Analysis of Inter-flow Network Coding in Lossy Wireless Networks

Alireza Shafieinejad and Faramarz Hendessi

Abstract— **This paper addresses the problem of inter-flow network coding for unicast sessions in lossy channel wireless networks. In spite of decreasing the number of transmissions, network coding intuitively increases the sensitivity of nodes to lost packets. First of all, coded packets carry more information than native packets and thus losing a coded packet prohibits a series of dependent nodes from decoding their intended packets. Secondly, for the scheme with opportunistic listening, it is necessary for some of the nodes to overhear the transmission of their neighbors. Thus, successful decoding requires overhearing of the corresponding packet(s) in addition to correct reception of unicast and broadcast transmissions.**

In this paper, we study the effect of lossy channel on the aggregate network throughput in the presence of network coding. We provided a linear programming formulation to compute the throughput performance of network coding for a general lossy wireless network. Further, we consider a retransmission mechanism for both unicast and broadcast. Our LP system supports both COPE and Star coding schemes. The advantages of the proposed NC schemes over the non-NC ones are shown through simulations and theoretical analysis. Results show that network coding can boost the capacity of wireless network up to 40% under lossy channel condition.

***Index Terms*— Wireless network coding, Lossy wireless networks, Reliable broadcasting.**

I. INTRODUCTION

Wireless networks provide means for mobility, internet connectivity and distributed sensing. However, the throughput limitation of these networks imposes many practical problems. Recently, network coding has been applied to wireless networks and received significant attention as a means of improving network capacity and coping with unreliable wireless links. In fact, the unreliability and broadcast nature of wireless links make wireless networks a natural setting for network coding.

Network coding for unicast traffic began by a prominent work by Katti et al. in [1] named as COPE. It is a packet encoding scheme via XOR operation. The authors studied certain basic topologies such as chain, cross and wheel in a unicast traffic model and reported the throughput gain as the first testbed deployment of wireless network coding. In the follow up, Sengupta et al. extended the use of COPE in wireless network with any patterns of multiple concurrent unicast transmissions [2]. From a theoretical perspective, the authors provided two linear programming formulations for measuring the throughput improvements of COPE-type NC scheme for both with and without opportunistic listening. By these formulations, the authors advocated the idea of *coding-aware routing*, i.e., the routing which selects the paths with the awareness of NC opportunities.

In this paper, we study the performance gain of inter-flow network coding for a non-ideal MAC. We assume that the transmissions are unreliable and thus the packet reception on each link is successful by a specific probability. The goal is study the impact of link quality on the network coding gain. In spite of decreasing the number of transmissions, network coding intuitively increases the sensitivity of nodes to lost packets. Since coded packets are combination of multiple native packets, losing one of them during a transmission prohibits a series of dependent nodes from decoding their packets. Further, for opportunistic listening paradigm, some of nodes require to overhear the transmission of the neighbors. Therefore, decoding is dependent on both the reception of unicast/broadcast packets and overhearing of the corresponding transmissions.

The previous work mainly studied network coding gain under ideal assumption for MAC layer. Our analysis is based on two metrics. The first is the average number of lost packets and the second is the aggregate network throughput. These metrics are evaluated for a portion of network related to a coding and provide a comparison between network coding scheme and standard routing. We consider both COPE and STAR coding schemes and tries to clarify the effect of lossy channel on coding gain according to these metrics.

A. Prior Work on NC in lossy channel System

The concept of *loss-aware network coding* was introduced in [3]. The authors proposed a type of redundancy by means of network coding for multiple unicast sessions. They argued that in lossy wireless environments, a better use of local network coding is to provide higher levels of redundancy even by the cost of increasing the number of transmissions. They presented a set of algorithms, called CLONE, which can lead to further throughput gains in multi-hop wireless scenarios. The main drawback of the CLONE is that it focuses 1) only on one coding structure 2) on individual packets, instead of flows, and

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accordingly does not propose a general solution for a network of multiple coding structures.

Against the CLONE, the work in [4] has a more comprehensive scenario for the network coding in lossy wireless network. The authors studied the capacity region of the 2-hop relay network (i.e., coding structure) with packet erasure channels. The capacity characterization is in terms of linear equations. Further, they extend the 2-hop relay networks results to multi-hop wireless networks by providing a linear program that can perform the superposition optimally. Further, work [5], characterizes the capacity region of the COPE principle for 2-flow wireless butterfly networks under packet erasure channel model. The main difference of this work with previous ones are 1) allowing for random overhearing with arbitrary overhearing probabilities and 2) the potential use of non-linear network codes.

In this paper, we provided a linear programming formulation, like [4], to compute the throughput performance of network coding for a general lossy wireless network. Opposite to the works in [3] and [5], the network is not limited to a single coding scheme and can have any topology with multiple coding structures. Further, we consider sessions instead of individual packets. Moreover, we provided three LP systems, respectively for COPE, Star, joint Star and COPE coding schemes.

B. Contributions of the Work

The main contributions of this paper can be summarized as follows:

- We study various inter-flow NC schemes including COPE and Star-NC. We compare these schemes over standard routing from network throughput aspect. We propose a formulation for both successful overhearing and reliable broadcasting in wireless lossy network.
- We provide a set of linear programming formulation to evaluate throughput performance of NC scheme for any configuration of wireless network, traffic model and routing method. Our LPs allows computing the performance of COPE, Star and joint COPE and Star coding schemes in lossy wireless networks. We consider retransmission mechanism to ensure reliable broadcasting for coded packets in our LP systems.

C. Roadmap

The rest of this article is organized as follows. In Section II, we present a background for network coding for multiple unicast sessions. In particular, COPE-type coding scheme along with Star-NC, is introduced. Next, in section III, we develop a theoretical formulation to study the benefits of NC schemes over non-coding schemes in a general wireless network. Then, in section IV, we evaluate the benefit of the collaboration scheme with various network topologies and routing strategies. Finally, section V concludes the paper.

II. BACKGROUND AND RELATED WORK

We are concerned with the unicast flows between any two nodes in a desired wireless network. When two or more unicast flows cross each other, a coding opportunity at the crossing point, hereafter referred to as the *relay node*, is created. Based on the number of unicast transmissions and the way of crossing, various structures and topologies can be generated. We study these structures in two basic categories. The first is the COPE-type structure which is a two-hop coding scheme [6][7]. The second is star-structure network coding which is a two/three-hop coding scheme.

A. COPE-type Network Coding

COPE has the following features:

1) **Opportunistic Coding**: Each wireless node uses only the packets in its local queues for coding. This allows each note to take the benefit of network coding through a local decision without requiring any form of coordination with the other nodes.

