Photovoltaic Cells Modeling via Artificial Neural Square Fuzzy Inference System

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Abstract

The present article investigates the application of high order TSK (Takagi Sugeno Kang) fuzzy systems in modeling photo voltaic (PV) cell characteristics. A method has been introduced for training second order TSK fuzzy systems using ANFIS (Artificial Neural Fuzzy Inference System) training method. It is clear that higher order TSK fuzzy systems are more precise approximators while they cover nonlinearities better than zero and first order systems with the same number of rules and input membership functions (MF). However existence of nonlinear terms of the rules' consequent prohibits use of current available ANFIS algorithm codes as is. This article aims to give a simple method for employing ANFIS over a class of simplified second order TSK systems and applies the proposed method on the nonlinear problem of modeling PV cells. Error comparison shows that the proposed method trains the second order TSK system more effectively.

Key Words: High Order TSK, ANSFIS, PV Cell, Fuzzy Systems.

1. Introduction

Photovoltaic Cells (abbreviated as PV Cells) are the leading technology in clean electricity generation. Controlling PV cells to perform in their most optimum situation is an important subject [1]. Having a precise model of the PV cells helps controlling them with more advanced methods [2]. TSK fuzzy systems are of the most popular fuzzy systems in modeling nonlinear functions [3]. The consequent of each fuzzy rule in these systems is a constant value or a linear combination of input variables that are called order and first order TSK fuzzy systems respectively. TSK fuzzy systems aim to simplify and speed-up defuzzification calculations, which is a bottle-neck in real time control systems [4] [5]. However this leads to loss of some key properties of native fuzzy systems (namely Mamdani Systems) [4] [6].

Also using traditional TSK systems in modeling complex problems including multivariable nonlinear functions leads to fuzzy systems with many rules and membership functions (MFs) which is undesirable [7]. High order TSK fuzzy systems have been introduced to overcome this problem and to increase the transparency and interpretability of TSK systems while preserving computational advantages [7] , [8], [9], [10], [11], [12]. Applications of high order TSK systems are wide spread for example in [13] and [14] an enhancement of neuro-fuzzy systems is developed using high-order TSK systems, an application of these systems for simple modeling and local models needed in transductive systems are addressed in [15] and [16] respectively, another application in decision making is given in [17] and [18], an advanced clustering method is used for fuzzy modeling by high order systems in [19], in the field of economy and price forecasting refer to [20], and also many applications of the proposed systems in the field of control engineering is studied [21], [22] and [23]. High order TSK systems will be much more useful if one finds a good training method for them similar to ANFIS for zero and first order TSK systems [24]. In this paper first the structure of a second order TSK system is explained and a method (named Artificial Neural Square Fuzzy Inference System - ANSFIS) is proposed to represent the second order TSK system as a first order system, so it will be possible to train it as it was possible to train zero and first order TSK systems by ANFIS.

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2. Second Order TSK System Modification

2.1. Original Second Order TSK System

Considering the general structure of a fuzzy system as Figure **1**, the rule base is the most different block in a second order TSK system.
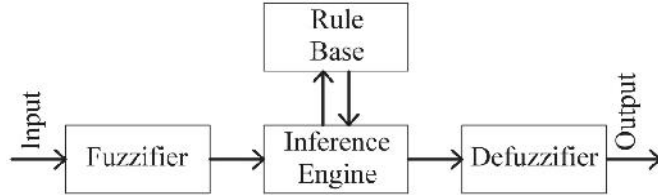

Figure 1. Structure of a fuzzy system

Supposing a two input TSK system, the *lth* rule will be:

$$IF\ x_1 \in A_1^l\ AND\ x_2 \in A_2^l\ THEN \quad (1)$$
$$y^l = a_{00}^l + a_{10}^l x_1 + a_{20}^l x_2 + a_{11}^l x_1^2 + a_{22}^l x_2^2 + a_{12}^l x_1 x_2$$

And the output will be the weighted average of rules' outputs:

$$y = \frac{\sum_{l=1}^r w^l y^l}{\sum_{l=1}^r y^l} \quad (2)$$

Where x_1 and x_2 are input variables, r is the total number of rules and w^l is firing strength of each rule determined by input membership function and the fuzzification block. A suitable training process must tune input membership functions and determine values of rules' coefficients (e.g. $a_{00}^l, a_{11}^l, \dots$) such that the overall fuzzy system performs desirably.

