Analysis the Performance of Controllers for He-Ne Laser Stabilization by Combination of Frequency Locking and Power Balanced Methods for Nano-Metrology Applications

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Abstract
In this paper, analysis the performance of PI, PI-like fuzzy, and parallel fuzzy P+ fuzzy I controllers for He-Ne lasers frequency stabilization by combination of frequency locking and power balanced methods is presented. He-Ne lasers can be attributed to an unstable system due to the influence of environmental factors on its' frequency. Therefore, the stabilization of He-Ne laser is so important in sensitive applications such as laser interferometers and nanometrology systems. The simulation results of controllers by powerful software MATLAB/SIMULINK-GUI show that parallel fuzzy P+ fuzzy I controller has better stabilization performance and integrated absolute error (IAE) than others. Also, frequency fluctuations of He-Ne laser is about 2×10^{-11} by parallel fuzzy P+ fuzzy I controller.

Keywords: Cavity Length, Controller, He-Ne Lasers, Nanometrology, Thermal Instability.

1. Introduction
He-Ne lasers have been noticed because of their capabilities, abilities and advantages in many fields such as industry, medical and laboratory researches in recent years. In spite of having these features due to very complexity and influenced by environmental conditions is an unstable system. The frequency of He-Ne lasers tend to fluctuate because of thermal instability and this factor changes laser cavity length. Therefore, stability of output power of He-Ne lasers is necessary in sensitive applications such as laser interferometers and nanometrology systems.

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Generally, in using of interferometry for measuring any physical quantity, changing unwanted frequency causes changing unwanted phase difference and as a result, limits the measurement accuracy.

The PI control loop and fuzzy controller is widely used in many industrial applications. The PI controller is the most popular controller among industry process that includes a proportional gain K_p, an integral time constant T_i. The main reason for using PI controller in industrial applications is due to low cost, simplicity of performance and ease of design [1-4]. Recently, a number of control structures have been researched by real-time tuning the parameters of controllers [3, 4].

Fuzzy logic controller has emerged as one of the most useful research areas in the fuzzy control theory. Some tuning methods have been published before to obtain appropriate controller parameters based on system model. Internal model control structure provides a successful response based on its estimated model [5].

Several methods have been proposed for the stabilization of He-Ne lasers. At first, Suh *et al.* presented three-mode He-Ne laser stabilizer in 1993 [6]. The high frequency stabilization method was introduced by Yeom and Yoon in 2005 [7].

This paper proposes the parallel fuzzy P+ fuzzy I controller for stabilizing He-Ne lasers and then compares the performance of this controller to PI, and PI-like fuzzy controller controllers. In section 2, we describe basis of He-Ne lasers operation. In section 3, analysis and simulation results of parallel fuzzy P+ fuzzy I controller performance for stabilizing He-Ne lasers compared to other controllers is expressed. Finally, the conclusion is discussed in section 4.

2. He-Ne Laser Principles
The internal-mirror gas laser is a tube that filled with proper atoms and enclosed by two highly reflective mirrors and produces coherent light (e.g. wavelength λ=633nm for He-Ne laser) [8-10]. The mirrors form a resonant cavity whose length L determines the exact frequencies of emitted light through the condition that only exact integer multiples of a half-wavelength can resonate [8]. The relationships between mode wavelength λ and cavity length of laser L, mode frequency ν and cavity length of laser L are described as follow:

$$L = \frac{n l}{2} \qquad (1)$$

$$n = \frac{nC}{2L} \qquad (2)$$

The difference in frequency Δν, between successive resonant modes is given by:

$$\Delta n = n_{n+1} - v_n = \frac{C}{2L} \qquad (3)$$

According to Eq. (1), by increasing cavity length of laser, the difference in frequency decreases. In the

absence of some sort of stabilizing circuit the frequency range produced by a He-Ne laser is determined by the Doppler frequency shifts (this caused by the motion of the neon atoms in the hot plasma for He-Ne laser) [11]. The intrinsic width of any one mode is set by quantum mechanical effects and is much smaller. The Doppler-broadened laser gain as a function of frequency can be represented by a Gaussian function [9]:

$$g(w) = (\frac{4\ln 2}{p\Delta w_d^2})^{\frac{1}{2}} \exp[-(4\ln 2)(\frac{w - w_0}{\Delta w_d})^2] \qquad (4)$$

where, ω_0 is the center frequency and $\Delta\omega_d$ is the full width at half maximum.

