

# An indirect adaptive fuzzy sliding mode controller for stabilization of multi-machine power systems

Mehdi Siahi<sup>1\*</sup>, Hossein Soghrati<sup>2</sup>

Received: 2015/6/7

Accepted: 2015/7/11

## Abstract

This paper introduces an indirect adaptive fuzzy sliding mode controller as a power system stabilizer for damping local and inter-area modes of oscillations of multi-machine power systems. This controller is designed based on the combination of sliding mode controller and the fuzzy logic systems. The fuzzy systems are used to approximate the unknown functions of power system model. Generator speed deviation and accelerator power are selected as fuzzy logic system inputs. A new sliding mode control law achieved by changing the sliding condition and the undesirable chattering has been removed by using of a continuous function. Based on the Lyapunov synthesis, adaptation laws are developed. Performance of the proposed stabilizer is studied for a two-area four-machine power system. Simulation results show the effectiveness of the proposed controller in comparison with multi-band power system stabilizer (MB-PSS), classical adaptive fuzzy sliding mode stabilizer and adaptive fuzzy sliding mode stabilizer with a proportional integral function (PI).

**Keywords:** Sliding mode control, chattering elimination, adaptive fuzzy control, multi-machine power system, power system stabilizer

## 1. Introduction

Power systems become larger and more complex, every day. Their behavior is nonlinear and the occurrence of disturbances such as a short circuit and load change causes the electromechanical oscillations in the synchronous generators. These low-frequency oscillations are an inherent problem in the power systems that can lead to instability and Loss of synchronism and separation of interconnected networks. Power system stabilizer provides an auxiliary control signal for synchronous generator excitation system to increase the stability and performance of the power system.

Conventional power system stabilizer (CPSS) is the first type of power system stabilizer which includes Lead-Lag Phase controllers with fixed structure and parameters [1]. These parameters are calculated based on the mathematical model of the power system and operating point. Due to the nonlinear behavior of the power system and the uncertainty of parameters, the stabilizers cannot achieve good results in a wide range of operating points and need to be reorganized. Multi-band stabilizer (MBPSS) is the latest and best stabilizer of this type which has more appropriate response, described in [2] and [3].

To track the changes in the power system, adaptive stabilizers such as [4], [5] and [6] have been proposed. These stabilizers require identification of power system parameters and its states estimation. The sliding mode controllers are some of the most robust control methods for nonlinear systems that can make a good stabilization even with changes in the system parameters. This type of stabilizers is presented in [7], [8] and [9]. This control method requires mathematical model, but it is complicated to provide this model for the power systems.

For years, fuzzy logic systems are used in the controller design. Fuzzy logic systems can perform controlling act without having any knowledge of the non-linear functions of the system. In recent years, there have been attempts to use fuzzy logic systems in the power system stabilization. Implementation of a fuzzy logic based power system stabilizer is described in [10]. It is a model-free approach and its parameters considered fixed. Therefore, changes in the characteristics of the power system, causes the decrease in stabilizer efficiency. The Design of a type of fuzzy PID-like controller with a mechanism to predict error and adjust the coefficients of the controller as a power system stabilizer is presented in [11]. In [12], a self-learning fuzzy PD controller is presented which

1. Assistant Professor, Department of Electrical Engineering, Islamic Azad University, Garmsar Branch, Garmsar. Iran. mahdi\_siahi@yahoo.com

2. M. Sc. Student, Department of Electrical Engineering, Islamic Azad University, Garmsar Branch, Iran, h.soghrati@chmail.ir.

uses steep decrease algorithm to identify system and make changes in input and output of membership functions and fuzzy rules in order to adaptation. The model-free controllers have limited ability in control of nonlinear systems. Direct adaptive fuzzy controller in [13] and [14] has been used as the power system stabilizer. This means that the fuzzy logic system is used to approximate the feedback control law. Indirect methods will be obtained if the fuzzy approximation be used to approximate unknown system functions. Indirect adaptive fuzzy power system stabilizers using feedback linearization control law have been proposed in [15],[16] and [17]. The main difference is in the mechanism of adaptation and robustness of controller. They have attempted to make the controllers robust, because the approximation error always exists in them.

The sliding mode control (SMC) is an effective robust control to deal with parameter uncertainties and disturbance for nonlinear system [18], it is widely used for the nonlinear control scheme. The combination of sliding mode controller and fuzzy approximation creates an indirect adaptive fuzzy sliding mode controller. However, there is severe chattering in the output of classical sliding mode controller. The cause is the existence of sign function in its control law, and it makes the implementation and practical application of this type of stabilizer hard or impossible. In the [19], [20] and [21] some methods have been proposed to chattering elimination. The proposed adaptive fuzzy sliding mode controller in [21] used a proportional integral (PI) control law instead of switching section of sliding mode controller. According to this method a power system stabilizer is provided in [22] and [23]. The steady state error of PI controller can reach to zero. By using of PID control function on sliding surface (S(t)) instead of switching term of sliding mode control law, finally the states of the system is placed on the sliding surface and the steady state error becomes zero, without chattering in control signal. Nevertheless, if there is a disturbance in the system, the steady state error would not be zero. In the power systems, small and large disturbances such as faults, load and generator operating point changes, continuously occur. In this system, the goal of the controllers is reducing

the stabilization time and amplitude of rotor speed error into an acceptable range.

An idea to remove chattering by making changes in the sliding condition is discussed in [20]. However, the obtained control law is incorrect. In this paper, the considered idea in [20] to eliminate the chattering of sliding mode controller output, is corrected, and an indirect adaptive fuzzy sliding mode stabilizer for multi-machine power systems is presented base on it. In Section 2, a generator dynamic model for sliding mode control law is given, and in Section 3, the sliding mode control law with different sliding condition is studied. In Section 4, an adaptive fuzzy system applied to approximate unknown functions of the power system, and the adaptive fuzzy sliding mode control law is obtained. In Section 5, a description of the test power system is given, and in Section 6, the effectiveness of the proposed stabilizer in damping of oscillations under various disturbances and simulation results are shown. Conclusions are stated in Section 7.

## 2. Power system model

In order to use of the sliding mode controller for second order systems, system equations must be expressed as the follow [18]:

$$\begin{cases} \dot{x}^{(n)} = f(x) + g(x)u \\ y = x \end{cases} \quad (1)$$

Speed deviation and electrical power are easily measurable directly. By choosing of speed deviation and power accelerator of synchronous generator as the state variables,  $[x_1, x_2] = [\Delta\tilde{S}, \Delta P]$ , and speed deviation as the output of power system,  $y = \Delta\tilde{S}$ , the equation set for  $i$  th generator is represented as follow [23], [24]:

$$\begin{cases} \Delta\dot{\tilde{S}}_i = \frac{1}{2H_i} \Delta P_i \\ \frac{1}{2H_i} \Delta\dot{P}_i = f_i(\Delta\tilde{S}_i, \Delta P_i) + g_i(\Delta\tilde{S}_i, \Delta P_i)u_i \\ y_i = \Delta\tilde{S}_i \end{cases} \quad (2)$$

where  $u_i$  is output of stabilizer and the control signal that will be designed in later sections,  $f_i$  and  $g_i$  are uncertain nonlinear functions of power system.  $x_{i1} = \Delta\tilde{S}_i = \tilde{S}_i - \tilde{S}_0$  is the speed deviation and  $x_{i2} = \Delta P_i = P_{mi} - P_{ei}$  is the accelerating power,  $\tilde{S}_i$  is the rotor angular speed,

$\dot{S}_0$  is the synchronous speed,  $P_{mi}$  is the input mechanical power of synchronous generator which is assumed constant in the controller design,  $P_{ei}$  is the output electric power and  $H_i$  is the machine inertia constant. The positive values of the stabilizer signal ( $u_i > 0$ ) lead to an increase in generator output voltage and therefore positive change in the output power ( $\dot{P}_{ei} > 0$ ) which sign is opposite to  $\Delta\dot{P}_i$ . In other words,  $g_i$  is a negative function [23], [24].