2.2. Square TSK System (S-TSK)

Considering a simplified second order TSK system by omitting second order terms which consist of more than one variable (like $x_1 x_2$ in (1)), the typical form of the consequent of a rule say in a two input system will be:

$$y^l = a_{00}^l + a_{10}^l x_1 + a_{20}^l x_2 + a_{11}^l x_1^2 + a_{22}^l x_2^2 \quad (3)$$

Now one might consider:

$$z_1 = x_1^2 \quad (4)$$
$$z_2 = x_2^2$$

And a new TSK system with four variables: x_1, x_2, z_1 and z_2, which its *lth* rule's consequent will be:

$$y^l = a_{00}^l + a_{10}^l x_1 + a_{20}^l x_2 + a_{11}^l z_1 + a_{22}^l z_2 \quad (5)$$

Which is a simple first order TSK rule and its coefficients can be found by ANFIS. This is a simple idea behind defining such simplified second order TSK system called in this paper Square TSK (or S-TSK) system. Generally a system with n input variables will have a typical consequent as follows:

$$y^l = a_{00}^l + a_{10}^l x_1 + \dots + a_{n0}^l x_n + a_{11}^l z_1 + \dots + a_{nn}^l z_n \quad (6)$$

Where $x_i{}^2 = z_i$ for $i = 1,2,\dots,n$. Let input sets be $A_{k_i,i}$ and $B_{k_{n+i},i}$ defined by membership functions $\mu_{k_i,i}(x_i)$ and $\mu_{k_{n+i},n-i}(z_i)$ respectively, where i is as defined and k_{n+i} and k_i equal $1,2,\dots,p$ that p is the number of input membership functions for each input variable. So a rule of such system looks as follows:

$$IF\ x_1 \in A_{k_1,1}\ AND\ x_2 \in A_{k_2,2}\ AND\ \dots\ AND\ x_n \in A_{k_n,n}\ AND \quad (7)$$
$$z_1 \in B_{k_{n+1},1}\ AND\ z_2 \in B_{k_{n+2},2}\ AND\ \dots\ AND\ z_n \in B_{k_{2n},n}$$
$$THEN\ y^l = a_{00}^l + a_{10}^l x_1 + \dots + a_{n0}^l x_n + a_{11}^l z_1 + \dots + a_{nn}^l z_n$$

Where $k_i = 1,2,\dots,p$ for each $i=1,2,\dots,n$. It must be noted that for a specific index i we have $x_i{}^2 = z_i$ and if $x_i \in A_{k_i,i}$ then the set $B_{k_{n+i},i}$ (that is supposed to contain z_i) cannot be chosen arbitrarily (for a specific k_i the index k_{n+i} cannot take any value from 1 to p). For a rule to b valid, $A_{k_i,i}$ and $B_{k_{n+i},i}$ must have large enough intersection otherwise the proposed rule will not be fired at all and can be eliminate from the rule base.

3. Artificial Neural Square Fuzzy Inference System - ANSFIS

As mentioned previously the main advantage of S-TSK is the possibility of being trained similar to first order TSK by ANFIS [24]. Suppose X is an m by n matrix containing the input values of a set of data including m points from an n-input system which is to be modeled, and Y is an m by one vector containing corresponding output values. Consider X and Y as following:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} \quad (8)$$

Now define X^* as follows:

$$X^* = \begin{bmatrix} x_{11} & \cdots & x_{1n} & x_{11}^2 & \cdots & x_{1n}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} & x_{m1}^2 & \cdots & x_{mn}^2 \end{bmatrix} =$$
$$\begin{bmatrix} x_{11} & \cdots & x_{1n} & z_{11} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} & z_{m1} & \cdots & z_{mn} \end{bmatrix}$$

Feeding X^* as input and Y as desired output to ANFIS for training results a first order TSK system with $2n$ inputs and one output. To minimize fuzzy inference engine calculations it is better to eliminate invalid rules form the rule base (as discussed in 2.2).

