New Approach to Determine 6DOF Position and Orientation of a Non-Orthogonal Coordinate System on the Object Using its Image

Mahdieh Hosseingholizadeh Alashti*[1], Jafar Heyrani Nobari[2], Abdolhamid Bahjat Panah[3]

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Abstract

In this paper, a new method for determining position and orientation of a coordinate system using its image is presented. This coordinate system is a three dimensional non-orthogonal system in respect to the two dimensional and orthogonal camera coordinate system. In real world, it's exactly easy to select three directions on an object so that they don't be orthogonal and on a plane. The image of this non-orthogonal coordinate system on the camera image plane is a two dimensional coordinate system. This image is obtained by a nonlinear mapping between three dimensional worlds coordinate and two dimensional image coordinate. In this paper, we review geometric relationships between a direction of a vector on the object and its image that was presented in paper [18] months ago. Then, using these relationships for three arbitrary non-orthogonal directions which are not on a plane, a system of 15 nonlinear equations is established, and by solving it, nine unknowns are extracted. Because of the importance of the sign of these unknowns to determine true lengths and angels, it's essential to run this system of nonlinear equations in eight cases and then best answer with right signs can be extracted. The results of this theory have been examined using simulation and programs.

In paper [18] we have to select three orthogonal vectors on an object. Since world is 3D, in some cases it is exactly difficult to choose all three directions with proper length and maybe we have to choose third vector (which is in depth) with a short length and it increase errors in finding position and orientation. But in this paper we don't limit directions to be orthogonal, so all three directions can be in proper length and it decreases the errors.

Keywords: Position and orientation determining, non-orthogonal coordinate system, nonlinear mapping between real world frame and camera frame.

List of symbols

C	Rotation matrix between CCD coordinate system and object coordinate system
p	The distance between an object and camera lens
q	The distance between the image and camera lens
a	CCW transition angel around Z axis
b	CCW transition angel around Y axis
g	CCW transition angel around X axis
f_x	Focal length at X direction
f_y	Focal length at Y direction
l	Length of the object
l'	The object's image length placed on the camera sensor
l''	The length of the projected object on the plane which is parallel to the camera sensor and is in d meter from camera lens
x'	The first component of an object with first component x
x''	The first component of the projected object on the plane which is parallel to the camera sensor and is in d meter from camera lens
Δx	The distance between initial point of the object in X direction and a point in the d meter away from the lens, along the main axis of the camera
$\Delta x'$	The distance between initial point of the object in X direction and a point in the f meter away from the lens, along the main axis of the camera
$-m$	A vector which connect the end point of the projected object on a plane, which is parallel to the camera sensor, to the camera lens
x_e	The difference between first component of the object and the projected object on a plane which is parallel to the camera sensor
y_e	The difference between second component of the object and the projected object on a plane which is parallel to the camera sensor
l_e	The difference between the length of the object and the projected object on a plane which is parallel to the camera sensor

1 . M.Sc. in Electrical Engineering - Control, Department of Electrical Engineering, mah.gholizadeh@gmail.com

2 . Assistant Professor, Department of Electrical Engineering, nobari@eetd.kntu.ac.ir

3. Ph.D. student in Electrical Engineering - Control, Department of Electrical Engineering, 8700396@ee.kntu.ac.ir

$\vec{r}_{AB}$	2D coordinate of AB object in respect to the reference
$\vec{r}_{A'B'}$	2D coordinate of the image of AB object in respect to the reference system
x_{AB}	first coordinate of AB object which is expressed in a known system
y_{AB}	second coordinate of AB object which is expressed in a known system
z_{AB}	third coordinate of AB object which is expressed in a known system
$x_{A'B'}$	first coordinate of the image of AB object which is expressed in a known system
$y_{A'B'}$	second coordinate of the image of AB object which is expressed in a known system
d	The distance between the plane which is parallel to the camera sensor and camera lens
y'	The second component of an object with second component y
y''	The second component of the projected object on the plane which is parallel to the camera sensor and is in d meter from camera lens
Δy	The distance between initial point of the object in Y direction and a point in the d meter away from the lens, along the main axis of the camera
$\Delta y'$	The distance between initial point of the object in Y direction and a point in the f meter away from the lens, along the main axis of the camera
c	A vector which connect the end point of the object to the camera lens
c'	A vector which connect the end point of the object to the camera lens
Ψ	The angel between the object image with x direction and also the angel between the projected object with x direction

1. Introduction

One of the issues considered by many researchers and academics is determining the position and orientation of an object (object Pose) using feature points on it. In truth, the issue is determining position and orientation of this object in respect to the camera frame, using corresponding points of the object on its image while the object model is specified. In order to gain it, many methods with different advantages, disadvantages and applications are presented.