### 3. Indirect adaptive fuzzy sliding mode controller for power system stabilizer

#### 3.1. Sliding mode control design

The goal of sliding mode control is to force the output of system ( $y$ ) to track a bounded desired trajectory ( $y_d$ ), under uncertainties and disturbances, so that all signals in the closed-loop system and system states are bounded. For a second order system, the tracking error defined as  $e = y - y_d = [e, \dot{e}]^T$  and sliding surface is:

$$s = \dot{e} + \lambda e \tag{3}$$

If  $s(e, t) = 0$ , then the error moves toward zero, exponentially. For this purpose, in the sliding mode control the Lyapunov function defined as:

$$V = \frac{1}{2} s^2 \tag{4}$$

and the derivative of the Lyapunov function is generally considered as follow:

$$\dot{V} = s\dot{s} \leq -\gamma |s| \tag{5}$$

where  $\gamma$  is a strictly positive constant. Inequality (5) is called the sliding condition. The discrete control function ( $u_s = K \times \text{sign}(s)$ ) exists in classical sliding mode control law which causes chattering in the control output signal. The frequency and the time constant of controller are limited in the implementation of sliding mode control. Therefore, the error of the system output will not become zero, and even the chattering may appear in the system output. The chattering could be eliminated through a continuous approximation of the discontinuous control ( $u_s = K \times \text{sat}(s/\lambda)$ ) in a boundary layer around the sliding surface

( $B(t) = \{x : |s(x, t)| < \lambda\}$ ) [18], where  $\lambda$  is boundary layer thickness which determines the accuracy of the controller, and  $0 < \lambda < 1$ . In this paper, the sliding condition is defined as:

$$\dot{V} = s\dot{s} \leq -\frac{\gamma}{\lambda} s^2 \tag{6}$$

If the sliding condition (6) is satisfied outside the boundary layer, then it is guaranteed that after finite time  $|e(t)| \leq \nu$ , where  $\nu = \{\lambda/\gamma\}^{n-1}$  is width of the boundary layer. Derivative of the sliding surface is:

$$\begin{aligned} \dot{s}(e) &= \ddot{x} - \ddot{x}_d + \lambda \dot{e} \\ &= f(x) + g(x)u - \ddot{x}_d + \lambda \dot{e} \end{aligned} \tag{7}$$

Eq. (6) is written as follow:

$$s(f(x) + g(x)u - \ddot{x}_d + \lambda \dot{e}) \leq -\frac{\gamma}{\lambda} s^2 \tag{8}$$

where  $f(x)$  and  $g(x)$  are unknown functions. The sliding mode control law for the second order system is proposed as follow:

$$\begin{aligned} u &= \frac{1}{\hat{g}(x)} [-\hat{f}(x) + \ddot{x}_d - \lambda \dot{e} - u_s] \\ &= \frac{1}{\hat{g}(x)} [\hat{u} - u_s] \end{aligned} \tag{9}$$

where  $\hat{f}(x)$  and  $\hat{g}(x)$  are estimations of nonlinear functions of system, and equivalent control law is as  $\hat{u} = -\hat{f}(x) + \ddot{x}_d - \lambda \dot{e}$ , and  $u_s$  will be designed in the below.

**Assumption.** The estimation error of system functions is limited to  $F(x)$  and  $S(x)$ , and sign of  $g(x)$  and  $\hat{g}(x)$  are known and same. In other words:

$$|f(x) - \hat{f}(x)| \leq F(x) \tag{10}$$

$$0 < s^{-1} \leq \frac{\hat{g}(x)}{g(x)} \leq s \tag{11}$$

Substituting (9) into (8) leads to:

$$s \left( f(x) + \frac{g(x)}{\hat{g}(x)} [\hat{u} - u_s] - \ddot{x}_d + \lambda \dot{e} \right) \leq -\frac{\gamma}{\lambda} s^2 \tag{12}$$

$-\hat{f}(x) + \hat{f}(x)$  is added and the inequality (12) is written as below:

$$\begin{aligned} -\frac{g(x)}{\hat{g}(x)} su_s + s \left( \frac{g(x)}{\hat{g}(x)} \hat{u} + f(x) - \hat{f}(x) + \hat{f}(x) - \ddot{x}_d + \lambda \dot{e} \right) \\ \leq -\frac{\gamma}{\lambda} s^2 \end{aligned} \tag{13}$$

$$\begin{aligned} -\frac{g(x)}{\hat{g}(x)} su_s + \left( \frac{g(x)}{\hat{g}(x)} \hat{u} + (f(x) - \hat{f}(x)) - \hat{u} \right) s \\ \leq -\frac{\gamma}{\lambda} s^2 \end{aligned} \tag{14}$$

and so:

$$su_s \geq \frac{\hat{g}(x)y}{g(x)\xi} s^2 + \left( \frac{\hat{g}(x)}{g(x)} (f(x) - \hat{f}(x)) + \left( 1 - \frac{\hat{g}(x)}{g(x)} \right) \hat{u} \right) s \quad (15)$$

In order to satisfy (15) we proposed  $u_s$  as:

$$u_s = (s[y + F(x)] + (s - 1)|\hat{u}|) \times \frac{s}{\xi} = K \times \frac{s}{\xi} \quad (16)$$

So, substituting (16) into (15) leads to:

$$\frac{\hat{g}(x)y}{g(x)\xi} s^2 + s \frac{\hat{g}(x)}{g(x)} (f(x) - \hat{f}(x)) + s \left( 1 - \frac{\hat{g}(x)}{g(x)} \right) \hat{u} \leq \frac{1}{\xi} s y s^2 + \frac{1}{\xi} s^2 s F(x) + \frac{1}{\xi} s^2 (s - 1) |\hat{u}| \quad (17)$$

If  $|s| \geq \xi$  (out of boundary layer) then  $s \leq (s^2/\xi)$ , and according to (10) and (11):

$$\frac{\hat{g}(x)y}{g(x)\xi} s^2 \leq \frac{1}{\xi} s y s^2 \quad (18)$$

$$s \frac{\hat{g}(x)}{g(x)} (f(x) - \hat{f}(x)) \leq \frac{1}{\xi} s^2 s F(x) \quad (19)$$

$$s \left( 1 - \frac{\hat{g}(x)}{g(x)} \right) \hat{u} \leq \frac{1}{\xi} s^2 (s - 1) |\hat{u}| \quad (20)$$

Therefore, the sliding condition (6) for  $|s| \geq \xi$  is satisfied. The (17) shows the boundary layer is attractive. The proposed control law is similar to the sliding mode control with the saturation function, in the boundary layer ( $|s| < \xi$ ). If the system affected by unknown disturbances, the system states get out of the boundary layer and the error increases in the system output. The saturation function for out of boundary layer ( $|s| \geq \xi$ ) is constant ( $u_s = \pm K$ ), but the control function in (16) increases proportional to the distance of the system states from the sliding surface ( $u_s = K \times (s/\xi)$ ) and therefore the amplitude of the error and time to reach the boundary layer after removing the disturbance, reduce. This feature discussed in below.

The  $t_{reach}$  is the time required to reach the sliding surface ( $s = 0$ ) or the boundary layer, and by integration of the sliding condition between  $t = 0$  and  $t = t_{reach}$ , is calculated [18]. For classical sliding mode control with the sliding condition (5), and  $s(t=0) > 0$ :

$$\int_0^{t_{reach}} \dot{s} dt \leq \int_0^{t_{reach}} -y dt \quad (21)$$

$$s(t = t_{reach}) - s(t = 0) = 0 - s(t = 0) \leq -y(t_{reach} - 0) \quad (22)$$

In conclusion,  $t_{reach} \leq s(t=0)/y$ . A similar result for  $s(t=0) < 0$  can be obtained. Finally,  $t_{reach} \leq |s(t=0)|/y$  for all  $s(t=0)$ . If the boundary layer is considered,  $|s(t=t_{reach})| = \xi$ , then the time required to reach the boundary layer is:

$$t_{reach} \leq \frac{|s(t=0)| - \xi}{y} \quad (23)$$

However, in this paper, the sliding condition is defined as (6). Thus:

$$\int_0^{t_{reach}} \frac{\dot{s}}{s} dt \leq - \int_0^{t_{reach}} \frac{y}{\xi} dt \quad (24)$$

$$\ln|s| - \ln|s(t=0)| \leq - \frac{y}{\xi} t \quad (25)$$

and therefore:

$$|s| \leq |s(t=0)| e^{-\frac{y}{\xi} t} \quad (26)$$

In other words,  $s$  decreases exponentially. As a result:

$$t_{reach} \leq \frac{\ln \left( \frac{|s(t=0)|}{\xi} \right)}{y} \quad (27)$$

It is clear that:

$$\frac{\ln \left( \frac{|s(t=0)|}{\xi} \right)}{y} < \frac{|s(t=0)| - \xi}{y} \quad (28)$$

In other words, the new sliding condition reduces time to reach the boundary layer.