2) **Opportunistic Listening**: Exploiting the broadcast nature of the wireless medium, COPE allows the nodes to overhear all of the packets communicated by its neighbors. The overheard packets are subsequently used in the coding decisions.

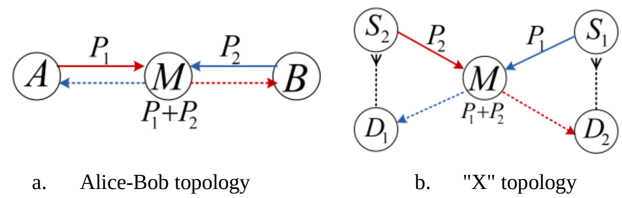


FIG. 1. COPE CODING STRUCTURES

In general, each NC opportunity has one of these scenarios: the information exchange, the opportunistic listening and the hybrid paradigm. These paradigms for COPE-type coding scheme are respectively shown in

FIG. 1*(a), (b) and (c). The information exchange refers to a situation which some nodes around the relay node have packets to destination of each others. The scheme in*

FIG. 1*(a) shows this situation for two nodes around a relay node which is known as Alice-Bob topology. Lemma 1 of [6] states that this is the only scenario for packet exchange paradigm, i.e., the scenario is only occurred for two nodes around a relay node. The opportunistic listening paradigm with four nodes is shown in*

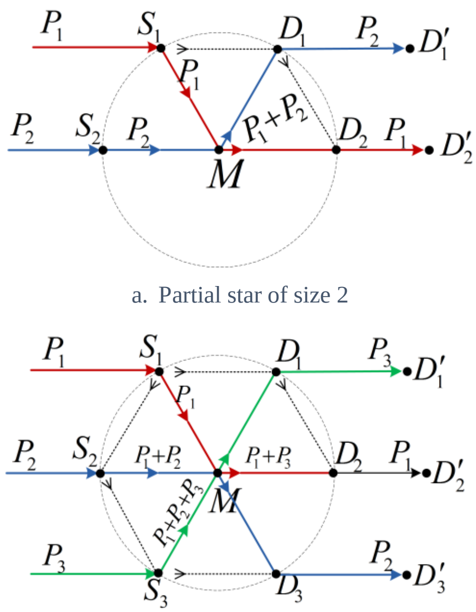
FIG. 1*(b) wherein D_1/D_2 can overhear transmission of S_2/S_1. This scheme is known as X topology. The hybrid paradigm, depicted in*

FIG. 1(c) with three nodes, takes the advantage of both packet exchange and opportunistic listening. Sengupta et al. [6] introduced the concept of *coding structure* to formulize different scenarios of COPE-type network coding scheme.

B. Star-Structure Network Coding

The concept of *star-structure network coding* (Star-NC) is introduced in [8]. It considers multiple unicast flows intersecting each other at a relay node. The relay node can decrease the required transmissions by mixing the packets of different flows. This reduction is due to the opportunistic listening of the nodes at the proximity of the relay node. The key idea of our scheme is the generality and flexibility of the opportunistic listening among the nodes around the relay node.

It aims the node to exploit more coding opportunities, thereby increasing the aggregated network throughput.



a. Partial star of size 2

b. Full star of size 3 and π=(3,1.2)

FIG. 2. STAR-NC SCHEMES.

Star-NC is centered on star topology as a basic element of network coding in wireless networks. A star structure of size n is composed of n input nodes, a relay node and n output nodes wherein every pair of input and output nodes belong to a specific unicast session. The basic idea was explained in FIG. 2(a) and (b) wherein the transmitted and overheard packets are indicated by the solid and dotted lines, respectively. We denote the input nodes by S_1 through S_n, the output nodes by D_1 through D_n and the relay node by M. The scheme in FIG. 2 (a) is *partial-star* structure while FIG. 2 (b) depicts a *full-star* structure.

The goal is the route of incoming packets to each input node to the corresponding output node. For instance, in FIG. 2 (b), the incoming packets (P_1, P_2, P_3) must be routed, in sequence, to (D_2, D_3, D_1). The same must be occurred for the incoming packets (P_1, P_2) in FIG. 2 (a) and the nodes (D_2, D_1). Note that, these packets belong to different independent unicast sessions which include the corresponding paths (e.g., S_1-M-D_2, S_2-M-D_3,S_3-M-D_1 in Fig. 2(b)). In regular routing, the relay node M sends exactly n packets to the output nodes. However, for the schemes depicted in Fig. 2(a) and Fig. 2(b), Star-NC can do this by sending only a single coded packet from M to the output nodes. Therefore, two transmissions are saved for the scheme in FIG. 2 (b) and one transmission for the NC scheme in Fig. 2(a). Precisely, Star-NC consists of three steps:

1. ***Coding at input nodes:*** S_i ($1\leq i \leq$n) sends its packet (likely encoded) to the relay node.
2. ***Coding at relay node:*** M broadcasts its encoded packets (e.g. $P_1 + P_3$ in the above example) to output nodes.
3. ***Decoding at output nodes:*** D_i ($1\leq i \leq$n) decodes desired packet using both encoded packets received from M and the overheard packets from the neighbors.

Note that, for some schemes such as Fig. 2(a), the first step is not needed since no overhearing is done by the input nodes. Thus Star-NC without/with coding at input nodes will be a *two*/*three-hop* coding scheme. Each scenario of Star-NC scheme corresponds to a specific routing pattern of flows between input and output nodes. The routing pattern is identified by a unique-spanning mapping from the input nodes to the output nodes. We define *target permutation* of Star-NC as a permutation of (*1*,*2*,..,*n*) that respectively identifies the indexes of the packets received by D_1 through D_n, e.g., (3,1,2) in Fig. 2(b). Generally, Star-NC only applies network coding to the flows going through the paths identified by the target permutation. The packets of other flows are processed via regular routing.

III. ANALYSIS FOR A GENERAL LOSSY WIRELESS NETWORK

In this section, we try to analyze a general lossy-link network consist of multiple coding structure as well as regarding practical issues of MAC. The road map is based on a linear programming formulation which computes the throughput performance of network coding for a general network configuration including various topology, routing method, traffic model and coding scheme.

We formulate a linear programming (LP) framework to find the maximum throughput of the network using Star-NC scheme. The framework uses an LP technique similar to ones used in[2; 9; 10]. The difference is that our scheme is based on opportunistic listening that is missing in [9] and[10]. Further, our framework significantly differs from [2] in which it has a two-hop coding scheme with opportunistic listening while Star-NC has both two and three hop coding schemes.