A main property of He-Ne lasers, especially those with relatively short tubes that only fit two or three modes at once inside the gain curve, is that adjacent modes are polarized orthogonally as a result of mode competition [12]. The He-Ne laser supports at most three modes at once, and the orthogonal polarization of these provides a very convenient way to determine their relative intensity, and hence their relative positions on the gain curve [12].

3. Analysis the Performance of Controllers for He-Ne Laser Frequency Stabilization

Here, the basis of He-Ne lasers frequency stabilization is based on combination of frequency locking and power balanced methods, as shown in Fig. 1. We assume that the cavity length of laser is 35cm. The optical head of this combination method is shown in Fig. 2. According to Fig. 2, the beam of three-mode He-Ne laser is separated by a beam splitter. The reflected beam through 45° linear polarizer is focused on the first avalanche photodiode (APD.1). A signal having primary beat frequencies is produced by avalanche photodiode resulting from interference of the electrical fields of three modes after passing through the linear polarizer. The passed beam of beam splitter is directed towards the polarizing beam splitter. As a result, two photo-currents in accordance with amplitude of the electrical fields are produced [13].
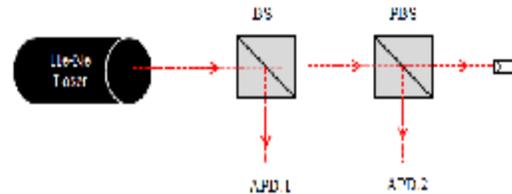

Figure 1. The structure of laser frequency stabilization

Figure 2. The optical head of laser frequency stabilization system

As shown in Fig. 1, the output current signal of APD. 1 through current to voltage converter is sent to the band pass filter. The interference of modes produces intermode beat frequencies and DC components. So, the double balanced mixer and band pass filter are needed for extracting the secondary beat frequency. After that, the signal is sent to a Frequency to Voltage converter (F/V). The output signal of F/V is a part of error signal. The differential amplifier produces another part of error signal. Two error signals with a DC signal produce control signal. Changing in the cavity length of laser produces error signal. Then, the control signal is sent to a Voltage Controlled Oscillator (VCO) and through a phase/frequency detector and loop filter is sent to a Pulse-Width Modulation (PWM) controller [13]. When the laser turns on, the switch is locked on preheating loop. After reaching the temperature of laser cavity to suitable value, the switch is locked on stabilization loop. In this paper, the loop filter is chosen as PI, self tuning fuzzy-PI, and parallel fuzzy P+ fuzzy I controllers individually in which can be adjusted the transient time and steady-state response. Finally, the performances of PID, fuzzy, fuzzy-PID, and fuzzy P+ fuzzy I+ fuzzy D controllers for stabilizing frequency of He-Ne laser and comparing their operation and effectiveness for laser process are discussed.

However the thermal behavior of laser cavity and its enclosure is so complex, it can be modeled as a low pass filter with a time constant of RC [14]. Transfer function of a gas laser, such as He-Ne laser, is estimated by a first order function as:

$$G(S) = \frac{A}{1 + tS} \qquad (5)$$

where, A and τ are the transfer gain of power driver and time constant of laser cavity, respectively. We assume that A, and τ are considered equal to 65 and 4×10^{-7}, respectively.