### 3.2. Fuzzy Logic System

Fuzzy logic system consists of a set of IF-THEN rules as:

$$R(l) : \text{If } x_1 \text{ is } A_1^l \text{ ... and } x_n \text{ and } A_n^l \text{ Then } B^l \quad (29)$$

A fuzzy logic system contains rules base, fuzzifier inference engine and defuzzifier. It can be viewed as a nonlinear mapping from inputs to outputs. By using the singleton fuzzifier, product inference and center average defuzzification, the output value of the fuzzy logic system can be formulated as [25]:

$$y(x) = \frac{\sum_{l=1}^m \left( \prod_{i=1}^n \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^m \left( \prod_{i=1}^n \mu_{A_i^l}(x_i) \right)} \quad (30)$$

where  $\tilde{\mu}_{A_i^l}(x_i)$  is the membership function of the linguistic variable  $x_i$ , and  $y \in R$  is output of the fuzzy logic system,  $A_i^l$  and  $B^l$  are the input and output fuzzy sets, respectively.  $i = 1, \dots, n$  denotes the number of input of fuzzy logic system and  $l = 1, \dots, m$  denotes the number of fuzzy IF-THEN rules.  $u_i$  is the point that the maximum value of output of  $l$ th rule, achieves in. Eq. (30) can be rewritten as:

$$y(x) = \sum_{l=1}^m u_l W_l(x) = u^T W(x) \quad (31)$$

Based on the universal approximation theorem in [26], unknown functions  $f(x)$  and  $g(x)$  can be approximated by (31) as:

$$\hat{f}(x|u_f) = u_f^T W(x) \quad (32)$$

$$\hat{g}(x|u_g) = u_g^T W(x) \quad (33)$$

where  $u_f = [u_f^1, \dots, u_f^m]^T$  and  $u_g = [u_g^1, \dots, u_g^m]^T$  are the parameters vectors of the fuzzy approximations, and  $W(x) = [W_1(x), \dots, W_m(x)]^T$  is the fuzzy basis functions vector that:

$$W_l(x) = \frac{\prod_{i=1}^n \tilde{\mu}_{A_i^l}(x_i)}{\sum_{l=1}^m \left( \prod_{i=1}^n \tilde{\mu}_{A_i^l}(x_i) \right)} \quad (34)$$

### 3.3. Indirect adaptive fuzzy sliding mode control design

Because the system functions  $f(x)$  and  $g(x)$  are unknown, so we replace  $f(x)$  and  $g(x)$  by the fuzzy estimates  $\hat{f}(x|u_f)$  and  $\hat{g}(x|u_g)$  which are in the form of (32) and (33). These functions are used for implementation of sliding mode control law. Adaptive fuzzy sliding mode control law, according to (9) defined as:

$$u = \frac{1}{\hat{g}(x|u_g)} \left[ -\hat{f}(x|u_f) + \ddot{x}_d - \lambda \dot{e} - u_s \right] \quad (35)$$

The above equation is rewritten as:

$$\lambda \dot{e} = -\hat{f}(x|u_f) + \ddot{x}_d - u_s - \hat{g}(x|u_g)u \quad (36)$$

Substituting (36) into (7) leads to:

$$\begin{aligned} \dot{s} &= \left( f(x) - \hat{f}(x|u_f) \right) \\ &+ \left( g(x) - \hat{g}(x|u_g) \right) u - u_s \end{aligned} \quad (37)$$

**Definition.**  $u_f^*$  and  $u_g^*$  are defined as optimal parameters vectors for fuzzy approximation that conclude the smallest estimation error.

$$u_f^* = \arg \min_{u_f \in \Omega_f} \left( \sup_{x \in R^n} \left| \hat{f}(x|u_f) - f(x) \right| \right) \quad (38)$$

$$u_g^* = \arg \min_{u_g \in \Omega_g} \left( \sup_{x \in R^n} \left| \hat{g}(x|u_g) - g(x) \right| \right) \quad (39)$$

where  $\Omega_f$  and  $\Omega_g$  are constraint sets for  $u_f$  and  $u_g$ , respectively. Assuming that the fuzzy approximation parameters are bounded. The difference between optimal values and real values of parameters are defined as:

$$\begin{cases} w_f = u_f^* - u_f \\ w_g = u_g^* - u_g \end{cases} \quad (40)$$

The fuzzy approximations (32) and (33) can be separated based on the optimal value and the approximation error (40) as follow:

$$\begin{aligned} \hat{f}(x|u_f) &= u_f^{*T} W(x) - w_f^T W(x) \\ &= \hat{f}(x|u_f^*) - w_f^T W(x) \end{aligned} \quad (41)$$

$$\begin{aligned} \hat{g}(x|u_g) &= u_g^{*T} W(x) - w_g^T W(x) \\ &= \hat{g}(x|u_g^*) - w_g^T W(x) \end{aligned} \quad (42)$$

So, equation (37) becomes:

$$\begin{aligned} \dot{s} &= f(x) - \hat{f}(x|u_f^*) + w_f^T W(x) \\ &+ \left( g(x) - \hat{g}(x|u_g^*) + w_g^T W(x) \right) u - u_s \end{aligned} \quad (43)$$

Define the minimum estimation error:

$$E = f(x) - \hat{f}(x|u_f^*) + \left( g(x) - \hat{g}(x|u_g^*) \right) u \quad (44)$$

Thus, the equation (43) can be written as:

$$\dot{s} = w_f^T W(x) + w_g^T W(x) u - u_s + E \quad (45)$$

The Lyapunov function is selected as follow:

$$V = \frac{1}{2} s^2 + \frac{1}{2\chi_1} w_f^T w_f + \frac{1}{2\chi_2} w_g^T w_g \quad (46)$$

and its time derivative is:

$$\begin{aligned} \dot{V} &= s\dot{s} + \frac{1}{\chi_1} w_f^T \dot{w}_f + \frac{1}{\chi_2} w_g^T \dot{w}_g \\ &= s \left[ w_f^T W(x) + w_g^T W(x) u - u_s + E \right] \\ &+ \frac{1}{\chi_1} w_f^T \dot{w}_f + \frac{1}{\chi_2} w_g^T \dot{w}_g = s w_f^T W(x) + \frac{1}{\chi_1} w_f^T \dot{w}_f \\ &+ s w_g^T W(x) u + \frac{1}{\chi_2} w_g^T \dot{w}_g - s u_s + s E \\ &= \frac{1}{\chi_1} w_f^T \left[ \chi_1 s W(x) + \dot{w}_f \right] \\ &+ \frac{1}{\chi_2} w_g^T \left[ \chi_2 s W(x) u + \dot{w}_g \right] - s u_s + s E \end{aligned} \quad (47)$$

The derivative of (40) is  $\dot{w}_f = -\dot{u}_f$  and  $\dot{w}_g = -\dot{u}_g$ .