A. Reliable broadcasting

The 802.11 MAC has two modes: unicast and broadcast. In the 802.11 unicast mode, packets are immediately ack-ed by their intended nexthops. The 802.11 protocol ensures reliability by retransmitting the packet at the MAC layer for a fixed number of times until a synchronous ack is received. Lack of an ack is interpreted as a collision signal, to which the sender reacts by backing off exponentially, thereby allowing multiple nodes to share the medium.

Suppose that the probability of a successful unicast transmission on a specific link is equal to p. The number of transmission required to send a packet is a random variable with geometric distribution. Thus the average number of required transmissions is equal to:

$$E(X) = \sum_{i=1} i.P(X = i) = \sum_{i=1} i.p(1 - p)^{i-1} = \frac{1}{p} \quad (1)$$

It means that successful transmission consumes a bandwidth equal to $\frac{1}{p}$ of packet size. Note that, we ignore from the bandwidth which is used by control packet during retransmissions.

In contrast, IEEE 802.11 broadcast mode lacks both reliability and backoff. Since a broadcast packet has many intended receivers, it is unclear who should ack. In the absence of the acks, the broadcast mode offers no retransmissions and consequently very low reliability. Further, a broadcast source cannot detect collisions, and thus does not back off. If multiple backlogged nodes share the broadcast channel, and each of them continues sending at the highest rate, the resulting throughput is therefore very poor, due to high collision rates.

COPE presents two distinct solutions for this problem; pseudo-broadcast and hop-by-hop ack. The former piggybacks on IEEE 802.11 unicast and benefits from its reliability and backoff mechanism. Pseudo-broadcast unicasts packets that are meant for broadcast. The link-layer destination field is set to the MAC address of one of the intended recipients.

In previous section, we did not consider the retransmission mechanism, i.e., the coding opportunity is missed in the case of reception error in the relay node and the remained packets are forwarded to next-hop using standard routing. By regarding retransmission mechanism, we can assume the packets are reliably transmitted to the intended next-hop except that an additional bandwidth is consumed due to the lossy channel. Obviously, this extra bandwidth is dependent the rate of packet loss which is different for each network link. Now, we want to provide a theoretical formulation for the amount of bandwidth required for successful broadcasting a packet to *n* neighbors.

Consider that node M, e.g. in Fig. 1(a), is to broadcast a coded packet to both A and B. Assume that the probability of successful transmission on the link *(M,A)* and *(M,B)* are respectively equal to p_1 and p_2. Further suppose M schedules a retransmission mechanism to resend the packet in the case of not receiving ACK from either A or B. Let $X_{A,B}$ denotes the number of transmissions required for a successful broadcasting to both A and B. Nguyen showed in [11] that:

$$\mathrm{E}(\mathrm{X}_{\mathrm{A,B}}) = \frac{1}{\mathrm{p}_1} + \frac{1}{\mathrm{p}_2} - \frac{1}{1-(1-\mathrm{p}_1)(1-\mathrm{p}_2)} \tag{2}$$

Similarly, the average number of transmissions for successful broadcasting to *n* nodes, namely A_1 through A_n , is equal to:

$$E(X_{A_1,A_2,..,A_n}) = \sum_{m=1}^{n} \sum_{1\le i_1<i_2<..<i_m\le n} \frac{(-1)^{m+1}}{1-\prod_{k=1}^{m}(1-p_{i_k})} \tag{3}$$

In which p_i denotes the success probability of transmission on link (M, A_i). In our optimization framework, we use N_{Bcast} to denote the average number of transmissions for successful broadcasting to *n* nodes. The parameters of N_{Bcast} are the success probabilities of transmission on each link, that is:

$$N_{Bcast}(p_1, p_2, .., p_n) = E(X_{A_1,A_2,..,A_n}) \tag{4}$$

B. Overhearing of unicast transmission

In our loss-aware coding scheme, we are engaged in computation the probability of overhearing a unicast transmission to the relay node. Note that under the assumption of backoff and retransmission, like in IEEE 802.11, a native packet can be sent more than one times, thereby increasing the chance of a third node to overhear the packet.

Suppose A sends a packet to M while B tries to overhear this packet. Assume the success probability of packet transmission on links (A,M) and (A,B) equal to p_o and p_u , respectively. The overhearing could be done either in the first transmission of A or in the successive retransmission of A. Let X_A denotes the number of required transmissions from A to successfully send the packet to M. Thus, we have:

$$\begin{aligned} P_{Over}(B \leftarrow AM) &= \sum_{n=1} P(Over_B | X_A = n) \\ &= \sum_{n=1}^{\infty} p_u(1-p_u)^{n-1}[1-(1-p_o)^n] \\ &= \sum_{n=1}^{\infty} p_u(1-p_u)^{n-1} \\ &\quad - p_u(1 - p_o) \sum_{n=1}^{\infty} [(1-p_u)(1-p_o)]^{n-1} \\ &= 1 - \frac{p_u(1-p_o)}{1-(1-p_u)(1-p_o)} \\ &= \frac{p_o}{1-(1-p_u)(1-p_o)} \end{aligned} \tag{5}$$

Note that if $p_u = 1$ then $P(Over_B) = p_o$ which confirms with the above equation. We can extend the formula, from listener aspect, to obtain the probability of overhearing for two or more nodes. Suppose both B_1 and B_2 try to overhear the transmission of node A. We can show that:

$$\begin{aligned} \mathrm{P}(Over_{B_1,B_2\leftarrow \mathrm{AM}}) &= \sum_{\mathrm{n}=1} P(Over_{B_1,B_2} | X_A = \mathrm{n}) \\ &= 1 - \frac{\mathrm{p_u}(1-\mathrm{p}_{\mathrm{o}_1})}{1-(1-\mathrm{p_u})(1-\mathrm{p}_{\mathrm{o}_1})} \\ &\quad - \frac{\mathrm{p_u}(1-\mathrm{p}_{\mathrm{o}_2})}{1-(1-\mathrm{p_u})(1-\mathrm{p}_{\mathrm{o}_2})} \\ &\quad + \frac{\mathrm{p_u}(1-\mathrm{p}_{\mathrm{o}_1})(1-\mathrm{p}_{\mathrm{o}_2})}{1-(1-\mathrm{p_u})(1-\mathrm{p}_{\mathrm{o}_1})(1-\mathrm{p}_{\mathrm{o}_2})} \end{aligned} \tag{6}$$

And in a similar manner, we can obtain a formula for general case:

$$\mathrm{P}(Over_{B_1,B_2,..,B_n\leftarrow \mathrm{AM}}) = \sum_{m=1}^{n} \sum_{1\le i_1<i_2<..<i_m\le n} \frac{(-1)^m p_u \prod_{k=1}^{m}(1-p_{o_{i_k}})}{1-(1-p_u)\prod_{k=1}^{m}(1-p_{o_{i_k}})} \tag{7}$$