Pose computation is the process of extracting information about the position and orientation of an object using the corresponding points of its model and its image, which is obtained using optical sensors such as a camera [9].

With determining position and orientation of an object we can display it in a specified Pose or in other words, virtual objects can be displayed in real world. It is the meaning of augmented reality. Also, sometimes the only information about a fixed or moving object which is available is a picture or a film of it, for example, we have a film of a moving object but it is unclear how it moved. So, for hunting a flying object or motion analysis of a missile, taking a photo or video is done. Such issues show the importance and applications of Pose determining.

How many points a method need to determine Pose, how much precise is it and what restriction it apply on the size and type of the object and on the feature points selecting, can be considered as an advantage or disadvantage of a method.

At first, Pose determining using Perspective three-point Problem is proposed by Lacroix in 1795 [1]. For the first time, German mathematician, Grunert, solved this problem in algebraic way [4]. The problem of algebraic solving is that if a little error in values obtained from image is happened, then this way has no answer. Since in practice data are so noisy, algebraic way is not suitable.

To overcome this drawbacks, for the first time Fischer and Bolles have introduced the term “n-point projection problem” for finding position and orientation of an object with n feature point [3].

The easiest way, for the first time was proposed by Roberts in 1965, which is finding elements of perspective projection matrix with solving a system of linear equations. Indeed this matrix is a projection between feature points and their images in a Homogeneous coordinate system. Eleven unknown elements can be determined using at least six feature points and their corresponding point on the image [5]. The problem of this ways is their needing to a lot of feature points.

Another important method between proposed methods in determining Pose is the way which is presented by Tsai in 1987, Lowe in 1985 and Yuan in 1989 [7],[5],[10]. When camera focal length and image center is not known Tsai way is useful. Of course, this way is a good Pose estimation when 7 corresponding are available [7]. If camera focal length and image center are calibrated before, Yuan and Lowe’s way is useful [8] , [9]. Nevertheless both ways depend on Newton-Raphson method that has 2 major objections. First objection is that for starting an iterative process there should be an estimation of Pose and second one is that in any iteration pseudo-inverse of $2N\times 6$ Jacobean matrix (N is the number of feature points) in Lowes’ way or $N\times 6$ in Yuan’s way should be determined which is computationally very expensive.

To use the advantages of algebraic and optimal way, we can combine them. This algorithm is named combined algorithm. A good combined way, combine numerical stability and robust computation of optimal algorithms with the speed of algebraic algorithms.

Thus, in 1992 POSIT algorithm was proposed by Daniel F.DeMenthon and LaryS.Davis [2]. The name of their article “model based Pose determining of an object in 25 line of Mathematica” was so curious. POSIT is a combined way to find Pose. The base of this algorithm is linear algebraic ways and is an iterative algorithm like Lowe and Yuan methods but it doesn’t need any initial pose estimation and matrix inverse in its iterations. Of course POSIT is very fast for 8 feature point and 4 iteration. This method needs little computation in comparison to Lowe and Yuan’s ways. There is only one thing in applying this method. The size of the object that its pose estimation is our goal must be very shorter than the distance between the object and camera or the object depth must change a few around a plane.

In 1999, an algorithm which name was the family of linear algebraic algorithms was proposed by Quan and Lan. This algorithm is an algebraic method for Pose estimation. This method forms a linear system and then applies eigenvalue decomposition to these equations [6].

After expression of Pose determination problems using object feature points and their corresponding point in an image, many researchers work on finding the best answer for them under different conditions. For example, in 2001 Hu and et al. presented an answer for P4P (4 corresponding point problem) and in the same year Wu and Hu presented an answer for P5P. In 2010 Bujnak and et al. expressed a general solving for P4P problem for a camera with an unknown focal length [13]. Indeed their work is a robust solving of a system of algebraic equations and used special variables and methods to solve this system. The difference between their work and the others work like Abidi and Chandrra [14] is selected points in their methods which are arbitrary and without any limitation. Also their answer is complicated but is completely general.

In 2012, Yang Guo proposed a new way for P4P problem for a no calibrated camera [15]. Indeed this method is useful when we use camera’s auto setting and in each frame setting will change according to the conditions. The method they used is not special and is a common algebraic way which can obtain the answer with a geometric view to the problem and rebuilding some parameters.