Now it is possible to use the trained system, however this system can be represented as an n input system by replacing z_i by x_i^2 and merging corresponding sets of x_i and x_i^2 by a t-norm. The following example illustrates this procedure.

Suppose estimating the single input function ($n=1$), $y=sin(x_1)$, $x_1 \grave{e} [-f,f]$ by 12 equally spaced points of real data. Following the proposed procedure using 2 Gaussian MFs for each input ($p=2$), after 100 epochs the ANFIS algorithm returns a 2 input first order TSK system (Figure **2**) with following rules and MFs (Figure **3**):
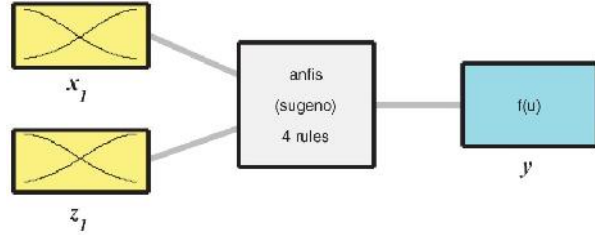

Figure 2. Trained TSK system

If x_1 $\grave{e}$ $A_{1,1}$ AND z_1 $\grave{e}$ $B_{1,1}$ THEN $y^1=0.8194x_1+0.3218z_1-0.4205$

If x_1 $\grave{e}$ $A_{1,1}$ AND z_1 $\grave{e}$ $B_{2,1}$ THEN $y^2=0.6466x_1+0.2601z_1-0.3037$

If x_1 $\grave{e}$ $A_{2,1}$ AND z_1 $\grave{e}$ $B_{1,1}$ THEN $y^3=0.8194x_1-0.3218z_1+0.4205$

If x_1 $\grave{e}$ $A_{2,1}$ AND z_1 $\grave{e}$ $B_{2,1}$ THEN $y^4=0.6465x_1-0.2601z_1+0.3037$

$$\mu_{1,1}(x_1) = exp\left(-\frac{(x_1+3.1710)^2}{2\times2.5981^2}\right), \mu_{2,1}(x_1) = exp\left(-\frac{(x_1-3.1710)^2}{2\times2.5981^2}\right)$$

$$\mu_{1,2}(z_1) = exp\left(-\frac{(z_1-0.1904)^2}{2\times4.3176^2}\right), \mu_{2,2}(z_1) = exp\left(-\frac{(z_1-9.8450)^2}{2\times4.3023^2}\right)$$
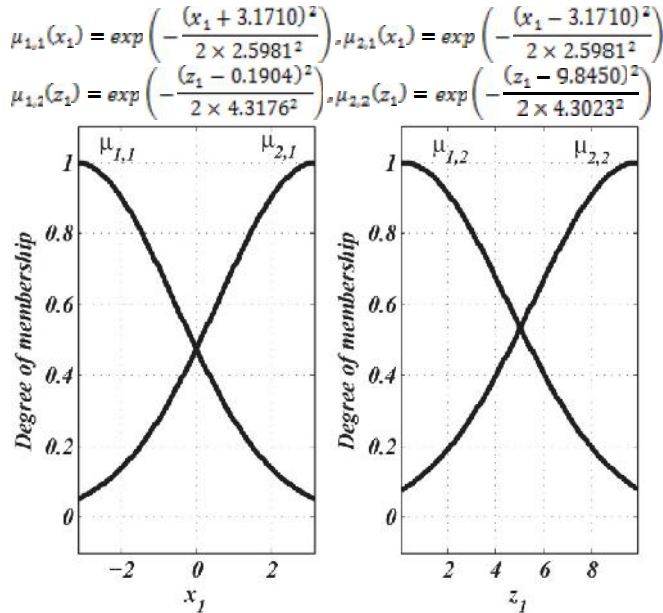
(10)