C. Notation and modeling assumptions

1) COPE structure modeling

COPE coding structure is modeled by the pair $(\mathcal{SD}, \Omega)$. The former denotes a set containing the links of the structure while the latter associates with the overhearing information. An element of $\mathcal{SD}$ is described by a triple $(\mathrm{e}_1, \mathrm{e}_2, \mathrm{t})$ in which e_1 and e_2 respectively denote input and output link of the coding structure and *t=N,C* identifies the input traffic type which is either native (N) or coded(C). The coded traffic refers to the set of packet which is encoded by other coding structures. This definition of $\mathcal{SD}$ is on the basis of Sengupta's formulation of COPE-type coding structure [6]. For some coding structure such as 'X' topology, it is necessary for the input traffic to be native. For example, 'X' topology only have a single choice for $\mathcal{SD}$ as $\{(S_2M, MD_1, N), (S_1M, MD_2, N)\}$. However, Alice-Bob topology, which is based on information exchange paradigm, is more flexible than 'X' topology and accepts traffic of both native and coded type. Such flexibility allows a single Alice-Bob topology to have four options of coding structures with distinct $\mathcal{SD}$ as the followings:

$$\mathcal{SD}_{NN} = \{(AM, MB, N), (BM, MA, N)\}$$
$$\mathcal{SD}_{VC} = \{(AM, MB, N), (BM, MA, C)\}$$
$$\mathcal{SD}_{CN} = \{(AM, MB, C), (BM, MA, N)\}$$
$$\mathcal{SD}_{CC} = \{(AM, MB, C), (BM, MA, C)\}$$

On the other side, Ω consists of elements such as (e,V_o) in which e denotes the link whose transmissions are overheard by the members of set V_o. For example, for 'X' topology, we will have:

$$\Omega = \{(S_2M, \{D_1\}), (S_1M, \{D_2\})\}$$

For unified modeling of coding with and without opportunistic listening, we further consider the Alice-Bob topology in native-input-traffic mode, in which incoming traffic to the structure is native, as a *self-overhearing* case. That is, the coding structures related to Alice-Bob topology are described as:

	$\mathcal{SD}$	Ω
ψ_{NN}	$\{(AM, MB, N), (BM, MA, N)\}$	$\{(AM, \{A\}), (BM, \{B\})\}$
ψ_{NC}	$\{(AM, MB, N), (BM, MA, C)\}$	$\{(AM, \{A\})\}$
ψ_{CN}	$\{(AM, MB, C), (BM, MA, N)\}\}$	$\{(BM, \{B\})\}$
ψ_{CC}	$\{(AM, MB, C), (BM, MA, C)\}$	$\{\}$

For example, entry $(AM, \{A\})$ denotes that the node A overhears its transmissions to node M, i.e., saves its outgoing packets destined to M. Next, we will see that such notation help us to simply represent the *overhearing constraint* in our LP formulation. In LP formulation, we refer to component x of ψ as $x(\psi)$. Additionally, for the unification of both COPE and Star-NC modeling, we redefine the notations $\mathcal{S}$, $\mathcal{D}$ and N of Star-NC for COPE structure. That is, $\mathcal{S}(\psi)$ refers to input links of structure ψ, $\mathcal{D}(\psi)$ points to its output links and $N(\psi)$ refers to a constant value equals to 1. Further, denotes the size of coding structure (number of elements in $\mathcal{SD}$).

2) Other Definitions and assumptions

The notations and modeling assumptions are listed in TABLE 1. We use the protocol model of interference introduced by Gupta and Kumar[12], i.e., two nodes have a link if their distance is less than *communication range* and are interfered if their distance is less than *interference range*. Also, two links $e_1 = (i_1, j_1)$ and $e_2 = (i_2, j_2)$ are interfered either as j_1 is within the interference range of i_2 or j_2 is within the interference range of i_1.

The success probability of transmission on link e is denoted by p_e. The directional link e can be denoted by the node pair *(u, v)* in which u and v, respectively, identify the sender and receiver of the link. For a specific coding structure, N_{Bcast} identifies the average number of transmissions required for successful broadcasting a coded packet to output nodes. It is defined as a function of the success transmission probability of the output links:

$$N_{Bcast}(\psi) = N_{Bcast}\left(p_{\mathcal{D}(\psi)_1}, p_{\mathcal{D}(\psi)_2}, \ldots, p_{\mathcal{D}(\psi)_n}\right) \quad (8)$$

Further, for a specific link e of coding structure, $P_{Overhear}$ identifies the probability of successful overhearing the transmission on link e by the listener nodes:

$$P_{Overhear}(e, V_o) = P\left(over_{v_1,..,v_k \leftarrow e}\right) \ : V_o = \{v_1, \ldots, v_k\} \quad (9)$$

D. Optimization framework for COPE coding schemes

LP formulation is defined by the constraints in (11) through (22). This system is derived from Sengupta's formulation [6] by extending it in order that covers lossy links. The formulation is done in format close to Star-NC modeling. The traffic is modeled as a set of traffic demand denoted by D. The demand k corresponds to $D(k)$ amount of traffic (e.g. in Mbps) requested by source node *s(k)* to be routed to destination node *d(k)*. As in[2; 9; 10], we define the throughput as a multiplier λ such that for each demand k, at least $\lambda D(k)$ amount of requested traffic is guaranteed to be routed by the network. This definition holds the linearity of the system while provides a means of fairness for MAC scheduling. Note that the aggregated network throughput is equal to the sum of all routed traffic for each demand, i.e. $\sum_{k\epsilon D} D(k)\lambda$. We have the following set of constraints:

Fairness constraints: The first constraint is about definition of throughput in order that takes the system as linear at the same time as considers fair bandwidth allocation. We consider multi-path routing in our modeling. Let P_k be the set of available paths for the routing demand k from *s(k)* to *d(k)*. Assume $F_k(P)$ denotes the amount of traffic on path P for routing demand k, where $P\epsilon P_k$. Thus, the total traffic routed for demand k equals $\sum_{P\epsilon P_k} F_k(P)$. On the other hand, this amount of traffic must be equal to value of demand k multiplied by throughput, i.e. $D(k)\lambda$. This is stated by the constraint in (11).