This paper is in P4P problems category. Maybe we can say that the main difference between this article and the others proposed up to now is in most of them the relationships that have been used for camera is in matrix relations with camera’s parameter, but in this article all steps are geometric and there is no relation in matrix expression. First the relationship for determining Pose of a line is reviewed. Also these relationships are from our last paper [18] but the innovation in this article is generalizing feature point and removing the limitation on them to just be orthogonal. If feature point are not on a plane and are not on 3 orthogonal directions this paper should be really impressive.

2. Image formation in a camera

As you know, if an object be in a distance p of a lens with focal length f, its image is formed in distance q from the lens and equation (1) is true for camera.

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q} \quad (1)$$

If $p>>q$ then we can suppose that image is formed in focal length of lens (in another words $q=f$). Since focal length is always in mm range, it can be supposed that object is always in infinitive distance from camera lens and always image is formed in f distance from camera lens. This case is similar to the case that a pinhole is replaced with the lens and for each object placed in p distance of pinhole plane the image is formed in f distance from this plane. As shown in figure 1 if object AB be in front of a camera with focal length f and the distance between object and camera be p and the distance in which image is formed be q and p be much greater than q, then with a good approximation inverse image is formed in focal plane.

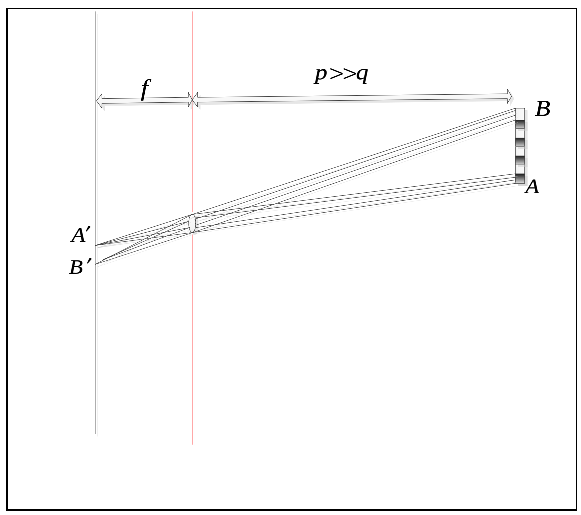

Figure 1. image formation for an object in infinitive using a lens

Equation (2) is correct for a pinhole camera. This equation is based on Thales theorem.

$$\vec{r}_{A'B'}=\frac{f}{p}\vec{r}_{AB} \quad (2)$$

In equation (2) f is camera focal length, p is object distance to camera, $\vec{r}_{AB}$ is 2D object coordinate that expressed in a specified 2D coordinate system and $\vec{r}_{A'B'}$ is 2D image coordinate that expressed in a specified 2D coordinate system. You must notice that if 2 expressions are in one coordinate system, this equation will be correct. Equation (3) is another expression of equation (2) in a specified 3D coordinate system.

$$\frac{\begin{pmatrix} x_{A'B'} \\ y_{A'B'} \end{pmatrix}}{\begin{pmatrix} x_{AB} \\ y_{AB} \end{pmatrix}} = \frac{f}{p+z_{AB}} \qquad (3)$$

In equation (3), x, y and z are first , second and third object coordinate in millimeter and all coordinate are expressed in a specified reference coordinate system.

3. Introduction of required reference coordinate systems and how to rotate according to the definition of Euler angels

In order to use relationships between corresponding point in 3D object and 2D image in a correct way, it is essential to introduce a reference coordinate system with 3 directions and 1 point (like figure 2). Since these 3 directions are attached to camera sensor or CCD, we entitle it as CCD attached coordinate system. Reference point which is considered for CCD reference system is o' that has minimum pixel value in both dimensions.

Also, object attached reference system have 3 non-orthogonal directions that are not on a plane and are specifies on the object. Because of non-orthogonality of object attached system definition of it is really easy for lots of objects. One point in an object is chosen as reference point, too. Notice that only images with clear initial and final points of all directions and clear reference point are applicable. Because we can attribute pixel value to these point.

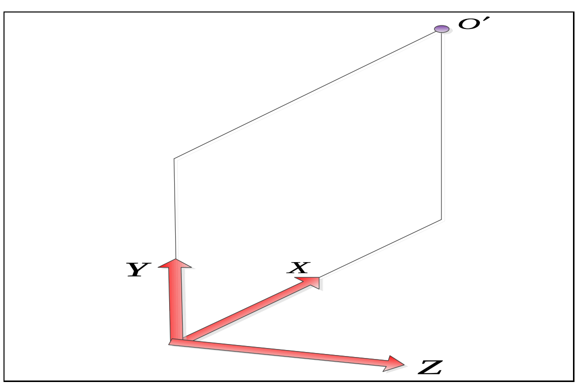

Figure 2.object attached reference coordinate system

Indeed, the meaning of non-orthogonal coordinate system specified on the object is 3 directions with known length that the angels between all 3 pairs of them are known. To simplify it relationships are extracted only for the case that one direction is rotated in respect to camera coordinate system. In another words the object is a line.

