elements: the players, the strategy space, and the payoff function, to measure the outcome of players. We can model the behaviors of cognitive radio network as a dynamic spectrum sharing game [14]. The users of secondary and primary network act as players in the game model. The variables which are introduced in prior section are the strategy space for the players. For example, which licensed spectrum of the primary network will be used, what transmission power will be applied and which price will be paid, are strategy space for secondary users. In primary network, the strategy space includes the sub-carrier which will be shared with secondary network and the prices which will be offered to secondary network for using that spectrum. Moreover, the payoff functions are the utility and revenue functions for secondary and primary network.

In this paper, we consider pricing the interference from the secondary network to protect the primary network. We applied the Stackelberg game model [15] to our proposed problem. In Stackelberg game which is a strategic game, there are a leader and several followers which try to obtain specific resources. Firstly, the leader does a certain action and based on that the followers move subsequently. In this scenario, the primary transmitter is the leader, and the secondary users are the followers. A set of prices for the received interference power from secondary network are offered by the primary transmitter. Then, based on the optimal interference prices, the secondary network update the power allocation strategy to maximize its utility function .

Problems (16) and (17) should be solved together by a Stackelberg game. In the this game formulation, s^j_n is a function of price. Since there is an interference threshold at the primary transmitter, it is required that primary network consider a appropriate price for secondary interference which maximize the primary network's revenue by considering that interference threshold. Then, based on this price, the secondary network can decide how much interference to buy from primary network.

For secondary network, the utility function consists of two parts, profit and cost [11]. By growing the transmit power of secondary network, the transmission rate enhances, and as a result the profit increases. When the transmit power increases, more interference is brought to the primary network by the secondary network. Hence, the secondary network has to pay more price to the primary network and the cost function will be increased. In the next stage, the optimal power allocation in the secondary network can be obtained by solving (16).

A. Stackelberg Equilibrium

Finding the primary's and secondary's equilibrium points is a objective of Stackelberg game. For our game, the Stackelberg Equilibrium (SE) is defined as follows:

Definition 1): If $\boldsymbol{\mu}^*$ is obtained from problem (17) and $\boldsymbol{s}^*$ and $\boldsymbol{\rho}^*$ obtained from (16), then the point $(\boldsymbol{\mu}^*, \boldsymbol{p}^*, \boldsymbol{\rho}^*)$ is a SE for a proposed game provided that any $(\boldsymbol{\mu}, \boldsymbol{s}, \boldsymbol{\rho})$ the conditions are satisfied in (27) on the top of next page, where $\boldsymbol{s_n}$ and $\boldsymbol{\rho_n}$ are vectors of power and subcarrier allocation for all users except user n, respectively, i.e., $\boldsymbol{s_{-n}} = [\boldsymbol{s_1}, \ldots, \boldsymbol{s_{n-1}}, \boldsymbol{s_{n+1}}, \ldots, \boldsymbol{s_N}]^T$ and $\boldsymbol{\rho_{-n}} = [\boldsymbol{\rho_1}, \ldots, \boldsymbol{\rho_{n-1}}, \boldsymbol{\rho_{n+1}}, \ldots, \boldsymbol{\rho_N}]^T$

In Stackelberg game, the Nash Equilibrium (NE) is a subgame which is used to find Stackelberg Equilibrium. To achieve this, the secondary users compete in a non-cooperative way where NE is a point at which there is no utility improvement for any player by changing its strategy where other players continues to use their current strategies [11]. For the primary network, the optimal solution can be obtained by solving Problem (17). To this end, the primary network can obtain the best revenue based on utility function of the secondary network which must be solved firstly. In this paper, to obtain the SE, in first step, we consider a set of interference price and based on them solve (16). In the next step, (17) is solved to obtain the optimal interference price based on result of the secondary network in pervious step.