Coding constraints: For this constraint, we need to know the amount of multiple unicast traffic intersecting each other at $M(\psi)$. To derive this condition, we use $z_e^k(P)$ to denote the portion of the traffic on path P for demand k that is transmitted as native from link e. Thus for each combination of incoming link e_1 and outgoing link e_2 at node M, the portion of transmitted traffic that received as native is equal to $\sum_{k\epsilon D}\sum_{P\epsilon P_k:P\ni e_1 e_2} z_{e_1}^k(P)$.

Now, based on the input traffic type of each elements of coding structure, i.e., parameter t in (e_1, e_2, t), we have different situations.

For the structure which needs the incoming traffic on link e_1 to be native, the rate of coded traffic must be less than or equal to total native-transmitted traffic on link pair (e_1, e_2), i.e., :

$$f^{NC}(\psi) \le \sum_{k\epsilon D} \sum_{P\epsilon P_k:P\ni e_1 e_2} z_{e_1}^k(P) \quad (9)$$

Here, $f^{NC}(\psi)$ denotes the rate by which the relay node of ψ generates coded traffic and broadcasts to output nodes. Constraint (14) states an extended form of the above condition since the pair $e_1 e_2$ may participate in more than one coding structure. Note that, Γ refers to set of all COPE coding structures in the network. In contrast, constraint (15) represents this condition for the structure whose incoming traffic into link e_1 has coded type, i.e., itself is coded by another structure.

We can write a balance constraint for $F_k(P)$ in terms of $z_e^k(P)$ and $f^{NC}(\psi)$ in (16) where the total transmitted traffic entering through link e_1 and exiting through link e_2 appears on LHS. The first portion on RHS, is the amount of traffic that participates in coding as native while the second denotes the amount that participates in coding as coded. The last portion is the amount of traffic that goes out as native, i.e., does not participate in any coding. Furthermore, $z_e^k(P)$ is bounded by constraints (17) and (18).

Overhearing constraint: For coding scheme with opportunistic listening, overhearing the required packets is a necessary condition for the output node to be able to decode the coded packet. This constraint can be satisfied by the following ways:

1. The relay node decides to encode when it is convinced about the successful overhearing of the corresponding packet(s) by output nodes.
2. The relay node decides to encode as soon as possible it collects enough packets without concerning about the successful overhearing of output nodes. If some of the output node cannot decode the coded packet as a result of missed overhearing, they request the transmission of original packet to the relay node.

The former strategy is used by COPE. Note that, the opportunistic listening in COPE structure is limited to the overhearing transmission of input nodes by output nodes, namely output-from-input. Thus the demanded overhearing is done before the relay node's decision about the encoding of the packets. In contrast, this method cannot be employed by star structure since it encounters some overhearing of the form output-from-output. It means that some of the overhearing is taken after generation of coded packet and exactly at the time that an output node transmits its decoded packet. Therefore, in some situation of star structure we are bound to use the second solution.

Note that, both solutions encounter some drawbacks. The first enforces every node to periodically inform the neighbors about the overheard packets. Obviously, a portion of bandwidth is consumed for transmission of such control messages. This mechanism is implemented in COPE protocol by adding a special block, so-called reception report, to COPE header. Each node periodically sends the reception reports either in a distinct packet or via piggybacking on other packet to the neighbors. The second solution wastes the bandwidth for sending the coded packet which is not decodable by the output nodes as a result of missed overhearing. This can diminish the coding gain especially when the overhearing packet loss is significantly increased due to the poor link quality.

Regarding the first solution in our LP system, bounds the encoding rate to the rate of successful overhearing. Thus for each coding structure $\psi \in \Gamma$ and each element of $\Omega(\psi)$, we have:

$$f^{NC}(\psi) \le \mathrm{P}_{\mathrm{Overhear}}(e, V_o) \sum_{k\in D} \sum_{P\in P_k: P\ni(e,\acute{e})} z_e^k(P) \quad \forall e : (e, V_o) \in \Omega(\psi) \ \wedge (e,\acute{e}) \in \mathcal{SD}(\psi) \tag{10}$$

Note that, we use $z_e^k(P)$ in RHS of the above constraint since the COPE opportunistic listening is limited to overhearing of the native packets. Since the links are lossy, we must consider the probability of successful overhearing. Thus for each link e we use the term $\mathrm{P}_{\mathrm{Overhear}}(e, V_o)$ which denotes the successful overhearing of unicast transmission on link e.

Further, as a link may be overheard in more than one coding structure, we extend the above inequality to constraint (17) in order that covers all the coding structures that overhears transmission of link e.

TABLE 1. SUMMARY OF NOTIONS USED IN THE SYSTEM MODELING.

V	The set of nodes in the network
E	The set of directed links in the network
(u,v)	The link from node u to node v.
$E^+(v)$	The set of incoming links incident on node v
$E^-(v)$	The set of outgoing links incident on node v
D	The set of traffic demands (related to unicast sessions)
$D(k)$	Traffic amount which requested by session k
λ_k	The end-to-end throughput for the demand k, i.e. a portion of $D(k)$ which can be routed by the network
$C(e)$	Capacity of data link e
$t(e)$	The transmitting node of the directed link e
$r(e)$	The receiving node of the directed link e
$s(k)$	Source node of traffic demand k
$d(k)$	Destination node of traffic demand k
$\mathcal{S}$	Data links of star structure from input nodes to relay node, i.e. $\mathcal{S} = \{S_1, .., S_n\} \times \{M\}$
$\mathcal{D}$	Data links of star structure from relay node to output nodes, i.e.$\mathcal{D} = \{M\} \times \{D_1, .., D_n\}$
$\xi = (M, \mathcal{S}, \mathcal{D}, \pi, m)$	Star coding structure of size n with five tuples: M is the relay node, $\mathcal{S}$ and $\mathcal{D}$, respectively, denote the input and output links to M, π is target permutation for routing and m is radius of overhearing ($n = \lvert\mathcal{S}\rvert = \lvert\mathcal{D}\rvert$).
$\psi = (\mathcal{SD}, \Omega)$	COPE coding structure, $\mathcal{SD}$ represents a set consists of link-pairs and Ω denotes the overhearing information
Ω	A set contains elements of (e,V_o), in which e denotes the link whose transmissions is overheard by each node of V_o.
$\mathcal{SD}$	A set contains elements of triple (e_1, e_2, t) in which e_1 and e_2 respectively denote input and output link of the coding structure and t=N,C identifies the input traffic type which is either native (N) or coded(C).
$N_{Bcast}(\psi)$	The average number of transmission which is required by the relay node of ψ to successfully broadcast a coded packet to output nodes of the structure.
Γ	Set of all Star and COPE coding structures in the network.
$f(e)$	Total flow rate passed through link e
$F_k(P)$	The flow rate of demand k over path P
P_k	The set of available paths for demand k
$f^{NC}(\psi)$	Flow rate of coded traffic for ψ which broadcast by M to output nodes
$z_e^k(P)$	The portion of the traffic on path P for demand k that is transmitted as uncoded from link e.
$E^C(e)$	The set links which conflict by link e
$\Gamma^C(e)$	The set Star structures which conflict by link e

Note that, the link e may also participate in a coding scheme with information exchange. In this case, the encoding rate of that structure is limited to the transmission of link e, too. To avoid the complexity of classification coding scheme to with and without overhearing, we consider the information exchange paradigm in native-input- traffic mode as a *self-overhearing* case which is described in previous section. This approach leads to a unified description of both the overhearing and packet exchange condition, as well as done in constraint (17).