Figure 3. Input MFs

Replacing z_1 by x_1^2 in (10) and plotting all MFs on a single plane gives:
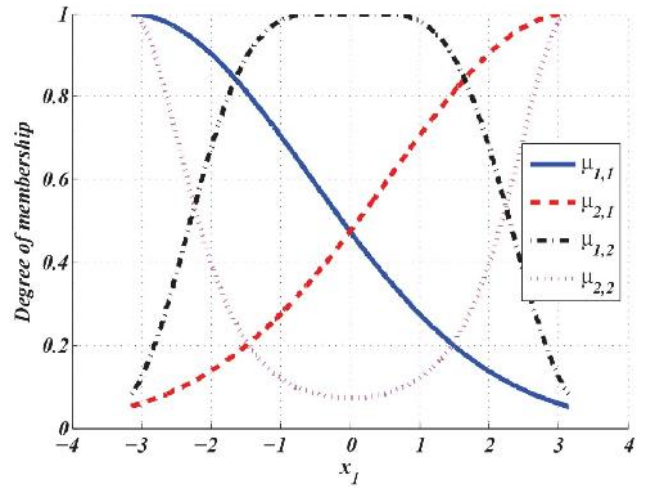

Figure 4. Input MFs after replacing z_1 by x_1^2

In Figure **4** $\sim_{1,2}$ and $\sim_{2,2}$ show the membership of x_1 to $B_{1,1}$ and $B_{2,1}$, so the rules can be rewritten:

If x_1 $\grave{e}$ $A_{1,1}$ AND x_1 $\grave{e}$ $B_{1,1}$ THEN $y^1=0.8194x_1+0.3218 x_1^2-0.4205$

If x_1 $\grave{e}$ $A_{1,1}$ AND x_1 $\grave{e}$ $B_{2,1}$ THEN $y^2=0.6466x_1+0.2601 x_1^2-0.3037$

If x_1 $\grave{e}$ $A_{2,1}$ AND x_1 $\grave{e}$ $B_{1,1}$ THEN $y^3=0.8194x_1-0.3218 x_1^2+0.4205$

If x_1 $\grave{e}$ $A_{2,1}$ AND x_1 $\grave{e}$ $B_{2,1}$ THEN $y^4=0.6465x_1-0.2601 x_1^2+0.3037$

Using production as the *AND* method, the above rules' antecedents will reduce to:

If x_1 $\grave{e}$ C_1 THEN $y_1=0.8194x_1+0.3218 x_1^2-0.4205$

If x_1 $\grave{e}$ C_2 THEN $y_2=0.6466x_1+0.2601 x_1^2-0.3037$

If x_1 $\grave{e}$ C_3 THEN $y_3=0.8194x_1-0.3218 x_1^2+0.4205$

If x_1 $\grave{e}$ C_4 THEN $y_4=0.6465x_1-0.2601 x_1^2+0.3037$ (11)

Where the membership functions defining C_1 to C_4 are as follows (illustrated in Figure **5**):

$$\mu_{C_1}(x_1) = \mu_{1,1}(x_1)\mu_{1,2}(x_1), \quad \mu_{C_2}(x_1) = \mu_{1,1}(x_1)\mu_{2,2}(x_1)$$
$$\mu_{C_3}(x_1) = \mu_{2,1}(x_1)\mu_{1,2}(x_1), \quad \mu_{C_4}(x_1) = \mu_{2,1}(x_1)\mu_{2,2}(x_1)$$
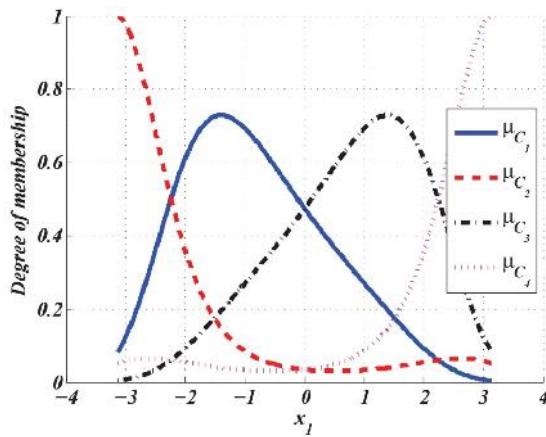(12)

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Figure 5. MFs of the S-TSK system

It is clear that rules mentioned in (11) and input sets C_1 to C_4 defined by $\sim_{C_1}$ to $\sim_{C_4}$ (Figure **5**) totally define a second order TSK system (or actually an S-TSK system) which has been originally created by ANFIS. The mentioned example was too simple, in a general case some MFs acquired by the *AND* operation between MFs defined as $A_{k_i,i}$ and $B_{k_{n+i},i}$ (namely C_i MFs in the previous example), have an ignorable value and can be eliminated, so the rule including such MFs will be eliminated from the rule base too (as discussed in 2.2).