Finally, the dual method is used to solve the problem. To achieve this end, the Lagrangian function L of this problem can be written as

$$\begin{aligned}
&L(\boldsymbol{\mu}, \boldsymbol{s}, \boldsymbol{\rho}) = \\
&\rho_n^j \big[\mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j)(r_{00,n}^j - \mu_n^0) \\
&+\mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j(r_{01,n}^j - \mu_n^0) \\
&+\mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)(r_{10,n}^j - \mu_n^1 h_{sp,n}\frac{s_n^{j0}}{\rho_n^j}) \\
&+\mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j(r_{11,n}^j - \mu_n^1 h_{sp,n}\frac{s_n^{j1}}{\rho_n^j})\big] \\
&-\lambda\big(\sum_{j=1}^{J}\sum_{n=1}^{N}\mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j)(s_n^{j0}) \\
&+\mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j(s_n^{j1}) \\
&+\mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)(s_n^{j0}) \\
&+\mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j(s_n^{j1}) - P^{max}\big). \qquad (28)
\end{aligned}$$

To solve the mentioned problem the Karush-Kuhn-Tucker (KKT) conditions must be satisfied. Accordingly, the solution of (16) can be obtained by solving the KKT conditions. For a given set of μ_n, the solution of (16) is obtained by $\boldsymbol{s}^*, \boldsymbol{\rho}^*$ where $s_n^{j0^*}$ and $s_n^{j1^*}$ are the roots of the following equations:

$$\begin{aligned}
&\frac{\partial L(\boldsymbol{\mu}, \boldsymbol{s}, \boldsymbol{\rho})}{\partial s_n^{j0}} = 0 \\
&a_1 . \frac{h_{ss,n}^j}{\rho_n^j \sigma^2 + h_{ss,n}^j s_n^{j0}} \\
&+b_1 . \big(\frac{h_{ss,n}^j}{\rho_n^j(h_{ps,n}^j p_p^n + \sigma^2) + h_{ss,n}^j s_n^{j0}} - \frac{h_{sp,n}\mu_n}{\rho_n^j}\big) \\
&-c_1 = 0, \qquad (29)
\end{aligned}$$

where $a_1 = \frac{\mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j)}{\ln(2)}$, $b_1 = \frac{\mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)}{\ln(2)}$ and $c_1 = \lambda\big(\frac{\mathscr{P}(H_{0,n}^j)(1-\mathscr{P}_{fa,n}^j)+\mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)}{\rho_n^j}\big)$.

$$\begin{aligned}
&\frac{\partial L(\boldsymbol{\mu}, \boldsymbol{s}, \boldsymbol{\rho})}{\partial s_n^{j1}} = 0 \\
&a_2 . \frac{h_{ss,n}^j}{\rho_n^j \sigma^2 + h_{ss,n}^j s_n^{j1}} \\
&+b_2 . \big(\frac{h_{ss,n}^j}{\rho_n^j(h_{ps,n}^j p_p^n + \sigma^2) + h_{ss,n}^j s_n^{j1}} - \frac{h_{sp,n}\mu_n}{\rho_n^j}\big) \\
&-c_2 = 0, \qquad (30)
\end{aligned}$$

where $a_2 = \frac{\mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j}{\ln(2)}$, $b_2 = \frac{\mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j}{\ln(2)}$ and $c_2 = \lambda\big(\frac{\mathscr{P}(H_{0,n}^j)\mathscr{P}_{fa,n}^j+\mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j}{\rho_n^j}\big)$ and λ is chosen such that power constraint is satisfied with equality.

Now for each subcarrier, the secondary user with the maximum SINR will be selected:

$$\rho_n^{j\,*} = \begin{cases} 1 & j^* = \arg\max_j \text{SINR}_n^j, \\ 0 & \text{o.w.} \end{cases} \qquad (31)$$

where SINR_n^j is the signal to noise of secondary user j on subcarrier n.

Substituting $p_n^{j\,*}, \rho_n^{j\,*}$ into Problem (32), the optimization problem of primary network can be formulated as

$$\begin{aligned}
\max_{\boldsymbol{\mu}} \quad & \sum_{j=1}^{J}\sum_{n=1}^{N} U_{\text{primary}}^{j,n}(\boldsymbol{\mu}, \boldsymbol{s}^*, \boldsymbol{\rho}^*) \\
s.t. \quad & \sum_{j=1}^{J} \rho_n^{j\,*} I_n^j(s_n^{j\,*}) \le Q_n, \quad \forall n. \qquad (32)
\end{aligned}$$