To determine position and orientation of a line we should choose a specified display. The display we select is extracted using spherical coordinate system. As you see in figure 3, first the line in XY plane of camera is rotated around z axis of the camera as much as a , then this rotated line is rotated around new y axis in XZ plane of camera as much as b . Of course to display these rotations in figure 3 better, second rotation is done as much as $-b$.

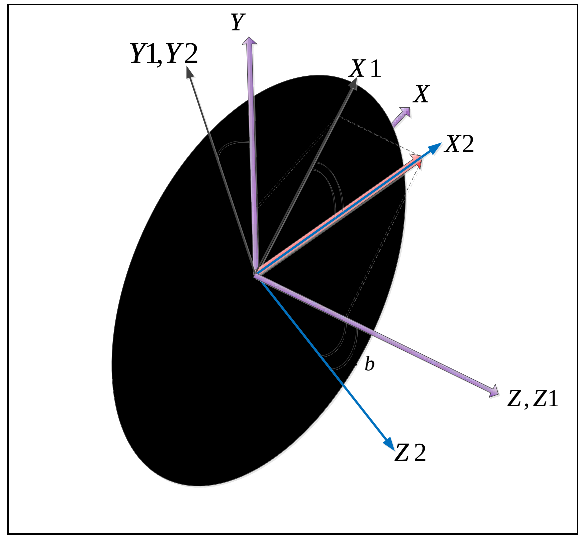

Figure 3. 2D rotation of a line with Euler angels

According to figure 3, first XYZ directions are rotated as much as a and become X1Y1Z1. Since rotation is around Z axis, Z1 is in Z direction. Then, X1Y1Z1 directions are rotated around Y1 as much as $-b$ and become X2Y2Z2. Since rotation is around Y1, Y2 is in Y1 direction.

Maximum rotation that a 2D object like a line can have is 2D (with 2 angels around 2 different axes).

4. Camera focal length determination using images of a known object

In equation (3), that is written based on Thales theorem, focal length is unknown. So f must be calculated before Pose determination. In another words once a camera is calibrated and then its setting does not change.

In order to calculate it in an easiest way, we place a 2D object (like a plane) in front of the camera in a Pose in which the image has pixel values in both direction of CCD. Then after specifying a special point (*A*) on it, measure the distance between this point and center of camera lens (*p*) in millimeter.

Then, specify another point on this object (*B*) in such a way that vector AB has an image $A'B'$ that have pixel values in both directions *x* and *y*.

Thus, with replacing measured value in equation (4) f_x and f_y are obtained.

$$\frac{\begin{pmatrix} x_{A'B'} \\ y_{A'B'} \end{pmatrix}}{\begin{pmatrix} x_{AB} \\ y_{AB} \end{pmatrix}} = \frac{1}{p}\begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{4}$$

In equation (4) f_x and f_y are focal length in *x* and *y* directions that used because of different pixel size (s_x, s_y) in *x* and *y* directions and their dimension is millimeter. Pixel size is written in datasheet of every camera. Indeed, since image coordinates are in pixel value, using these factors in equation (4) dimension of are elements become millimeter. Also, equation (5) is true for focal length in *x* and *y* directions.

$$f_x = \frac{f}{s_x}\ ,\ f_y = \frac{f}{s_y} \tag{5}$$

After replacing known values in equation (4) focal length is obtained in millimeter. In another words camera becomes calibrated. Since this focal length value is used in position and orientation determination equations, the other images that will take with this camera must be with the same setting. So, in following focal length is a known value.

5. Position and orientation determination of a line with known length and known distance up to camera

A line rotation can be 1D or with 1 angel (only *α* or *β*) or 2D with 2 angels (*α* and *β*). Initial point of the line can be in the camera main axis (the axis that pass through center of CCD and pinhole and is perpendicular to the CCD) or can be Δy away from main axis in *Y* direction (initial point is place on *YZ* plane) or can be Δx away from main axis in *X* direction and Δy away from main axis in *Y* direction (initial point is arbitrary). Of course rotation in *XZ* plane is similar to rotation in *YZ* plane. Also, we suppose that third coordinate of line is always *d* and *d* is a measured value. So, a line can be placed in 9 cases in front of camera.