4. PV Cell Behavior

A photovoltaic (PV) cell can be modeled like Figure **6** and the voltage-current characteristics can be obtained by following equations [25] [26]. This model includes 1) a current source (I_{ph}) which depends on solar radiation and cell temperature, 2) a diode in which the inverse saturation current I_0 depends mainly on the operating temperature and 3) a series resistance R_S represents the resistive losses. V_T is the thermal voltage.

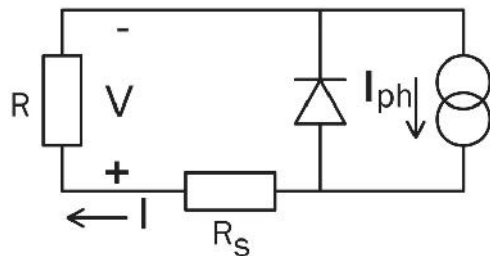$$V = V_T \ln[(I_{ph} - I + I_0)/I_0] - IR_s$$

Figure 6 . PV Model

In this section a set of data of the behavior of a 78W, 14.4V_{OC} (open circuit) and 7.24A_{SC} (short circuit) PV panel is considered as training data the S-TSK system. The set of data consists of normal irradiance (in KW per m^2), panel temperature (in C°), and panel output DC current (in Amperes) and voltage (in Volts). Irradiance, temperature and current are chosen as the input variables and voltage as the output variable. There exists total number of 855 points of data which 343 of them are used for training and the other 512 are used for validation. Training and validation points are distributed uniformly as: 0.1 < Irradiance < 1.2, 10 < Temperature < 70, 0 < Current < 7.24. Trained TSK system has 6 inputs (3 for first degree and 3 for second degree each has two Gaussian MFs equally 24 number nonlinear parameters. Not eliminating any rule, the rule base has a total number of 2^6=64 rules, each includes 7 coefficients so there exists 448 linear parameters to be determined and ANFIS runs for 100 epochs. Figure **7** shows the ANFIS progress applied to a first order TSK (FO-TSK) system and to a S-TSK system. Error is measured as Root Mean Square Error (RMSE) which is defined as (13).

Figure 7. ANFIS Progress
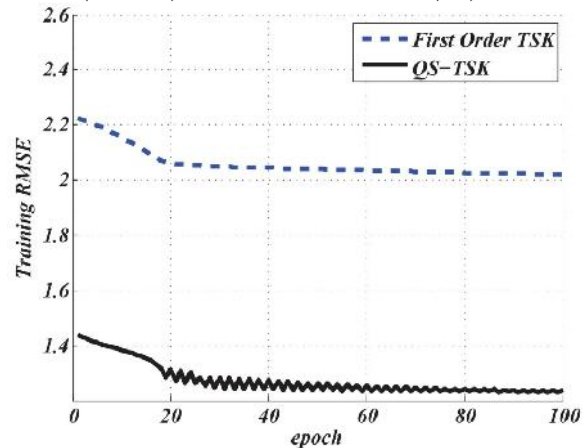
$$RMSE = \sqrt{\frac{\sum_{i=1}^{q}((y_i - \bar{y}_i)^2)}{q}} \tag{13}$$

Where y_i stands for the real and $\bar{y}_i$ for the estimated value. Table 1 summarizes the RMSE for training and validation of each method.