3.1. PI Controller Structure

Feedback circuits monitor the output of the system and then feed adjustments into the circuit that bring the output closer to the desired value. The process to be controlled produces an error signal which is the difference between the output value and the desired value. This error signal is then processed into an
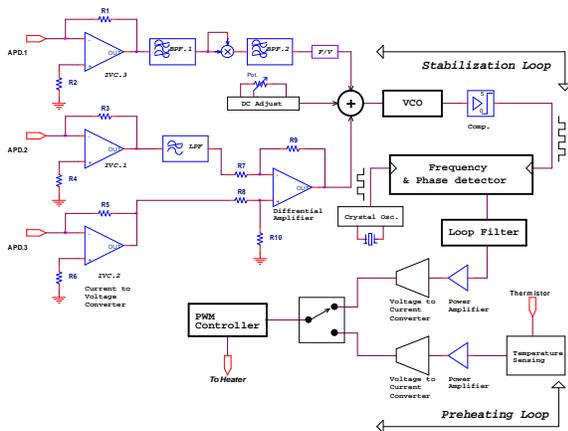
output signal that is sent back into the process to match its output to the desired value [15]. A widely implemented feedback system is proportional, integral control (PI Controller), which is used extensively in industry [16]. The two elements of PI control respond differently to the error signal [16]. The proportional component produces a response proportional to the magnitude of the error signal and is responsible for reducing much of the error signal. The final output of a proportional-only system results in a steady-state error, a constant difference between the desired value and the actual output, because the proportional component needs some error signal to function [15-16]. The integral component accumulates the error signal over time and removes the steady-state error [15-16]. The output of PI controller is proportional to coefficient of input error plus integral of input error. If e(t) is input error and C(t) is output value, PI controller can be modeled by:

$$C(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(t)dt \right] \qquad (6)$$

where, K_p and T_i are the proportional gain, and integral time constant, respectively. The block diagram of cavity laser and its stabilizer by using PI controller is shown in Fig. 3. The result of stabilization by PI controller to the step input is shown in Fig. 4. K1, and K2 are phase comparator gain and VCO gain, respectively. We assume that K1, and K2 are equal to 1.6 and 1M, respectively. The transfer function of PI controller is as follow:

$$\frac{C(s)}{E(s)} = K_p (1 + \frac{1}{T_i S}) \qquad (7)$$

The frequency instability can be estimated from the error signal fluctuations which it is confined to about 150kHz, theoretically. It means that the frequency instability is about 3×10^{-11}, approximately.

3.2. PI-like Fuzzy Controller

The PI-like fuzzy controller is the combination of PI and fuzzy controllers. The error and error changing rate are used as the input variables in control system. The block diagram of cavity laser and its stabilizer by using PI-like fuzzy controller is shown in Fig. 5. The input and output membership functions of fuzzy controller that defined in Simulink model as shown in Fig. 6 (N: negative; Z: zero and P: positive). Also, the result of stabilization by PI-like fuzzy controller to the step input is presented in Fig. 7.
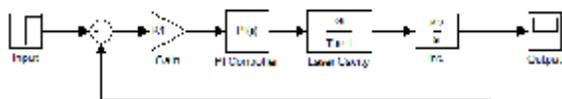

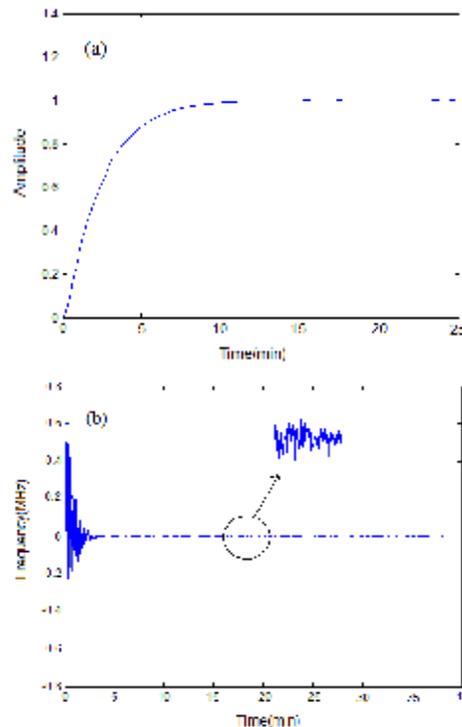**Figure 3.** The block diagram of laser cavity and its stabilizer by PI controller