The complex case of line position in front of a camera is the case in which rotation is 2D and its initial point is Δx away in *X* direction and Δy away in *Y* direction in respect to the main axis.

Geometric relationship for this case is expressed in equation (6).

$$\left.\begin{aligned} {}' &= \frac{d}{f}x' \\ {}' &= \frac{d}{f}y' \end{aligned}\right\} l'' = \frac{d}{f}l'$$

$$x = \frac{d}{f}\Delta x', \Delta y = \frac{d}{f}\Delta y' \tag{6}$$

$$\mathrm{n}(\Psi) = \frac{y'}{x'} = \frac{y''}{x''} \to x'' = \frac{l''}{\sqrt{1+\tan^2(\Psi)}}$$

In above equation x', y', $\Delta x'$, $\Delta y'$ and l' are known according to the image. Also, we suppose that d and f are known. So, x'', y'', l'', Δx, Δy and Ψ can be calculated. Using equation (6) a relation for x'' is obtained.

Also, with definition of f angel we have:

$$\tan(f) = \frac{y+\Delta y}{x+\Delta x} = \frac{y'+\Delta y'}{x'+\Delta x'} = \frac{y''+\Delta y''}{x''+\Delta x''} \to (y+\Delta y) = (x+\Delta x)\tan(f)$$

So:

$$y = x\tan(f) + \Delta x\tan(f) - \Delta y \to y = c_1 x + d_1 \tag{7}$$

In equation (7), c_1 and d_1 are known using equation (6) and a relation between first and second coordinate of a line (x and y) according to the camera attached coordinate system is obtained.

This position of a line in front of a camera is shown in figure 4. Figure 5 is a zoom view of important parts of figure 4. According to figure 4 we have equation (8).

$$\frac{\sqrt{(x''+\Delta x)^2+(y''+\Delta y)^2}}{\sqrt{(x+\Delta x)^2+(y+\Delta y)^2}} = \frac{d}{d+z} \tag{8}$$

The only thing we have is an image of the line. So we must try to import image coordinates in equation (8).

So, we have equation (9).