If the adaptation laws be selected as (48) and (49), the Eq. (47) could be simplified as (50):

$$\dot{u}_f = \chi_1 s W(x) \quad (48)$$

$$\dot{u}_g = \chi_2 s W(x) u \quad (49)$$

$$\dot{V} = s(E - u_s) \quad (50)$$

The sliding condition determined in section of the sliding mode control design, is again considered. So:

$$\dot{V} = s(E - u_s) \leq -\frac{\gamma}{\zeta} s^2 \quad (51)$$

In order to obtain  $u_s$ , the above inequality is used. So:

$$su_s \geq \frac{\gamma}{\zeta} s^2 + sE \quad (52)$$

On the base of the definition of  $E$  in (44):

$$su_s \geq \frac{\gamma}{\zeta} s^2 + \left[ (f(x) - \hat{f}(x|_{n_f})) + (g(x) - \hat{g}(x|_{n_g}))u \right] s \quad (53)$$

By using of (35), we can write the following inequality:

$$\begin{aligned} (g(x) - \hat{g}(x|_{n_g}))u = & \frac{g(x) - \hat{g}(x|_{n_g})}{\hat{g}(x|_{n_g})} (-\hat{f}(x|_{n_f}) + \ddot{x}_d - \dot{e}) \\ & - \frac{g(x) - \hat{g}(x|_{n_g})}{\hat{g}(x|_{n_g})} u_s \end{aligned} \quad (54)$$

Substituting (54) into (53) leads to:

$$\begin{aligned} su_s \geq & \frac{\gamma}{\zeta} s^2 + (f(x) - \hat{f}(x|_{n_f}))s \\ & + \frac{g(x) - \hat{g}(x|_{n_g})}{\hat{g}(x|_{n_g})} (-\hat{f}(x|_{n_f}) + \ddot{x}_d - \dot{e})s \\ & - \frac{g(x) - \hat{g}(x|_{n_g})}{\hat{g}(x|_{n_g})} su_s \\ su_s \geq & \frac{\hat{g}(x|_{n_g})}{g(x) + \hat{g}(x|_{n_g}) - \hat{g}(x|_{n_g}^*)} \left( \frac{\gamma}{\zeta} s^2 \right. \\ & \left. + (f(x) - \hat{f}(x|_{n_f}))s + \frac{g(x) - \hat{g}(x|_{n_g})}{\hat{g}(x|_{n_g})} s\hat{u} \right) \end{aligned} \quad (56)$$

The approximation error is more than the minimum error that is achieved by optimum parameters. So, according to (10) and (11), we can write the following inequalities:

$$|f(x) - \hat{f}(x|_{n_f})| \leq |f(x) - \hat{f}(x|_{n_f})| \leq F(x) \quad (57)$$

$$|g(x) + \hat{g}(x|_{n_g}) - \hat{g}(x|_{n_g}^*)| \geq |g(x)| \quad (58)$$

$$\frac{\hat{g}(x|_{n_g})}{g(x) + \hat{g}(x|_{n_g}) - \hat{g}(x|_{n_g}^*)} \leq \frac{\hat{g}(x|_{n_g})}{g(x)} \leq s \quad (59)$$

and because  $0 < s^{-1} < 1 < s$ , we can conclude:

$$\begin{aligned} 1 - s & \leq \frac{\hat{g}(x|_{n_g})}{g(x)} \times \frac{g(x) - \hat{g}(x|_{n_g}^*)}{\hat{g}(x|_{n_g})} \\ & \leq 1 - s^{-1} \leq s - 1 \end{aligned} \quad (60)$$

As a result, the following inequality can be considered instead of (56):

$$\begin{aligned} su_s \geq & \frac{\hat{g}(x|_{n_g})}{g(x)} \frac{\gamma}{\zeta} s^2 \\ & + \left( \frac{\hat{g}(x|_{n_g})}{g(x)} (f(x) - \hat{f}(x|_{n_f})) \right. \\ & \left. + \left( 1 - \frac{\hat{g}(x|_{n_g})}{g(x)} \right) \hat{u} \right) s \end{aligned} \quad (61)$$

which is similar to (15). So,  $u_s$  be considered as (16). Therefore, the inequality (51) for  $|s| \geq \zeta$  is satisfied. As it shown, the sliding mode control law of previous section has not changed, and just an adaptive mechanism is determined for fuzzy approximations. The designed controller structure is shown in Fig. 1.

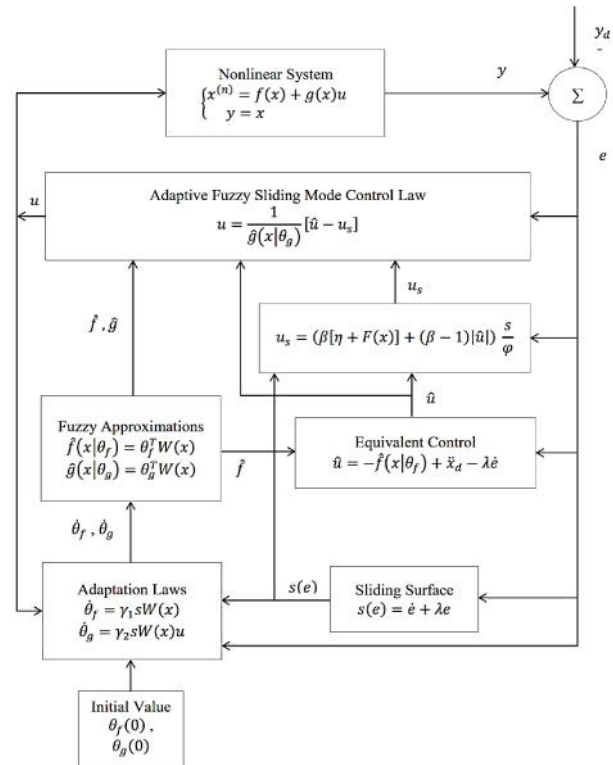


Fig. 1. The proposed indirect adaptive fuzzy sliding mode power system stabilizer.

#### 4. The procedure of stabilizer design

Stabilizer design steps can be summarized as follow:

##### 4.1. Of-line preprocessing

- Specify the coefficients of the sliding mode control law.  $[k, s, y, F(x)]$ .
- Specify the coefficients of the learning of fuzzy system.  $[x_1, x_2]$

4.2. Initial fuzzy system construction

- Define  $m_i$  fuzzy sets for input variables  $x_i$  so that the membership functions  $\mu_{A_i^l}$  cover uniformly input range. Here,  $i=1,2$  and  $m_1=m_2=7$  and  $l_i=1,\dots,m_i$ . The Input variables are considered as  $x_1=\Delta\dot{\delta}$  and  $x_2=\Delta P$  for power system stabilizer. The membership functions are Gaussian and it is defined in Appendix. The linguistic variables are labeled as: Negative Big (NB), Negative Medium (NM), Negative Small (N), Zero (ZR), Positive Small (PS), Positive Medium (PM), Positive Big (PB). The input range for the speed error ( $\Delta\dot{\delta}$ ) and for the power accelerator ( $\Delta P$ ) is  $[-1 \ 1]$  in per unit.

- Construct the fuzzy rule bases for the fuzzy approximations  $\hat{f}(x|_{n_f})$ ,  $\hat{g}(x|_{n_g})$ , which are consist of  $l=m_1 \times m_2=49$  rules. The initial value of  $n_f$  is chosen to be zero, but the initial value of  $n_g$  is chosen to be negative [22],[23], shown in Table 1.

$$R_f^{(l_1,l_2)}: \text{ If } x_1 \text{ is } A_1^{l_1} \text{ and } A_2^{l_2} \text{ Then } \hat{f}(x|_{n_f}) \text{ is } B^{(l_1,l_2)} \quad (62)$$

$$R_g^{(l_1,l_2)}: \text{ If } x_1 \text{ is } A_1^{l_1} \text{ and } A_2^{l_2} \text{ Then } \hat{g}(x|_{n_g}) \text{ is } B_g^{(l_1,l_2)} \quad (63)$$

- Construct the fuzzy systems  $\hat{f}(x|_{n_f})$  and  $\hat{g}(x|_{n_g})$  such as (32) and (33), and  $W(x)$  is given in (34).