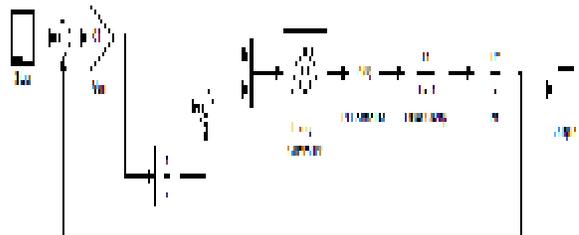**Figure 4.** The result of stabilization by PI controller, (a) response to the step and (b) laser frequency fluctuations

Figure 5. The block diagram of laser cavity and its stabilizer by PI-like fuzzy controller
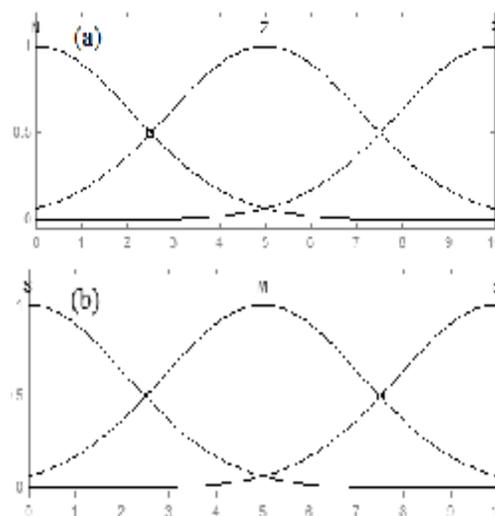

Figure 6. The membership functions, (a) input membership function and (b) output membership function

The frequency is confined to about 130kHz, theoretically. It means that the frequency instability is

about 2.3×10^{-11}, approximately.

3.3. Parallel Fuzzy P+ Fuzzy I Controller Structure

The fuzzy P+ fuzzy I control action is obtained by summing fuzzy P control action, and fuzzy I control action. Now, we review the fuzzy P control action and fuzzy I control action, respectively. The output of the conventional P controller is as follow:

$$u_P(t) = K_P e(t) \tag{8}$$

where, $u_P(t)$, K_P and $e(t)$ are output of conventional P controller, proportional constant and error signal, respectively. The discrete version of Eq. (8) is:

$$u_P(z) = K_P e(z) \tag{9}$$

By taking inverse z-transform of Eq. (9), we have:

$$u_P(nT) = K_P e(nT) \tag{10}$$

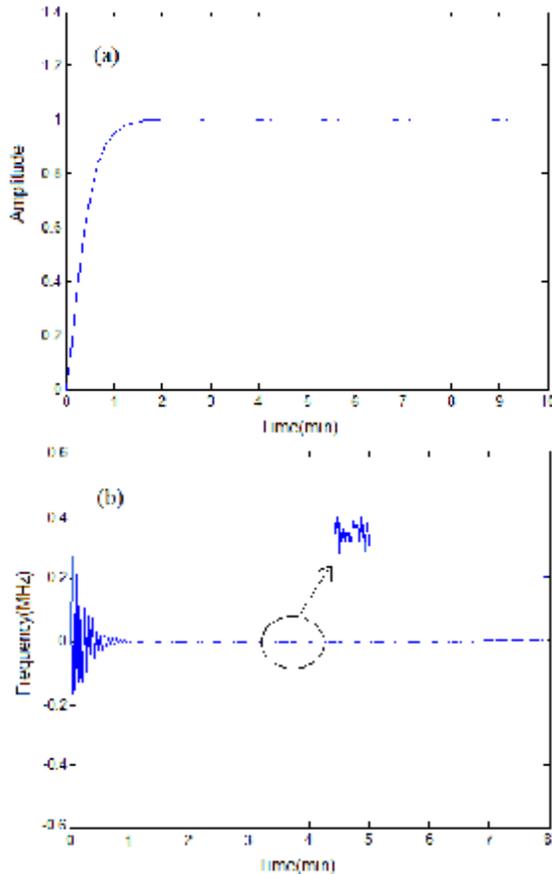The $K\Delta e(t)$ should be added to above equation. Then the Eq. (10) is converted to:

Figure 7. The result of stabilization by PI-like fuzzy controller, (a) response to the step and (b) laser frequency fluctuations

$$u_P(nT) = K_P e(nT) + K\Delta(nT) \tag{11}$$

where, $\Delta e(nT)=e(nT)-e(nT-t)$ and K is a constant. Therefore, we change K_P and K coefficient to K_{P1} and K_{P2}, respectively and rewrite above equation as:

$$u_P(nT) = K_P e(nT) + K_{P2}\Delta(nT) \tag{12}$$

In order to increase the degree of freedom, an additional gain G_P multiply in $u_P(nT)$. So, the fuzzy P

control action is as follow:

$$u'_P(nT) = G_P u_P(nT) \tag{13}$$

The output of the conventional I controller is as follow:

$$u_I(t) = K_I \int e(t)dt \tag{14}$$

where, $u_I(t)$, K_I and $e(t)$ are output of conventional I controller, integral constant and error signal, respectively. In frequency s-domain, the above equation is converted to:

$$u_I(s) = \frac{K_I}{s} e(s) \tag{15}$$

The discrete version of equation 15 by applying transformation s=(2/T)(z-1)/(z+1) is as follow:

$$u_I(z) = (-\frac{K_I T}{2} + \frac{K_I T}{1-z^{-1}})e(z) \tag{16}$$

where, T>0 is sampling time. By taking inverse z-transform of equation 16, we have:

$$u_I(nT) - u_I(nT-T) = K_I Te(nT)$$
$$-\frac{K_I T}{2}(e(nT) - e(nT-T)) \tag{17}$$

Therefore, we change $K_I T$ and $K_I T/2$ coefficient to K_1 and K_{I2}, respectively and rewrite above equation as:

$$u_I(nT) - u_I(nT-T) = K_I e(nT)$$
$$-K_{I2}(e(nT) - e(nT-T)) \tag{18}$$

By dividing the above equation to T, we have:

$$\frac{u_I(nT) - u_I(nT-T)}{T} = \frac{K_1}{T} e(nT)$$
$$-K_{I2}(\frac{e(nT)-e(nT-T)}{T}) \tag{19}$$

we rewrite the Eq. (19) as:

$$\Delta u_I(nT) = K_I e(nT) - K_{I2} r(nT) \tag{20}$$

where, $\Delta u_I(nT)$, e(nT) and r(nT) are incremental control output of conventional integral controller, error signal and rate of change of the error signal. K_{I1} is equal to K_1/T. According to Eqs. (19) and (20), $\Delta u_I(nT)$ is equal to $(u_I(nT)-u_I(nT-T))/T$. Then we have:

$$u_I(nT) = u_I(nT-T) + T\Delta u_I(nT) \tag{21}$$

The term $T\Delta u_I(nT)$ is replaced by the fuzzy control action as follow:

$$u_I(nT) = u_I(nT-T) + G_I\Delta u_I(nT) \tag{22}$$

where, G_I is a fuzzy I controller gain. Finally, the fuzzy P+ fuzzy I control action is:

$$u_{PI}(nT) = u'_P(nT) + u_I(nT)$$
$$u_{PI}(nT) = G_P u_P(nT)$$
$$+u_I(nT-T) + G_I\Delta u_I(nT) \tag{23}$$