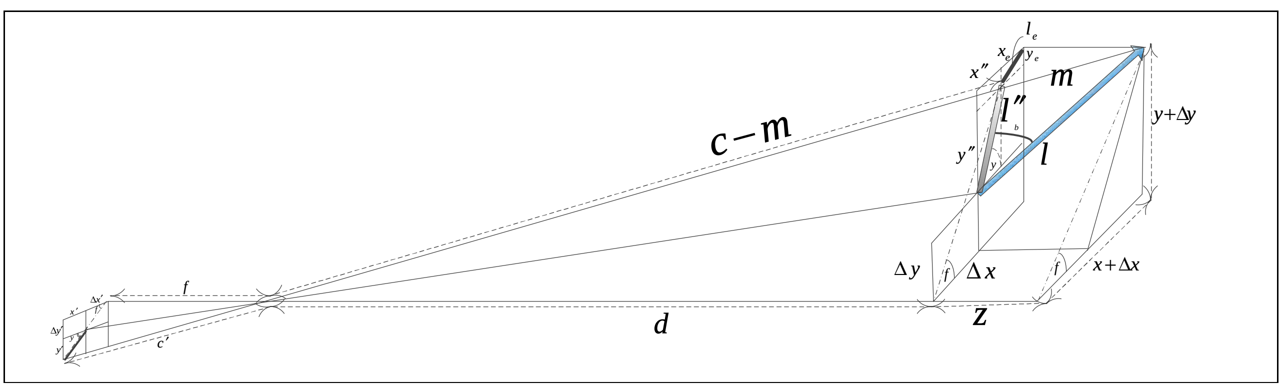

Figure 4. the complex position of a line in front of camera

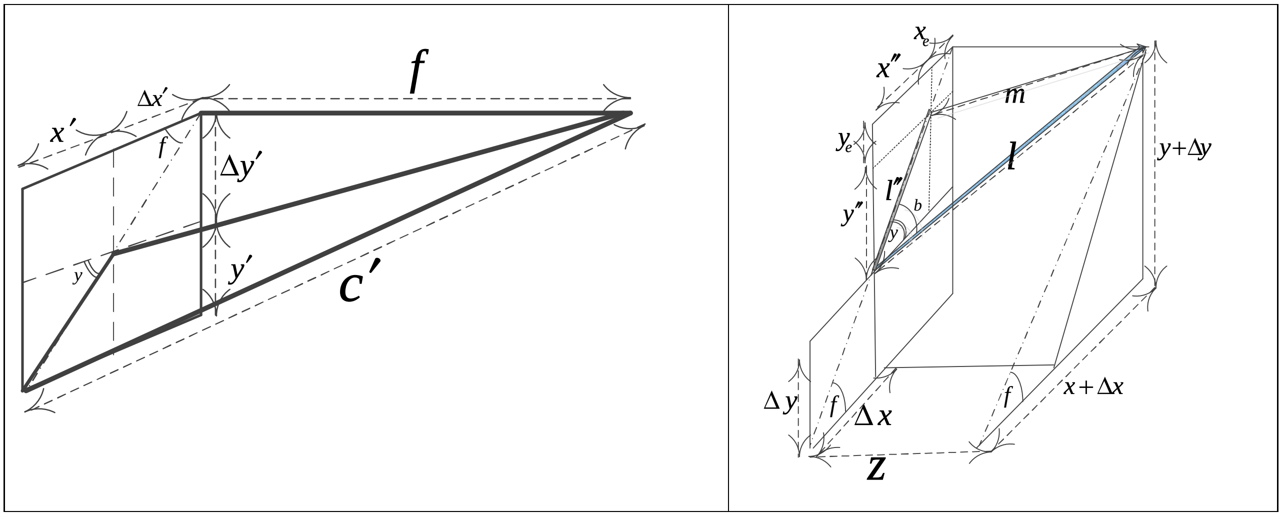

Figure 5. the zoom view of important part of figure 4

All parameters in equation (9) are known and x is the only parameter we are looking for. So, using equation (9) 2 values for first coordinate of line or its coordinate in first direction of camera attached coordinate system is obtained. Then, after replacing these values in equation (7) and (8) the others coordinate in second and third directions of camera coordinate system are obtained.

So, by knowing length of a line and its distance to camera, we can calculate its coordinates in camera attached coordinate system. Now we raise this problem to the problem in section 6.

$$
\begin{aligned}
&\xrightarrow{a=\sqrt{(x''+\Delta x)^2+(y''+\Delta y)^2}} \frac{a}{(x+\Delta x)\sqrt{1+\tan^2(f)}}=\frac{1}{1+\frac{z}{d}} \\
&\rightarrow a+\frac{a}{d}z=(x+\Delta x)\sqrt{1+\tan^2(f)} \\
&\xrightarrow{z=\sqrt{l^2-x^2-y^2}} a+\frac{a}{d}\sqrt{l^2-x^2-y^2}=(x+\Delta x)\sqrt{1+\tan^2} \\
&\xrightarrow{k=\sqrt{1+\tan^2(f)}} \sqrt{l^2-x^2-y^2}=\frac{d(x+\Delta x)k}{a}-d \\
&\rightarrow \sqrt{l^2-x^2-y^2}=\frac{dkx}{a}+\frac{dk\Delta x}{a}-d \\
&\xrightarrow{q=\frac{dk\Delta x}{a}-d} \sqrt{l^2-x^2-y^2}=\frac{dkx}{a}+q \qquad (9) \\
&\rightarrow l^2-x^2-y^2=(\frac{dkx}{a}+q)^2 \\
&\rightarrow l^2-x^2-y^2=\frac{d^2k^2x^2}{a^2}+q^2+2q\frac{dkx}{a} \\
&\xrightarrow{y=c_1x+d_1} l^2-x^2-(c_1x+d_1)^2=\frac{d^2k^2x^2}{a^2}+q^2+2q\frac{dk}{a} \\
&\rightarrow l^2-x^2-c_1{}^2x^2-d_1{}^2-2c_1d_1x=\frac{d^2k^2x^2}{a^2}+q^2+2q\frac{dk}{a} \\
&\rightarrow x^2(\frac{d^2k^2}{a^2}+1+c_1{}^2)+x(2q\frac{dk}{a}+2c_1d_1)+q^2-l^2-d_1{}^2
\end{aligned}
$$

6. Position and orientation determination of a non-orthogonal coordinate system specified on an arbitrary object

In this segment, 3 directions with known length for 3 directions of non-orthogonal coordinate system and also a special point on an object are specified. The distance between object and camera is unknown. The goal is achieving to the expression of these 3 directions in camera attached coordinate system and to the value of the distance between special point and camera lens. In other word, position and orientation of this non orthogonal system is obtained.