Hence, to solve the problem the Lagrangian function of this problem can be obtained as follows:

$$\begin{aligned}
&L_2(\boldsymbol{\mu}, \boldsymbol{s}^*, \boldsymbol{\rho}^*) = \\
&\rho_n^{j\,*}\big(\mu_n^0 \mathscr{P}(H_{0,n}^j)\big) \\
&+\mu_n^1\big(\mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j)h_{sp,n}s_n^{j0*} \\
&+\mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j h_{sp,n}s_n^{j1*}\big) \\
&-\nu_n\big(h_{sp,n}s_n^{j0*}\mathscr{P}(H_{1,n}^j)(1-\mathscr{P}_{d,n}^j) \\
&+h_{sp,n}s_n^{j1*}\mathscr{P}(H_{1,n}^j)\mathscr{P}_{d,n}^j - Q_n\big). \qquad (33)
\end{aligned}$$

This problem has the same structure as previous problem. Therefore, it can be solved by the same dual method. To solve it, the KKT conditions must be satisfied.

With the updated values of the prices, the resource allocation are evaluated again and this process continues iteratively within the desired accuracy.

V. Simulation Results

In this section, we present the numerical results to figure out the performance of the proposed interference pricing approach in cognitive networks under various system parameters. In this setup, the coverage area of secondary network is a circle with 1 Km diameter and secondary's users and primary's user

$$
\begin{aligned}
U^{j,n}_{\text{secondary}}(\boldsymbol{\mu}^*, \boldsymbol{s_n}^*, \boldsymbol{\rho_n}^*, \boldsymbol{s_{-n}}^*, \boldsymbol{\rho_{-n}}^*) &\geq U^{j,n}_{\text{secondary}}(\boldsymbol{\mu}^*, \boldsymbol{s_n}, \boldsymbol{\rho_n}, \boldsymbol{s_{-n}}, \boldsymbol{\rho_{-n}}) \\
U^{j,n}_{\text{primary}}(\boldsymbol{\mu}^*, \boldsymbol{s}^*, \boldsymbol{\rho}^*) &\geq U^{j,n}_{\text{primary}}(\boldsymbol{\mu}, \boldsymbol{s}^*, \boldsymbol{\rho}^*)
\end{aligned} \tag{27}
$$

are randomly distributed inside the coverage of secondary network. The number of secondary's users and sub-carriers are $M = 10$ and $N = 32$ and we set $P^{max} = 10$ dB. Furthermore, $h = \chi d^{-\beta}$ where $\beta = 2$ is the path-loss exponent, χ is the exponential random variable (i.e., representing the Rayleigh fading) with mean one and d is the distance between transmitter and receiver, and finally, we set $\sigma^2 = 1$.

The frame duration of the secondary network is set to 100 ms and the sampling frequency to 6 MHz and $\tau = 10$ ms . For all channels, we consider target detection probability as $\bar{\mathscr{P}}_d = 0.9$, and the worst-case received SNR from the primary user at the secondary detector on each channel equal to 12 dB. Moreover, the probability that the channel is idle supposed to be $\mathscr{P}(H_0^j) = 0.6$.

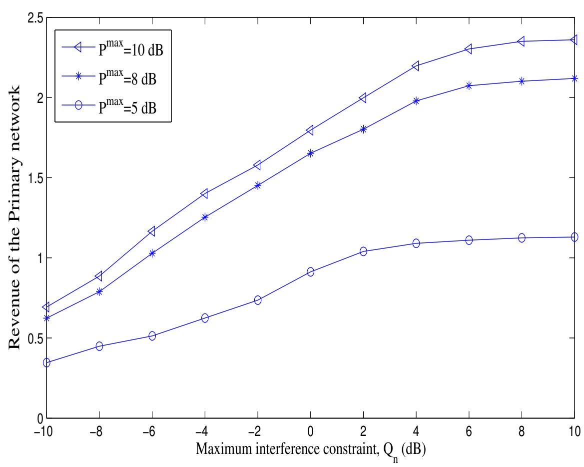

Fig. 2. Revenue of the primary network vs. the maximum tolerable interference margin Q_n.

First, we show the primary revenue versus the maximum interference threshold Q_n at the primary network for different values of power threshold in Fig. 2. As it shows when the interference margin grows, the primary's revenue increases. This can be explained as follows. By increasing Q_n, the feasibility set size is increased. It means that I increases and as a result the primary's revenue increase.