4.3. On-line adaptation.

- Apply the control law (16) and (35) as a power system stabilizer and fuzzy approximations  $\hat{f}(x|_{n_f})$  and  $\hat{g}(x|_{n_g})$  from above.
- Use the adaptation rules (48) and (49) to adjust the fuzzy approximation parameter vectors  $n_f$  and  $n_g$ , respectively.

Table 1. The fuzzy rule base and initial value of parameter vector [22]

	P						
	NB	NM	NS	ZR	PS	PM	PB
NB	-1	-2	-3	-4	-3	-2	-1
NM	-2	-3	-4	-5	-4	-3	-2
NS	-3	-4	-5	-6	-5	-4	-3
ZR	-4	-5	-6	-7	-6	-5	-4
PS	-3	-4	-5	-6	-5	-4	-3
PM	-2	-3	-4	-5	-4	-3	-2
PB	-1	-2	-3	-4	-3	-2	-1

5. The power system testing

In this paper, the two-area four-machine power system (Fig. 2) is used to evaluate the performance of the proposed stabilizer. It was specifically designed in [1] to study low frequency electromechanical oscillations in large interconnected power systems. This system is available in the Matlab Simpower [27] software as a demo for studying the dynamic stability of a small-size multi-machine system. It is different from [1], because the load voltage improved by installing 187 MVAR more capacitors in each area. In here, the power system used for simulations is although different from demo [27] in the model of excitation system and turbine. The generators have a rating of 20 kV/900 MVA and connected to the 230 kV transmission lines (220 km length) through a transformer. The area 1 transfers about 400 MW real power to area 2 under normal conditions. Dynamics equations of generators are presented in [28] and [29]. The generators have same stabilizer, speed governor, turbine, excitation system and AVR. The structure of power generation units and configuration of one generator equipped is described in [1], [2] and [29]. In Appendix described the parameters and models of the generator components, transmission lines and loads. The output of stabilizers is limited to the  $\pm 0.2$  pu.

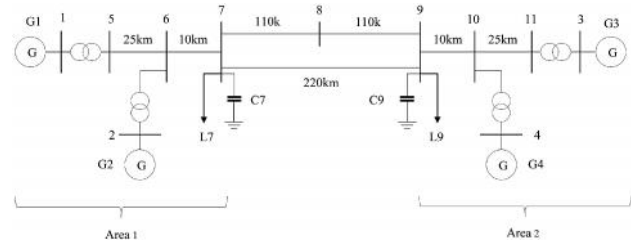


Fig. 2. Two-area four-machine test power system.

### 6. Simulation and Results

The performance of the presented adaptive fuzzy sliding mode power system stabilizer (AFSMPSS-Proposed) is compared to the multi-band power system stabilizer (MBPSS or IEEE PSS4B) [2], fuzzy adaptive sliding mode power system stabilizer with saturation function (AFSMPSS-Sat), and the adaptive fuzzy sliding mode power system stabilizer with PI control term (AFSMPSS-PI) [23]. Model and parameters of stabilizers expressed in Appendix. Power system simulation and performance evaluation of the controller is done under three types of disturbances.

Case 1: A three-phase fault at the middle of the upper tie line (bus-8) at  $t = 1$  s, for 12 cycles or 200 ms that is cleared by disconnecting the transmission line. The system continues to operate with one power transmission line.

Case 2: 20 % pulse disturbance in the AVR reference voltage of generator G1 for a period of 200 ms .

Case 3: 20 % pulse disturbance in the mechanical power of generator G1 for a period of 200 ms .

Case1 as a large signal disturbance to study the transient stability, and Case2 and Case3 as small signal disturbances for evaluating robustness of the designed PSS, are created. The speed difference between G1 and G2 ( $\dot{S}_2 - \dot{S}_1$ ) as local mode of oscillations and the speed difference between G1 and G3 ( $\dot{S}_3 - \dot{S}_1$ ) as inter-area mode of oscillations, are defined.

The Fig. 3, 4 and 5 present the simulation results for Case 1. Fig. 3 shows the local and inter-area modes of oscillations for multi-machine power system with the proposed PSS (AFSMPSS-Proposed), AFSMPSS-PI, AFSMPSS-Sat and MBPSS (PSS4B). As seen in Fig. 3 the proposed stabilizer has good performance in damping of oscillations, and the amplitude of inter-area mode of oscillation is slightly reduced. Fig. 4a shows the rotor speed of first generator (G1) by various stabilizers, and Fig. 4b shows the rotor speed of all generators with proposed PSS. Fig. 5 shows the variation of transferred power between two areas, and the field voltage of G1, for different types of PSSs. The excitation system saturation as highly nonlinear feature is seen in the field voltage.

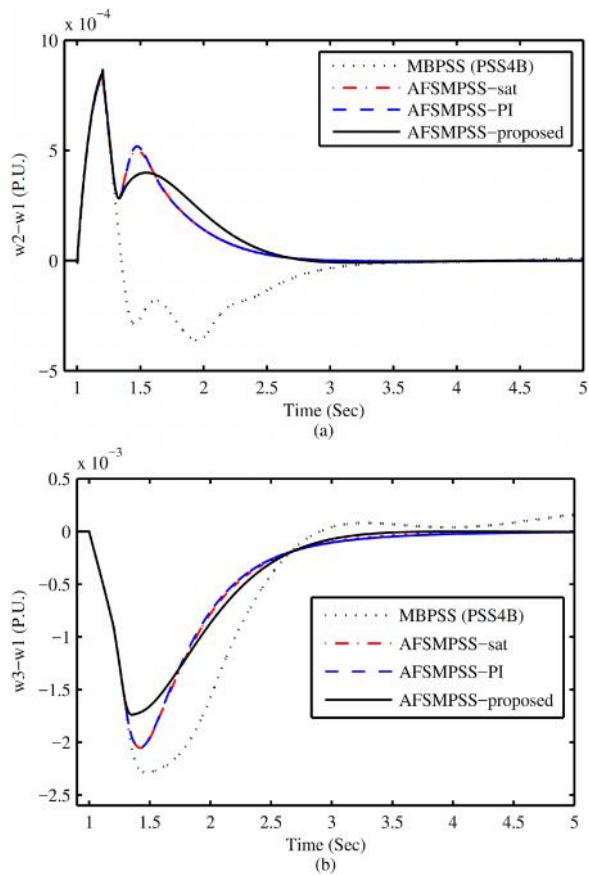
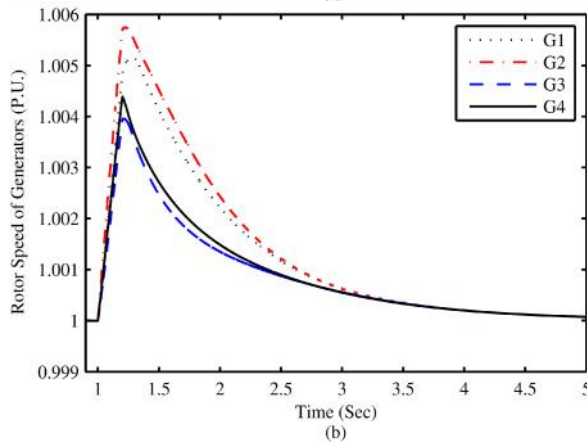
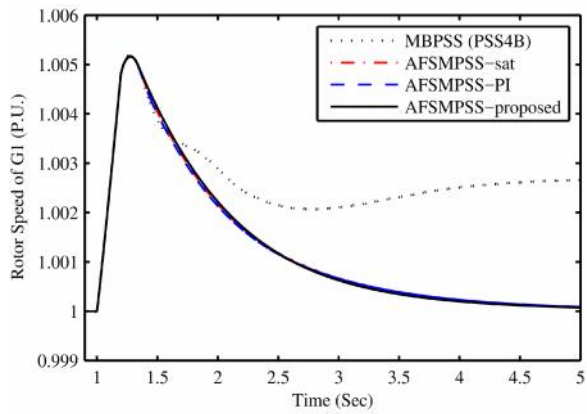