Interference constraint: We consider three different link and traffic combinations : i) the link which is not output link of any structures and thus always transmits unicast traffic, ii) the output link of a structures which transmits NC traffic and iii) the output link of star structures which transmits unicast traffic.

This classification identifies three types of interferences which respectively associated with three terms of constraint given in (19). In the first term, we seek link é which either resides outside of any coding structure or is an input link of a coding structure. In the second, we explore the structure ψ whose NC traffic is in the interference range of link e. This holds as e is interfered with any output links of ψ. In the last term, we look for the output link é of any ψ which interferes with link e and transmits unicast traffic. The amount of unicast traffic over link é is equal to the total traffic going through é minus the rate of flow which is decoded at the output node of é from the NC traffic, i.e. $\mathrm{f}(\acute{e}) - \frac{\mathrm{f}^{\mathrm{NC}}(\psi)}{\mathrm{N}(\psi)}$.

Maximize λ

Subject to

$$\sum_{P \epsilon P_k} F_k(P) = D(k)\lambda \quad \forall k \in D \tag{11}$$

$$z_e^k(P) \le F_k(P) \quad \forall k \in D, P \epsilon P_k : P \ni e \tag{12}$$

$$z_e^k(P) = F_k(P) \quad \forall k \in D, P \epsilon P_k : P \ni e\, , t(e) = s(k) \tag{13}$$

$$\sum_{\psi \in \Gamma:\, (e_1, e_2, N) \in \mathcal{SD}\,(\psi)} f^{NC}(\psi) \le \sum_{k \epsilon D} \sum_{P \epsilon P_k : P \ni e_1 e_2} z_{e_1}^k(P) \quad \forall M \in V, e_1 \in E^-(M \tag{14}$$

$$\sum_{\psi \in \Gamma:\, (e_1, e_2, C) \in \mathcal{SD}\,(\psi)} f^{NC}(\psi) \le \sum_{k \epsilon D} \sum_{P \epsilon P_k : P \ni e_1 e_2} (F_k(P) - z_{e_1}^k(P)) \quad \forall M \in V, \epsilon \tag{15}$$

$$\begin{aligned} \sum_{k \epsilon D} \sum_{\substack{P \epsilon P_k: \\ P \ni e_1 e_2}} F_k(P) = & \sum_{\substack{\psi \in \Gamma: \\ (e_1, e_2, N) \in \mathcal{SD}\,(\psi)}} f^{NC}(\psi) + \sum_{\substack{\psi \in \Gamma: \\ (e_1, e_2, C) \in \mathcal{SD}\,(\psi)}} f^{NC}(\psi) \\ & + \sum_{k \epsilon D} \sum_{\substack{P \epsilon P_k: \\ P \ni e_1 e_2}} z_{e_2}^k \quad \forall M \in V, e_1 \in E^-(M), e_2 \\ & \in E^+(M) \end{aligned} \tag{16}$$

$$\begin{aligned} \sum_{\substack{\psi \in \Gamma: (e, \acute{e}) \in \mathcal{SD}(\psi), \\ (e, V_o) \in \Omega(\psi)}} f^{NC}(\psi)/\mathrm{P}_{\mathrm{Overhear}}(e, V_o) \le \sum_{k \in D} \sum_{\substack{P \in P_k: \\ P \ni (e, \acute{e})}} z_e^k(P) \quad \forall M \\ \in V, e \in E^-(M), \acute{e} \in E^+(M) \end{aligned} \tag{17}$$

$$f(e) = \sum_{k \epsilon D} \sum_{P \epsilon P_k : P \ni e} F_k(P) \qquad \forall e \in E \tag{18}$$

$$\begin{aligned} & \sum_{\substack{\acute{e} \in E^C(e), \\ \nexists \psi \in \Gamma^C(e) : \acute{e} \in \mathcal{D}(\psi)}} f(\acute{e})/p_{\acute{e}} + \sum_{\psi \in \Gamma^C(e)} f^{NC}(\psi) . N_{Bcast}(\psi) \\ & + \sum_{\substack{\acute{e} \in E^C(e), \\ \exists \psi \in \Gamma^C(e) : \acute{e} \in \mathcal{D}(\psi)}} \left(f(\acute{e}) - \sum_{\substack{\forall \psi \in \Gamma^C(e), \\ \mathcal{D}(\psi) \ni \acute{e}}} f^{NC}(\psi) \right) \\ & /p_{\acute{e}} \le C(e) \quad \forall e \epsilon E \end{aligned} \tag{19}$$

$$0 \le f(e) \le p_e C(e) \quad \forall e \in \mathrm{E} \tag{20}$$

$$0 \le f^{NC}(\psi) \le N_{Bcast}(\psi) . C(e) \quad \forall \psi \in \Gamma \tag{21}$$

$$\sum_{e \epsilon E^+(v)} f(e) - \sum_{e \epsilon E^-(v)} f(e) = \begin{cases} 0 & \forall v \ne s(k), d(k), k \in D \\ \sum_{\substack{k \in D, \\ v = s(k)}} \lambda_k - \sum_{\substack{k \in D, \\ v = d(k)}} \lambda_k & \forall v = s(k), d(k), k \in D \end{cases} \tag{22}$$

An important feature of the current LP is that it considers the lossy channel which influences the interference constraint. For lossy channel, a transmission may be unsuccessful and thus the sender tries to retransmit it anyway. It means that the allocated bandwidth is more than the amount of traffic which is successfully transmitted. In the previous section, we stated that unicast transmission of a single packet on a link with success probability p needs $\frac{1}{\mathrm{p}}$ transmissions in average. Further, the average number of transmissions for a successful broadcasting can be computed by NBcast function. These formulations are appeared in constraint (17) where the amount of unicast traffic on link é is divided by $\mathrm{p}_{\acute{e}}$ and the broadcast traffic of structure ψ is multiplied by $\mathrm{N}_{\mathrm{Bcast}}(\psi)$.