For each direction that is placed in front of the camera we have 3 equations (7), (8) and (9). These 3 equations are shown as equation (10), a system of 3 known parameters and 3 unknown parameters.

$$
\begin{aligned}
&x-F(x',y',\Delta x,\Delta y,f)=0 \\
&y-G(x',y',\Delta x,\Delta y,f)=0 \\
&z-H(x',y',\Delta x,\Delta y,f)=0
\end{aligned} \qquad (10)
$$

So, for 3 directions we have equation (11).

$$
\begin{aligned}
&x_1-F(x_1',y_1',\Delta x_1,\Delta y_1,f)=0 \\
&y_1-G(x_1',y_1',\Delta x_1,\Delta y_1,f)=0 \\
&z_1-H(x_1',y_1',\Delta x_1,\Delta y_1,f)=0 \\
&x_2-F(x_2',y_2',\Delta x_2,\Delta y_2,f)=0 \\
&y_2-G(x_2',y_2',\Delta x_2,\Delta y_2,f)=0 \\
&z_2-H(x_2',y_2',\Delta x_2,\Delta y_2,f)=0 \\
&x_3-F(x_3',y_3',\Delta x_3,\Delta y_3,f)=0 \\
&y_3-G(x_3',y_3',\Delta x_1,\Delta y_3,f)=0 \\
&z_3-H(x_3',y_3',\Delta x_3,\Delta y_3,f)=0
\end{aligned} \qquad (11)
$$

Also, according to the first assumption the length of 3 directions which form non-orthogonal coordinate system and the angel between 3 pairs of them are known. Based on the inner product equation (12) is true.

$$\cos(q)=\frac{V.W}{|V||W|} \qquad (12)$$

So, the other system of equations that is true for the directions of non-orthogonal coordinate system is equation (13).

$$\begin{aligned}
&\frac{(x_1x_2+y_1y_2+z_1z_2)}{(\sqrt{x_1^2+y_1^2+z_1^2})(\sqrt{x_2^2+y_2^2+z_2^2})}-\cos(q_{12})=0\\
&\frac{(x_1x_3+y_1y_3+z_1z_3)}{(\sqrt{x_1^2+y_1^2+z_1^2})(\sqrt{x_3^2+y_3^2+z_3^2})}-\cos(q_{13})=0\\
&\frac{(x_3x_2+y_3y_2+z_3z_2)}{(\sqrt{x_3^2+y_3^2+z_3^2})(\sqrt{x_2^2+y_2^2+z_2^2})}-\cos(q_{23})=0
\end{aligned}\qquad(13)$$

Equation (14) is true with knowing the length of 3 directions, too.

$$\begin{aligned}
&\left(\frac{x_1}{l_1}\right)^2+\left(\frac{y_1}{l_1}\right)^2+\left(\frac{z_1}{l_1}\right)^2-1=0\\
&\left(\frac{x_2}{l_2}\right)^2+\left(\frac{y_2}{l_2}\right)^2+\left(\frac{z_2}{l_2}\right)^2-1=0\\
&\left(\frac{x_3}{l_3}\right)^2+\left(\frac{y_3}{l_3}\right)^2+\left(\frac{z_3}{l_3}\right)^2-1=0
\end{aligned}\qquad(14)$$

Using equation (11), (13) and (14) we can form a system of 15 nonlinear equations and 10 unknown.

By solving this system of equations 10 unknown are obtained. To do it we use *fsolve* function in MATLAB. Of course, we choose the value zero for the initial value of 9 unknown and initial value of the distance between object and camera is an estimation of this distance. To estimate this distance we suppose that if 2 directions of this coordinate system be in a plane which is parallel to the camera image plane, then what would be the value of distance. Then, for 3 possible case of it the value of ***d*** calculated. The value of ***d*** that is the same in 2 cases is used as estimated ***d***.

7. simulation

In this section we want to compare the method proposed for orthogonal coordinate system [18] and the method proposed in this paper. To compare them two cases will be considered. First, we add an error in mm to the value of first coordinate of image of each line and, second, we add an error in mm to the value of second coordinate of image of each line.

Figure 6 and 7 show the first state and figure 8 and 9 show the second state. In each figure we have four plots that three of them correspond to the value of three Euler angels and one of them correspond to the position of system. In fact we define $a = 68.32$, $b = 56.42$ and $g = 34.12$. Horizontal axis of each plot is case. In figure 6 and 7, cases correspond to the error values of $[\ -0.66\ \ -0.33\ \ 0\ \ 0.33\ \ 0.66\ \ 1\ \ 1.33\ ]$ in first coordinate and in figure 8 and 9, cases correspond to the error values of $[\ -0.66\ \ -0.33\ \ 0\ \ 0.33\ \ 0.66\ \ 1\ \ 1.33\ ]$ in second coordinate.

The third case in these figures is the case without error, in other word the correct Pose values are obtained in case three.