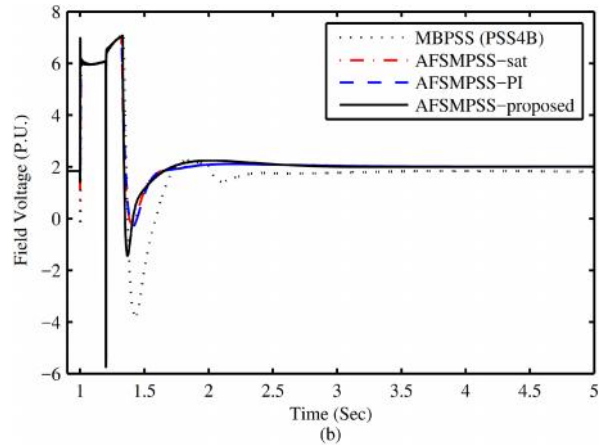
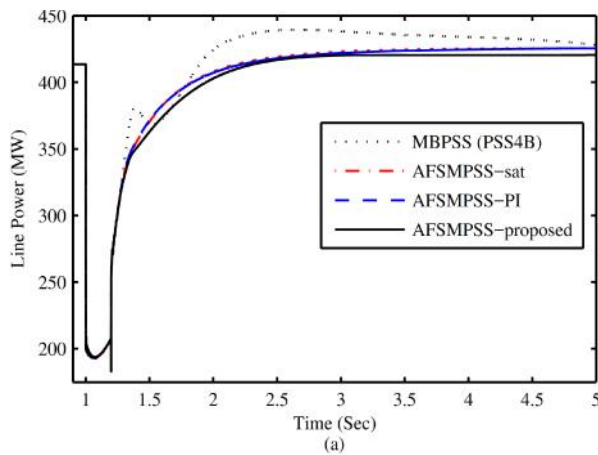
Fig. 3. Power System response for Case 1: (a) local mode of oscillations and (b) inter-area mode of oscillations.

The local and inter-area modes of oscillations and the variation of transferred power between two areas for Case 2 and 3 are drawn in Fig. 6 and 7. The performance efficacy of the proposed PSS is clear. When a large signal disturbance occurs (Case 1), excitation system reaches to saturation limits quickly, and increasing the signal of stabilizer does not have a significant impact on the speed deviation domain. Stabilizing is done after removing the disturbance and reducing the excitation signal to under the saturation point. In the small signal disturbance (Case 2, 3), the output amplitude of the excitation system does not reach to saturation limits and the stabilizer signal is effective. The effect of changes in the stabilizer signal amplitude proportional to distance between the system estates and the sliding surface, is seen in reducing the speed deviation domain (Fig. 6, 7).

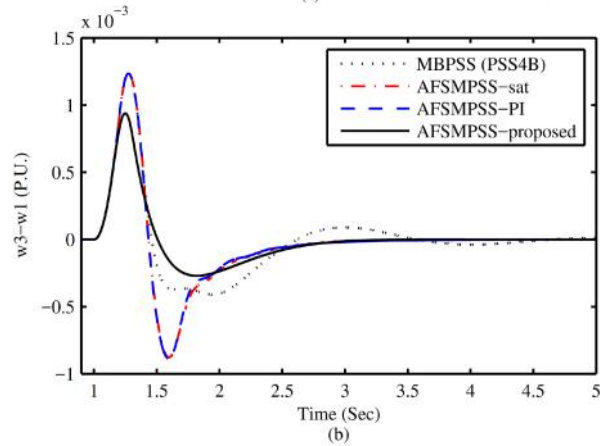
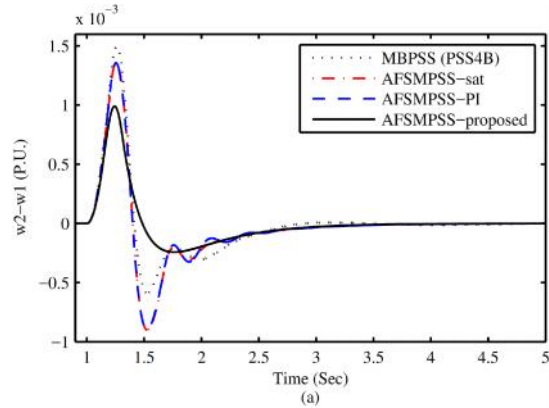




**Fig. 4.** Rotor speed for Case 1: (a) speed of the first Generator (G1) for different types of PSSs and (b) rotor speed of all generators with proposed PSS



**Fig. 5.** Comparative results for different types of PSSs (a) power transfer from area 1 to area 2 and (b) field voltage of G1



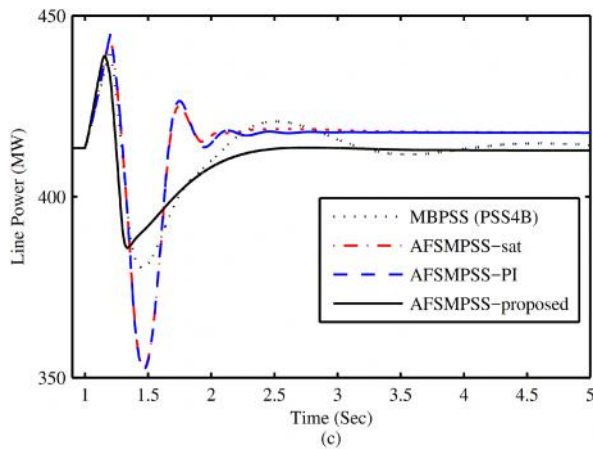


Fig. 6. Power System response for Case 2: (a) local mode of oscillations and (b) inter-area mode of oscillations and (c) power transfer from area 1 to area 2

The performance index is defined in order to evaluate the performance of the controllers. This

index is calculated by the following equation during the simulations:

$$J_p = \int_{t=0}^{t=t_1} |\Delta \tilde{S}_{i-1}| \quad (64)$$

The performance index comparison for different PSSs is shown in Table 2. It is clear that the less value of this index, the better is the stabilizer performance. Simulations period is 10 seconds ( $t_1 = 10\text{ s}$ ). Thus, the proposed indirect adaptive fuzzy sliding mode power system stabilizer has improved the system performance.

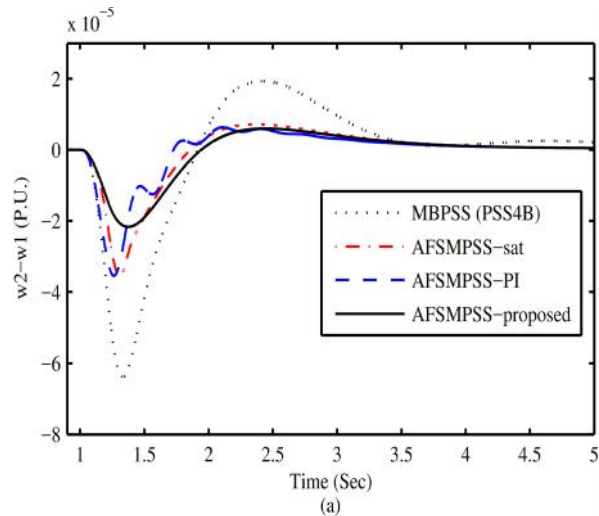
Table 2. The performance index comparison for different PSSs

		AFSMPSS Proposed	AFSMPSS PI	AFSMPSS Sat	MBPSS (PSS4B)
Case1	$J_{p(2-1)}$	0.0007897	0.0006716	0.0006648	0.001176
	$J_{p(3-1)}$	0.002783	0.003041	0.003012	0.006966
Case2	$J_{p(2-1)}$	0.0007139	0.001052	0.001053	0.001041
	$J_{p(3-1)}$	0.0006947	0.001011	0.00101	0.001315
Case3	$J_{p(2-1)}$	0.00004931	0.00004717	0.00005464	0.0001182
	$J_{p(3-1)}$	0.0000935	0.00008941	0.00009916	0.0002889

7. Conclusion

In this paper, an indirect adaptive fuzzy sliding mode power system stabilizer based on new sliding mode control law, is presented for damping of low frequency oscillations of multi-machine power systems. By changing the sliding condition of sliding mode controller, a new control law is obtained, so that the undesirable chattering is removed and the amplitude of error and stabilization time is reduced. Fuzzy systems, as universal approximations, have made it possible to approximate a highly nonlinear model of the power system without any knowledge of it. In scheme of this controller, the fuzzy systems are used to approximate the unknown functions of power system model. The adaptation laws are obtained so that global power system stability is guaranteed. The proposed stabilizer is able to consider all nonlinear features of the power system such as hard limits in the excitation system. Evaluating of the proposed stabilizer is done by

means of a two-area four-machine power system. Simulation results show the improvement in power system stability and robustness of the proposed PSS under different disturbances.