Fig. 3 shows the revenue of the primary network versus the different number of secondary users for various probability of the idle frequency band, $\mathscr{P}(H_0^j)$. As it is shown, by growing the number of secondary users and the probability of idle band, the revenue increases. In Fig. 4, the revenue of the primary network is shown versus the maximum tolerable interference margin Q_n for perfect and imperfect sensing. As can be seen, when the sensing error is considered, the interference to the primary band should be below the interference threshold. In other words, if perfect sensing is assumed, then the secondary network transmits the higher power that leads to the higher interference levels at the primary network, hence, the revenue increases.

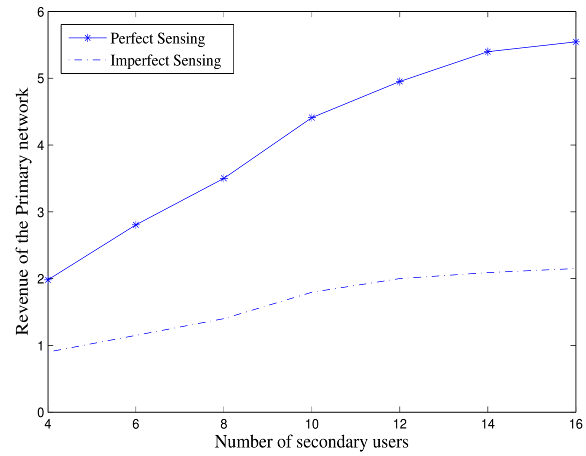

Fig. 3. Revenue of the primary network vs. different number of secondary users for different probability of the idle frequency band, $\mathscr{P}(H_0^j)$.

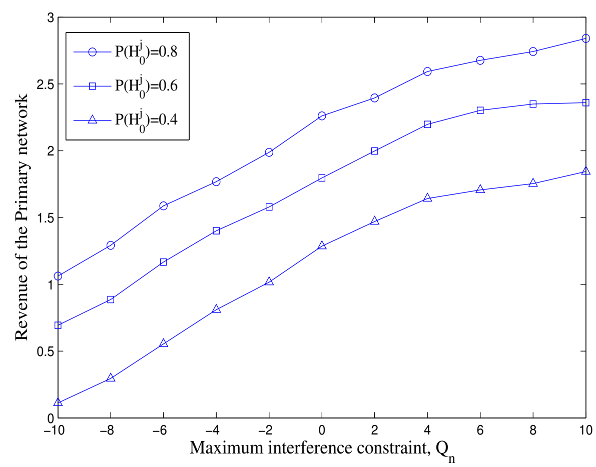

Fig. 4. Revenue of the primary network vs. the maximum tolerable interference margin Q_n for perfect and imperfect sensing.

Fig. 5 shows the sum rate of cognitive users versus the maximum interference threshold Q_n for different values of power threshold. As it can be seen, cognitive sum rate grows by increasing the interference margin. When interference margin is high, cognitive network can send information with the higher power and as a result the rate will be increased. Similar to the prior figure, when the power threshold increases, the sum rate growth is progressive for a given power since the

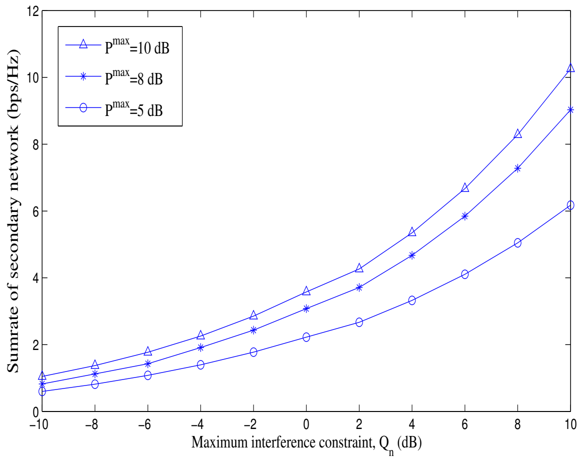

Fig. 5. Sum-rate of secondary network users vs. the maximum tolerable interference margin Q_n.