Routing constraint: The constraint given by (22) maintains the flow conservation at every node of the network. For the forwarder nodes which are neither source nor destination of any session, the difference of incoming and outgoing traffic is zero. For the other which is either sink or source or both, the difference becomes a non-zero value determined by RHS of (22).

Link capacity constraint: Equations (20) and (21) limit the flow rate of a link to its capacity, for unicast and NC traffic, respectively.

The above constraints form the linear programming setup to maximize λ. The complete LP formulation is shown in (11) through (22). It is worth noting that LP represents the general form of the throughput optimization for non-NC scheme in addition to COPE coding scheme. If we set $\Gamma = \emptyset$, then LP becomes a throughput optimization problem for the non-NC scheme.

IV. EVALUATIONS

In this section, we evaluate the proposed NC schemes compared to non-NC ones. First, we introduce the configurations and then, the evaluation results are presented.

A. *Configurations*

1) *Network topologies*

We use two target topologies consisting of 49 nodes located in a square of size 490m×490m. For the first network, the positions were randomly chosen while maintaining connectivity. The primary evaluation, under configuration OFDM-6mb/s, shows that the communication/interference ranges is approximately equal to 110m/220m for default noise level (7dBm). Increasing the noise to higher level makes the communication range smaller than 110m, according to the log-distance model depicted in FIG. 3. Note that the appearance of a link between a pair of nodes in the network topology is completely dependent on noise level. Our target topology regards the noise level equals to 10dBm. The resulted network is shown in Fig. 4 where the numbers in red font denote the success probability of the link while the others, without any number, denote the loss-free links. The second topology is a grid network where in each node has a distance equal to 70m from neighbors. Note that each node is in the transmission range of at most four neighbors from UP, RIGHT, LEFT and DOWN directions. Thus the nodes in diagonal directions cannot communicate to each other.

2) *Routing Strategies*

We consider three routing strategies: 1. single-path routing (SP), 2.optimized single path routing (OSP) and 3.multi-path routing (MP). In particular, the single-path routing can be obtained by Dijkstra’s algorithm and a metric which is to be minimized such as the Hop-count and joint Hop-count and physical distance. Note that the single-path routing is neither coding-aware nor interference-aware, i.e. none of coding opportunities or interference among the nodes is considered. To overcome these shortcomings, we lead to use multi-path routing.

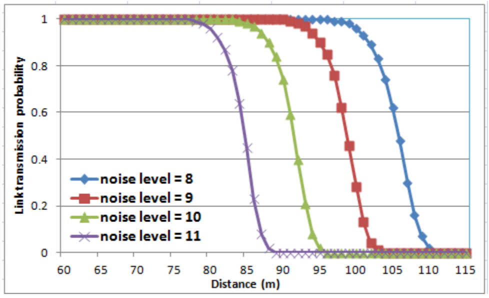


FIG. 3 SUCCESSFUL TRANSMISSION PROBABILITY FOR OFDM-6MB/S IN TERMS OF DISTANCE AND NOISE LEVEL

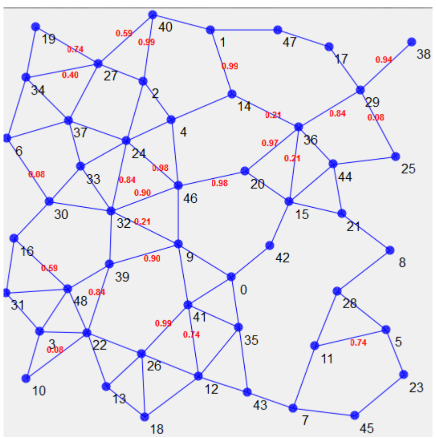


FIG. 4. NETWORK TOPOLOGY USED IN EVALUATION

As mentioned above, our LP formulation supports both the multi-path routing and single-path routing strategies. Multi-path routing considers *interference-aware* routing where the paths that minimize the interference are selected. The multi-path routing is implemented via *internally pairwise edge-disjoint paths*[13]. That is, for two nodes *s* and *d*, first we find the shortest single path between *s* and *d*. Then we remove links of this path and explore the possible single shortest path among remaining links. We repeat the procedure until no route exists between *s* and *d*. At the end, we remove the possible cycles from the paths found between *s* and *d* when all the links are present. The number of the paths found between *s* and *d* by this method depends on the *edge-connectivity* of the nodes in the graph, which equals to the *edge-cut* of *s* and *d*[13]. In our evaluation for the full grid network, this number is a variable between 3 and 8 depending on the position of the source and destination nodes.

However, in practice, multi-path routing is not applicable in most cases due to the high routing maintenance overhead. Thus, similar to [10], we consider *optimized single-path* routing in addition to single-path and multi-path routing strategies. The key idea of the optimized single-path routing is to select the path that provides the maximum flow in the multi-path routing for each session. The optimized single-path routing can be obtained by the following two steps. First, we solve the LP with multi-path routing. For each session, we select the path that achieves the highest flow, i.e., $P_{Opt} = Max_{P\in P_k} F_k(P)$. Second, these obtained optimized single-path routes for each session are then fed to the LP and solve the LP again.

Note that optimized single-path routing like multi-path routing tries to minimize interference among nodes, i.e. interference-aware routing. Intuitively, multi-path routing can provide more coding opportunities for the wireless NC schemes than single-path routing. On the other hand when the NC scheme employed in multi-path or optimized single-path routing method, another option for optimization is created. The routing protocol tries to transmit traffic from the paths which create more coding opportunities, i.e. *coding-aware* routing [2] in addition to paths that minimize interference among nodes. Hence, for multi-path routing, both the notion of coding-aware and interference-aware routing must be considered. In particular, our LP formulation provides a systematic approach for finding the routes that optimize the tradeoffs between the opposite effects of increased coding and increased interference and identifies the best routing choices.

3) Coding Strategies

Our evaluation covers two main NC schemes 1) the Star-NC 2) the COPE-type NC scheme which are referred to as NC(STAR) and NC(COPE) in the plots, respectively. Moreover, we consider joint Star and COPE type NC indicated by NC(STAR+COPE) in the plots. Note that, due to journal limitation, we only provide the LP system for COPE coding structure. The LP systems for both Star-NC and join Star and COPE can be found in [14]

4) Putting it all together

We developed a testbed tool which integrates all of the above modeling options. By setting the proper configurations about network topology, traffic model, coding scheme and routing strategy, our evaluation testbed generates the corresponding LP system. We solve this LP using AMPL [15] with the CPLEX solver [16] to obtain the theoretically optimized throughputs and the corresponding flows for the non-NC, Star-NC, COPE-type NC and joint Star and COPE NC schemes, respectively. For most often cases of multi-path routing, it is necessary to resolve the LP formulation since the load of some links become zero while the throughput is evaluated by considering corresponding interference constraints about these links. Thus, after each LP solution for multi-path routing, our testbed tool verifies whether zero-load links are found, if yes, then it removes the paths passing through these links and resolve the LP system.