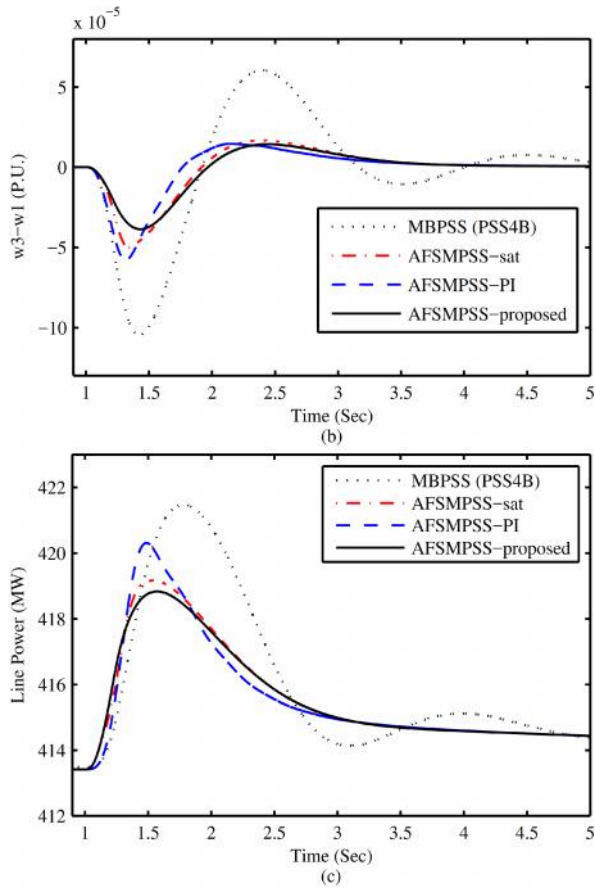


Fig. 7. Power System response for Case 3: (a) local mode of oscillations and (b) inter-area mode of oscillations and (c) power transfer from area 1 to area 2

Appendix

1. The parameters of Synchronous generators (per unit on 20 kV/ 900 MVA base) are [1]:

$$\begin{aligned}
 X_d = 1.8, X_q = 1.7, X_d' = 0.3, X_q' = 0.55, \\
 X_d'' = 0.25, X_q'' = 0.25, X_l = 0.2, R_a = 0.0025 \\
 T_{do}' = 8 \text{ s}, T_{qo}' = 0.4 \text{ s}, H = 6.5 \text{ (for G1 and G2)}, H = 6.175 \text{ (for G3 and G4)}
 \end{aligned}$$

2. The Generating units are loaded as follow:  
 G1:  $P_1 = 700 \text{ MW}, Q_1 = 91.887 \text{ MVAr}, E_t = 1$   
 G2:  $P_2 = 700 \text{ MW}, Q_2 = 117.67 \text{ MVAr}, E_t = 1$   
 G3:  $P_3 = 719 \text{ MW}, Q_3 = 82.281 \text{ MVAr}, E_t = 1$   
 G4:  $P_4 = 700 \text{ MW}, Q_4 = 82.738 \text{ MVAr}, E_t = 1$

3. Loads and shunt capacitors parameters:  
 BUS 7:  $P_L = 967 \text{ MW}, Q_L = 100 \text{ MVAr}, Q_c = 387 \text{ MVAr}$   
 BUS 9:  $P_L = 1767 \text{ MW}, Q_L = 100 \text{ MVAr}, Q_c = 537 \text{ MVAr}$

4. The transformers have an impedance of  $Z = 0 + j15$  per unit on 20/ 230 kV and

900 MVA base and transmission line parameters in base voltage  $V_b = 230 \text{ kV}$  and base power  $S_b = 100 \text{ MVA}$  are [1]:

$$\begin{aligned}
 r = 0.0001 \text{ pu/km}, L = 2.653e^{-6} \text{ pu/km}, \\
 C = 4.642e^{-6} \text{ pu/km}
 \end{aligned}$$

5. The simplified model of turbine and speed governor [1] shown in Fig. 8 and:

$$\begin{aligned}
 R = 0.05, T_G = 0.2 \text{ s}, F_{HP} = 0.3, T_{RH} = 7 \text{ s}, \\
 T_{CH} = 0.3 \text{ s}
 \end{aligned}$$

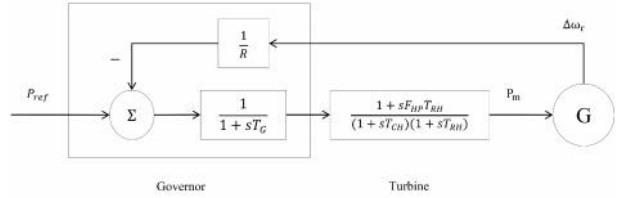


Fig. 8. Turbine and speed governor model [1]

6. The block diagram of thyristor excitation system model and transducer [2] is shown in Fig. 9 and:

$$\begin{aligned}
 k_a = 210, T_a = 0, k_c = 0.038, V_{R \max} = 6.43, \\
 V_{R \min} = -6, T_R = 0.01
 \end{aligned}$$

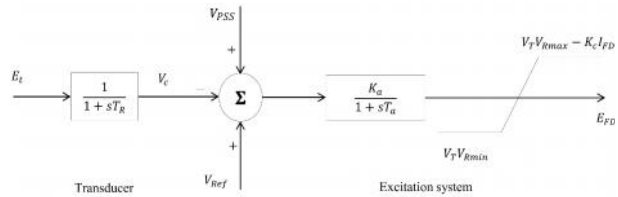


Fig. 9. Thyristor excitation system and transducer [2]

7. Gaussian membership function for fuzzy systems is defined as [25]:

$$\sim_j^i(x_i) = e^{-\left(\frac{x_i - m_j^i}{\dagger_j^i}\right)^2} \tag{65}$$

where the  $m_j^i$  is center of the  $j$ th membership function of the  $i$ th input, and  $\dagger_j^i = 0.6(m_{j+1}^i - m_j^i)$  is the constant expansion of membership [25].

8. Model and parameters of stabilizers:  
 The common term of adaptive fuzzy sliding mode control law is as (35), and adaptation laws is as (48) and (49). In the classical adaptive fuzzy sliding mode controller with saturation function,  $u_s = K \times \text{sat}(s/\{\})$ , and  $K$  is defined in (16) and:

$$\text{sat}\left(\frac{s}{\zeta}\right) = \begin{cases} -1 & \text{if } s \leq -\zeta \\ s/\zeta & \text{if } -\zeta \leq s \leq \zeta \\ +1 & \text{if } \zeta \leq s \end{cases} \quad (66)$$

The adaptive fuzzy sliding mode controller with PI controller, described in [21] and [23], and the main difference is:

$$u_s = u_p^T \mathbb{E}(s) = [k_p \ k_i] \left[ s \ \int s dt \right]^T = k_p s + k_i \int s dt .$$

The parameters values of designed PSS (AFSMPSS-Proposed), AFSMPSS-Sat and AFSMPSS-PI in the simulations expressed in the Table 3.

The estimation error of  $f(x)$  is defined proportional to its approximation. In other words,  $F(x) = F \times \hat{f}(x|_{u_f})$ . The sample time of sliding mode controllers in simulations is 0.001 s.