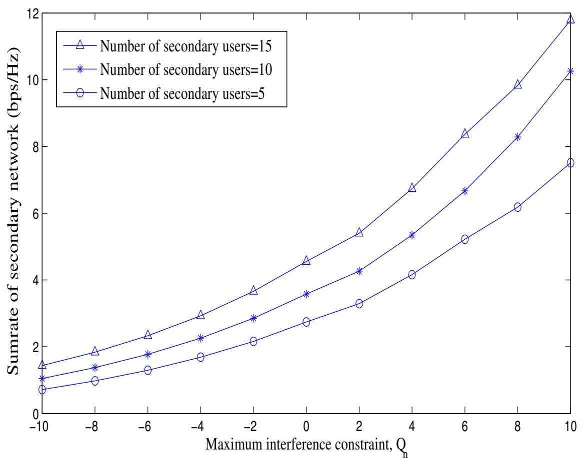

Fig. 6. Sum-rate of secondary network users vs. the maximum tolerable interference margin Q_n for different number of secondar y users.

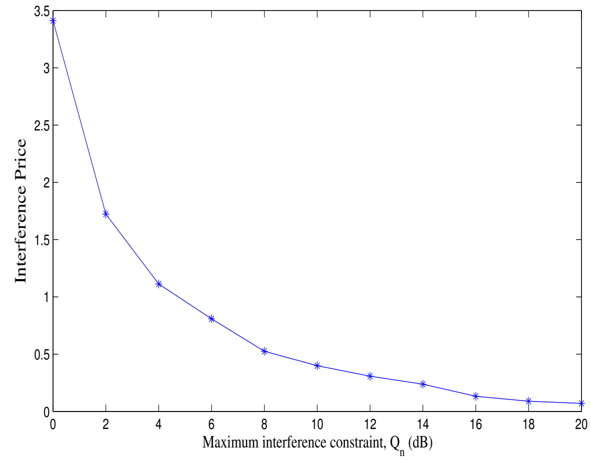

Fig. 7. Interference price for one of subcarrier vs. the maximum tolerable interference margin Q_n.

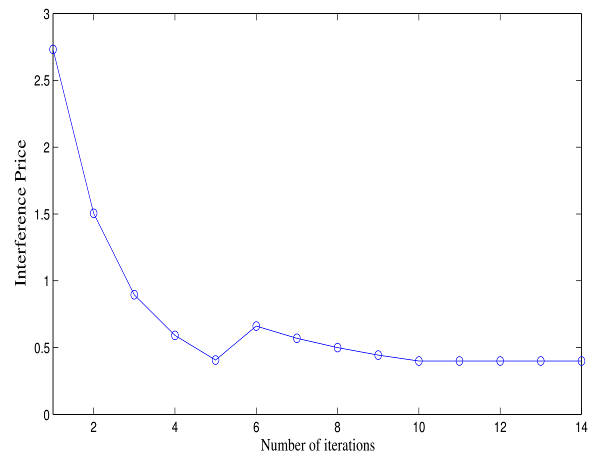

Fig. 8. Convergence performance of the proposed algorithm.

system performance is limited by the constraint on interference threshold.

Fig. 6, shows the secondary sum rate versus the maximum interference threshold Q_n for the different number of the secondary users. Obviously, cognitive sum rate enhances by growing the interference margin. As can be seen, secondary sum rate grows by increasing the number of users.

In Fig. 7, one of the interference price for one subcarrier is shown. It is observed that the interference prices decreases with the increasing of Q_n. It can be inferred that the PU would like to price lower if it has a large amount of spectrum to sell. In addition, the converges of the proposed algorithm is shown in Fig. 8.

VI. Conclusion

In this paper, we have investigated the price-based resource allocation strategies on the performance of an OFDMA spectrum sensing-based cognitive network using game theory. To guarantee the quality-of-service of the primary network, an interference constraint has been considered. A radio resource allocation problem including the interference prices and the power and sub-carrier allocation strategies with the aim of maximizing the utility functions of secondary and primary network has been proposed and solved by Stackelberg game model to jointly study the utility maximization of the secondary and primary networks. The impact of different system parameters such as maximum transmit power of secondary network and interference threshold at primary network on the achievable utility has been also investigated through simulations. It has been shown that this algorithm can be implemented with low complexity and requires minimal information exchange between the primary and secondary networks.

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