In this controller, $K_{P1}e(nT)$ and $K_{P2}\Delta e(nT)$ are the fuzzy p controller inputs, $K_{I1}e(nT)$ and $K_{I2}r(nT)$ are the fuzzy I controller inputs. $u_P(nT)$, and $\Delta u_I(nT)$ are outputs. The block diagram of cavity laser and its stabilizer by using parallel fuzzy P+ fuzzy I controller

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is shown in Fig. 8. The input and output membership functions of parallel fuzzy P+ fuzzy I controller that defined in Simulink model display in Fig. 9(n: negative; p: positive; on: output negative; oz: output zero; op: output positive). The fuzzy control rules for fuzzy P controller and fuzzy I controller are selected as shown in Table 1 and 2, respectively. Also, the result of stabilization by parallel fuzzy P+ fuzzy I controller to the step input is presented in Fig. 10.

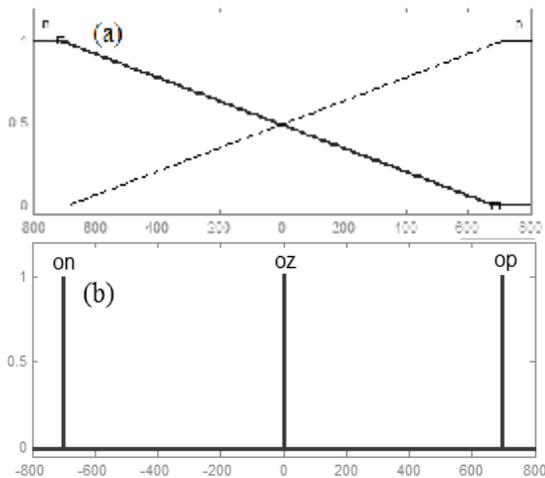**Figure 8.** The block diagram of laser cavity and its stabilizer by parallel fuzzy P+ fuzzy I controller

Figure 9. Membership functions, (a) input membership functions and (b) output membership functions

results show that the system modeled by parallel fuzzy P+ fuzzy I controller has better performance and IAE. The low IAE is the main factor in sensitive applications. The bode diagram of open-loop transfer function of designed system is shown in Fig. 12. As seen clearly in Fig. 12, the phase margin and gain margin are 55.2° and +∞dB, respectively.
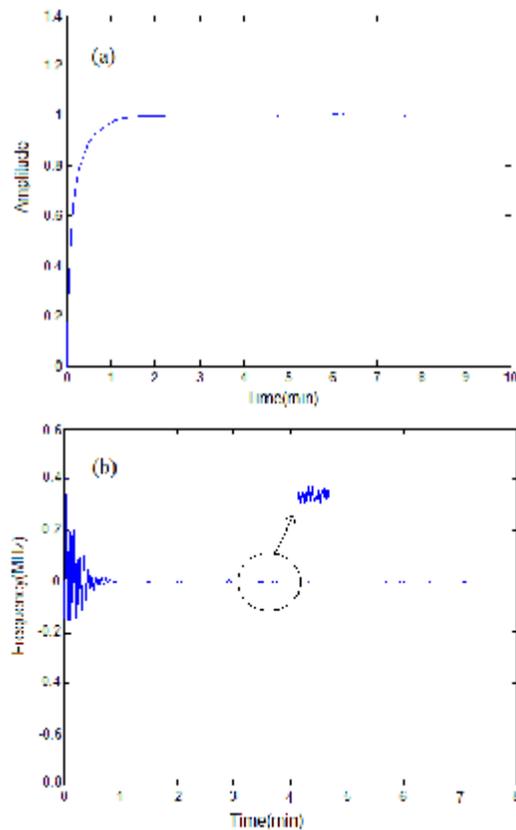

Figure 10. The result of stabilization by parallel fuzzy P+ fuzzy I controller, (a) response to the step and (b) laser frequency fluctuations
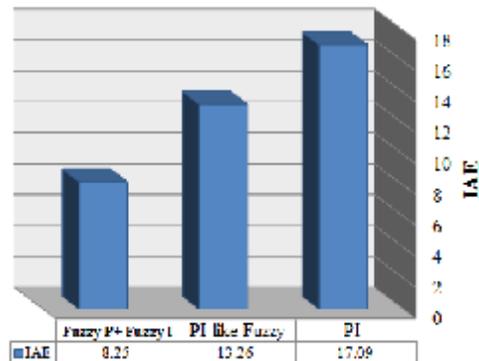
TABLE 1