B. Evaluation results

We developed a testbed tool in which by getting the proper configurations of network topology, traffic model, coding scheme and routing strategy, generates the corresponding LP system. We solve this LP using AMPL [15] with the CPLEX solver [16] to obtain the theoretically optimized throughput and the corresponding flows for the non-NC and joint Star and COPE NC schemes, respectively. Note that SP is the acronym of shortest path routing. Further, NC(STASR+COPE) means that we use both COPE and Star structures for coding opportunities.

Evaluation 1: Coding for random traffic in random network: The result for the throughput performance of network coding is shown in Fig. 5 for random network. We vary the number of demands from 10 to 100. Since the routes in single-path are fixed, the throughput improvement is based on the coding opportunities created by multiple unicast sessions crossed at a relay node. The improvement for STAR and COPE are approximately equal to 15%, 30% relative to non-coding scheme. Further joint star and COPE coding has performance improvement about 38% relative to single-path routing. This means that Star-NC creates coding opportunities different from COPE-type coding schemes which can improve the gain of coding up to 8% relative to COPE-Type NC schemes.

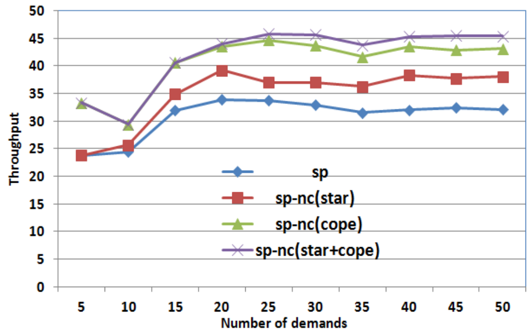


Fig. 5. Aggregate throughput for random topology (normalized to link capacity)

Evaluation 2: Coding for random traffic in grid network: We repeat the previous experiment for grid network. The result is shown in Fig. 6. The improvement for STAR and COPE are approximately equal to 25% and 34% relative to non-coding scheme. Further the joint coding scheme has performance improvement about 41% relative to single-path routing.

Evaluation 3: Coding for multi-path and optimized single-path routing in grid network: Multi-path routing selects the paths which minimize the interference among nodes. Since the average node degree in our network topology is relatively high, MP always finds multiple paths between each pair of nodes. Fig. 7 shows the results for multi-path and optimized single-path routing. As shown in the figure, MP itself increases throughput by 30% relative to SP. This is approximately equal to the performance of coding for single-path routing. By taking the advantage of coding to multi-path routing, joint COPE and STAR has approximate gain of 35% relative to MP.

Further, an important result is that the throughput performance for optimized single-path routing, for both coding and non-coding schemes, is close to multi-path routing. Precisely, the evaluation shows that the difference between MP and OSP is always less than 4% for different scenarios, i.e., there is no significant difference between gain of MP and OSP. This is a prominent result since the implementation of multi-path routing is so difficult for its heavy overhead in practice, while the optimized single-path routing can simply be implemented by means of source-routing protocols such as DSR.

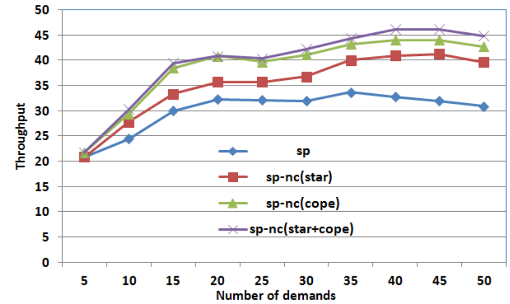


Fig. 6. Aggregate throughput for grid topology (normalized to link capacity)

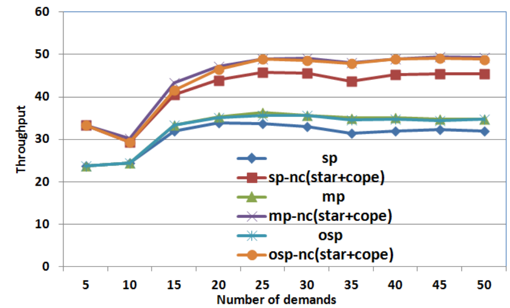


Fig. 7. Aggregate throughput for different routing in grid topology (normalized to link capacity).

Evaluation 4: Coding for multi-path and optimized single-path routing in random network: The experiment of evaluation 3 is repeated for random network. The result is shown in Fig. 8. We can see that the gain of network coding for different routing strategies, i.e., MP, OSP and SP, is close to each other and is equal to 40% over non-coding schemes. Similar to evaluation 3, the performance gain of MP and OSP is the same over SP. Moreover, this is true for network coding gain of MP and OSP over corresponding non-coding schemes.

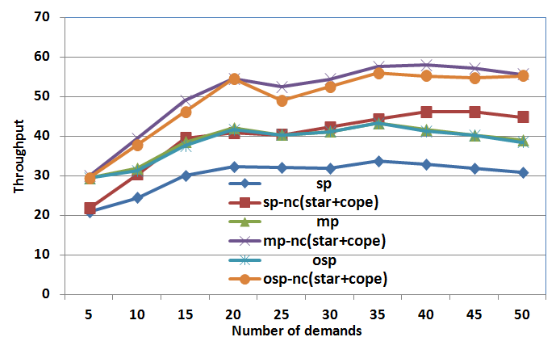


Fig. 8. Aggregate throughput for different routing in random topology (normalized to link capacity).

Against the grid network, we see that the benefit of MP is not higher than 7% over SP in the random network. This is because the average degree of node in random topology equals to 2.9 which is far from the average degree in grid network, i.e., equals to 8. Thus the number of edge-disjoint paths is dramatically reduced for random network relative to grid network, and accordingly, the opportunity of successful interference-aware routing is decreased.

V. Conclusion and Results

In this paper, we studied the throughput performance of network coding in a network with non-ideal MAC layer wherein the transmissions may encounter with error. The goal is studying the coding sensitivity to unreliable transmission. Since, coding scheme encounters broadcasting and overhearing in addition unicast transmissions, we considered the error modeling of such transmissions in distinct parameters.

We provided a linear programming formulation that computes the throughput performance of network coding for a general lossy wireless network. The result shows that the network coding can boost the aggregate throughput by a factor of up to 40% in lossy wireless network.

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