Fig. 10 shows the conceptual model of multi-band stabilizer (IEEE PSS4B). The structure of MBPSS is described in [2]. IEEE PSS4B parameters defined in Matlab Simpower demo [27] are used to simulation. The parameters values in the simulations are:

$$\begin{aligned} F_L &= 0.2 \text{ Hz} , K_L = 30 , F_I = 1.25 \text{ Hz} , K_I = 40 , \\ F_H &= 12 \text{ Hz} , K_H = 160 , V_{L \max} = 0.075 , \\ V_{L \min} &= -0.075 , V_{I \max} = 0.15 , V_{I \min} = -0.15 , \\ V_{H \max} &= 0.15 , V_{H \min} = -0.15 , V_{S \max} = 0.15 , \\ V_{S \min} &= -0.15 \end{aligned}$$

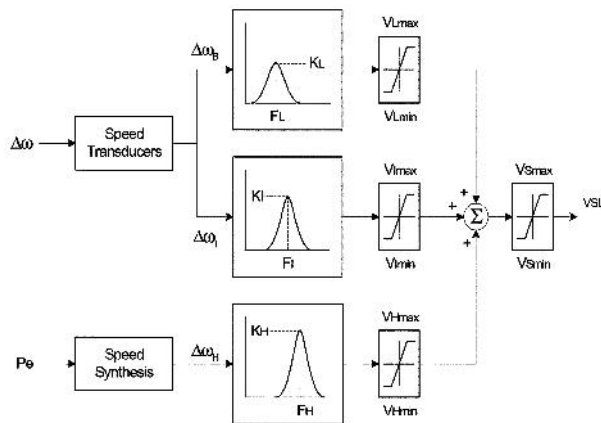


Fig. 10. Conceptual model of multi-band stabilizer (IEEE PSS4B) [2]

Table 3. The parameters values of PSS's

	AFSMPSS S Proposed	AFSMPSS S Sat	AFSMPSS S PI
	1	1	1
	0.01	0.01	--
	0.0001	0.0001	--
<b>F</b>	0.001	0.001	--
	1.001	1.001	--
$k_p$	--	--	1
$k_i$	--	--	4

References

- [1] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*: McGraw-Hill, 1994.
- [2] "IEEE Recommended Practice for Excitation System Models for Power System Stability Studies," *IEEE Std 421.5-2005 (Revision of IEEE Std 421.5-1992)*, pp. 0\_1-85, 2006.
- [3] I. Kamwa, R. Grondin, and G. Trudel, "IEEE PSS2B versus PSS4B: the limits of performance of modern power system stabilizers," *Power Systems, IEEE Transactions on*, vol. 20, pp. 903-915, 2005.
- [4] W. Bin and O. P. Malik, "Multivariable Adaptive Control of Synchronous Machines in a Multimachine Power System," *Power Systems, IEEE Transactions on*, vol. 21, pp. 1772-1781, 2006.
- [5] T.-L. Liao, "Design of an adaptive nonlinear controller to improve stabilization of a power system," *International Journal of Electrical Power & Energy Systems*, vol. 21, pp. 433-441, 8// 1999.
- [6] A. Karimi and A. Feliachi, "Decentralized adaptive backstepping control of electric power systems," *Electric Power Systems Research*, vol. 78, pp. 484-493, 3// 2008.
- [7] H. K. Pamsari, M. A. Bidgoli, M. Rajabzadeh, S. M. T. Bathaee, and S. Ozgoli, "Application of a new multivariable sliding mode controller for the single machine infinite bus systems," in *Power Electronics, Drive Systems and Technologies Conference (PEDSTC), 2011 2nd*, 2011, pp. 211-216.
- [8] S. S. Lee and J. K. Park, "Design of power system stabilizer using observer/sliding mode, observer/sliding mode-model following and H /sliding mode controllers for small-signal stability study," *International Journal of Electrical Power & Energy Systems*, vol. 20, pp. 543-553, 11// 1998.
- [9] A. Rajabi, G. Shahgholian, B. Karimi, and M. R. Yosefi, "Analysis and design of PSS based on sliding mode control theory for SMIB," in *IPEC*,

- 2010 *Conference Proceedings*, 2010, pp. 994-999.
- [10] K. A. El-Metwally, G. C. Hancock, and O. P. Malik, "Implementation of a fuzzy logic PSS using a micro-controller and experimental test results," *Energy Conversion, IEEE Transactions on*, vol. 11, pp. 91-96, 1996.
- [11] F. Mrad, S. Karaki, and B. Copti, "An adaptive fuzzy-synchronous machine stabilizer," *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on*, vol. 30, pp. 131-137, 2000.
- [12] T. Abdelazim and O. P. Malik, "An adaptive power system stabilizer using on-line self-learning fuzzy systems," in *Power Engineering Society General Meeting, 2003, IEEE*, 2003, pp. 1715-1720 Vol. 3.
- [13] A. L. Elshafei and K. El-Metwally, "Power system stabilization via adaptive fuzzy-logic control," in *Intelligent Control, 1997. Proceedings of the 1997 IEEE International Symposium on*, 1997, pp. 89-94.
- [14] T. Hussein and A. Shamekh, "Direct Adaptive Fuzzy Power System Stabilizer for a Multi-machine System," in *Computer Modelling and Simulation (UKSim), 2013 UKSim 15th International Conference on*, 2013, pp. 33-38.
- [15] T. Hussein, M. S. Saad, A. L. Elshafei, and A. Bahgat, "Robust adaptive fuzzy logic power system stabilizer," *Expert Systems with Applications*, vol. 36, pp. 12104-12112, 12// 2009.
- [16] T. Hussein, M. S. Saad, A. L. Elshafei, and A. Bahgat, "Damping inter-area modes of oscillation using an adaptive fuzzy power system stabilizer," *Electric Power Systems Research*, vol. 80, pp. 1428-1436, 12// 2010.
- [17] Z. Bouchama and M. N. Harmas, "Optimal robust adaptive fuzzy synergetic power system stabilizer design," *Electric Power Systems Research*, vol. 83, pp. 170-175, 2// 2012.
- [18] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*: Prentice Hall, 1991.
- [19] J. Wang, A. B. Rad, and P. T. Chan, "Indirect adaptive fuzzy sliding mode control: Part I: fuzzy switching," *Fuzzy Sets and Systems*, vol. 122, pp. 21-30, 8/16/ 2001.
- [20] K. Kyoung Joo, P. Jin Bae, and C. Yoon Ho, "Chattering Free Sliding Mode Control," in *SICE-ICASE, 2006. International Joint Conference*, 2006, pp. 732-735.
- [21] H. F. Ho, Y. K. Wong, and A. B. Rad, "Adaptive fuzzy sliding mode control with chattering elimination for nonlinear SISO systems," *Simulation Modelling Practice and Theory*, vol. 17, pp. 1199-1210, 8// 2009.
- [22] N. Hossein-Zadeh and A. Kalam, "An indirect adaptive fuzzy-logic power system stabiliser," *International Journal of Electrical Power & Energy Systems*, vol. 24, pp. 837-842, 12// 2002.
- [23] K. Saoudi and M. N. Harmas, "Enhanced design of an indirect adaptive fuzzy sliding mode power system stabilizer for multi-machine power systems," *International Journal of Electrical Power & Energy Systems*, vol. 54, pp. 425-431, 1// 2014.
- [24] N. Hosseinzadeh and A. Kalam, "A direct adaptive fuzzy power system stabilizer," *Energy Conversion, IEEE Transactions on*, vol. 14, pp. 1564-1571, 1999.
- [25] J. J. E. Oviedo, J. P. L. Vandewalle, and V. Wertz, *Fuzzy Logic, Identification and Predictive Control*: Springer, 2006.
- [26] L. X. Wang, *A Course in Fuzzy Systems and Control*: Prentice Hall PTR, 1997.
- [27] "Matlab SimPower Software," Ver 7.8 ed, 2011.
- [28] P. W. Sauer and A. Pai, *Power System Dynamics and Stability*: Prentice Hall, 1998.
- [29] M. Eremia and M. Shahidehpour, *Handbook of Electrical Power System Dynamics: Modeling, Stability, and Control*: Wiley, 2013.