Fuzzy control rules for the fuzzy P controller

$K_{P1}e(nT)$	$K_{P2}\Delta e(nT)$	$u_P(nT)$
n	n	z
n	p	p
p	n	n
p	p	z

TABLE 2

Fuzzy control rules for the fuzzy I controller

$K_{I1}e(nT)$	$K_{I2}r(nT)$	$\Delta u_I(nT)$
n	n	n
n	p	z
p	n	z
p	p	p

	Fuzzy P+ Fuzzy I	PI like Fuzzy	PI
■ IAE	8.25	13.26	17.09

Figure 11. Simulation result of IAE

The frequency is confined to about 100kHz, theoretically. It means that the frequency instability is about 2×10^{-11}, approximately. Also, the value of IAE by different controllers is shown in Fig. 11. The

We have designed the GUI of parallel fuzzy P+ fuzzy I controller for better interaction with the user and the preparation of technical reports. The designed GUI is shown in Fig. 13. As shown in Fig. 13, by changing the parallel fuzzy P+ fuzzy I controller

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parameters and simulation parameters and pushing simulate bottom, the users can analysis the step response of controlled system and tune it for achieving best IAE.

4. Conclusion

In this paper, the parallel fuzzy P+ fuzzy I controller for stabilizing He-Ne lasers has been proposed. Moreover, we compared this controller to other controllers. The results of simulation in SIMULINK showed that performance of system was better and IAE was less than other states. Also, the frequency fluctuations by parallel fuzzy P+ fuzzy I controller has been obtained as 2×10^{-11}.
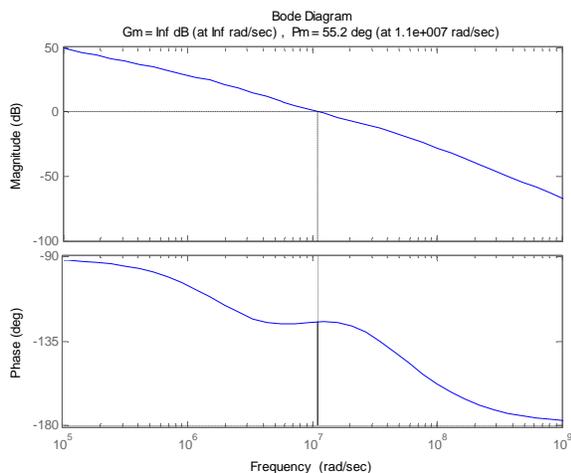

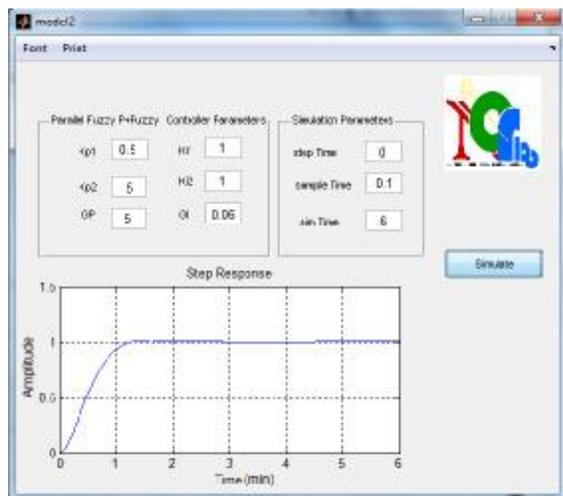**Figure 12.** The bode diagram of open-loop transfer function of designed system

Figure 13. The designed GUI of parallel fuzzy P+ fuzzy I controller

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