Figure 6 and 8 are based on this paper algorithm and figure 7 and 9 are based on the algorithm in [18]. As you see in figures, the algorithm proposed in this paper is so robust in appearance of error in pixel values of the image. Also, when we choose the value of errors greater than we choose in these figures, the algorithm of this paper was exactly so robust.

8. Conclusion

In this paper, a new method for Pose determining of an object using non-orthogonal feature points with known object model is proposed. First, to simplify it suppose that the object is a line that its distance to the camera is known. With the conclusion of the complex case of line position in front of the camera, fifteen equations for a 3D object are obtained.

With simulation the correction of these equations is confirmed. One of the important points of simulation was choosing a proper initial guess for 10 unknown. For distance initial guess we use estimation of it and for the other 9 unknown zeros was a good guess.

The advantage of the method proposed in this paper is that it is based on geometric relations and also uses optimization algorithms which in included in *fsolve* function in MATLAB.

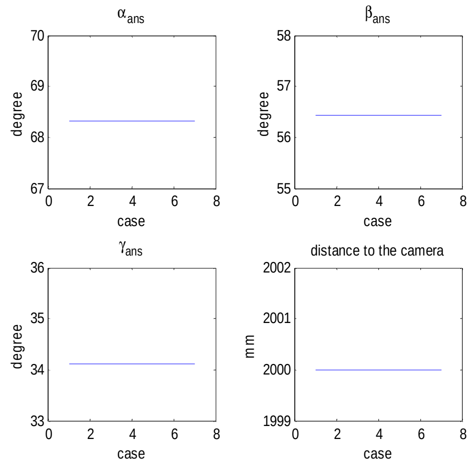

Figure 6. Pose values in respect to changes in the value of first coordinate of the image for non-orthogonal algorithm.

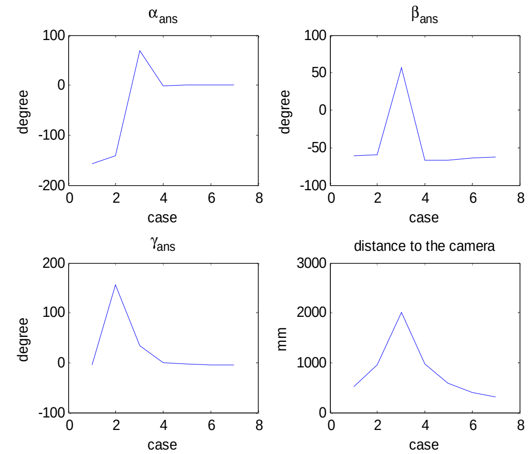

Figure 6. Pose values in respect to changes in the value of first coordinate of the image for orthogonal algorithm.

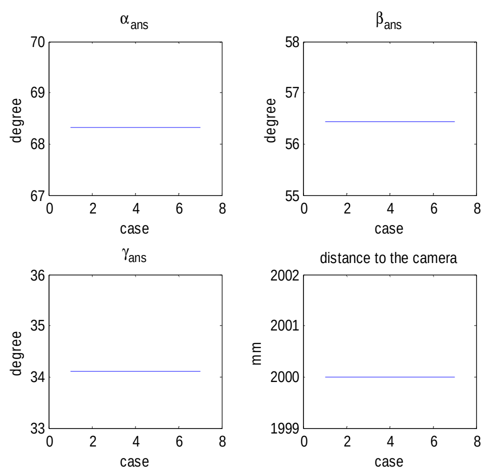

Figure 6. Pose values in respect to changes in the value of second coordinate of the image for non-orthogonal algorithm.

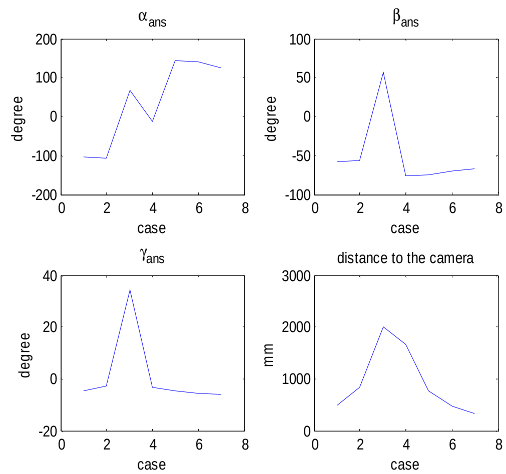

Figure 6. Pose values in respect to changes in the value of second coordinate of the image for orthogonal algorithm.

Also, this method does not limit feature points to be on orthogonal coordinate system and does not limit the size of the object. At last, by simulation it has been confirmed that this method is so robust in front of no precession data.

9.References

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