Table 1. RMSE Comparison

Method	ANFIS on FO-TSK	ANFIS on S-TSK
Training RMSE	2.0198	1.2319
Validation RMSE	2.1207	1.7352

The estimation of the voltage-current characteristics of the PV panel has been illustrated in Figure **8** for six situations.
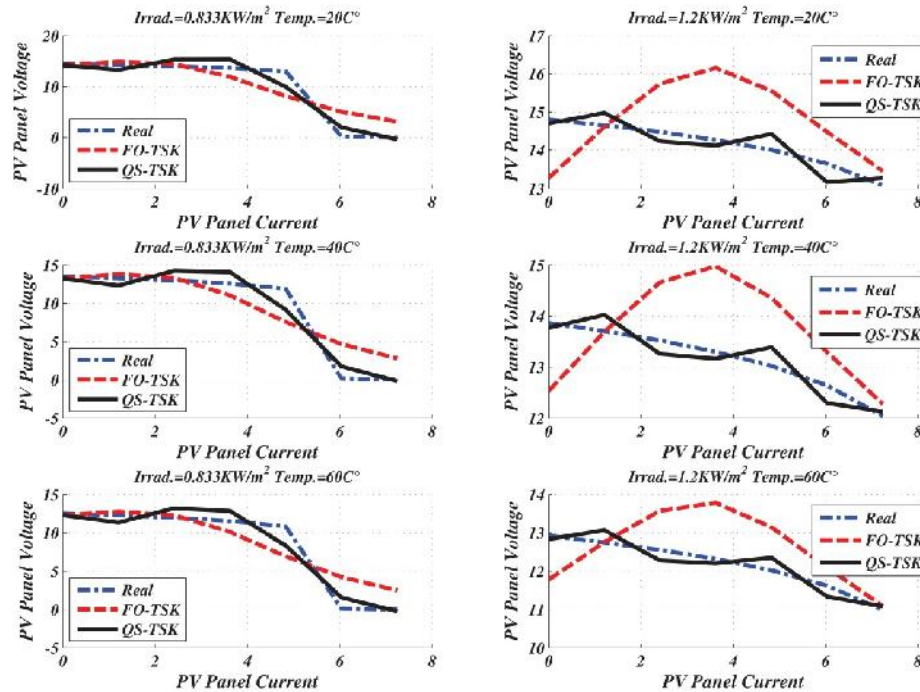

Figure 8: PV Voltage-Current Characteristics Estimation (Current is measured in Amperes and Voltage in Volts)

Table 2 compares the estimation RMSE between FO-TSK and S-TSK in these six situations.

Table 2 . RMSE Comparison for six situations of stimating voltage-current characteristics

Temperature→ Irradiance ↓	20°C	40°C	60°C	TSK Type
0.833 KW/m²	**2.9203**	**2.6608**	**2.4091**	FO-TSK
	1.6044	**1.4508**	**1.3274**	S-TSK
1.2 KW/m²	**1.2352**	**1.0792**	**0.9249**	FO-TSK
	0.3040	**0.2568**	**0.2382**	S-TSK

Considering the notation introduced in 2.2 for naming input MFs and considering x_1 as temperature, x_2 as Irradiance and x_3 as output current and remembering that all input MFs are Gaussian, the mean (C) and the variance (σ) of each input MF of both trained FO-TSK and S-TSK systems and their consequent coefficients are summarized in   **Table 3** to 8

Table 3. Input MF Parameters of FO-TSK

Input Variable Name→ MF Number↓	x_1	x_2	x_3	Parameter name
~_{1,i}	**10**	**0.0904**	**0.0090**	Mean
	25.4798	**0.2020**	**2.9522**	σ
~_{2,i}	**70**	**1.2392**	**7.4293**	Mean
	25.4798	**0.3581**	**2.7569**	σ

Table 4. Consequent Coefficients of FO-TSK

Output Parameter Name→ Rule Number↓	a_{10}	a_{20}	a_{30}	a_{00}
1	**-0.03324**	**21.12**	**-5.303**	**9.016**
2	**0.01552**	**-0.3425**	**-0.8524**	**8.444**
3	**-0.05372**	**-3.442**	**1.114**	**18.25**
4	**-0.02869**	**30.71**	**-0.5468**	**-19.03**
5	**-0.03368**	**16.65**	**-4.104**	**8.923**
6	**0.01577**	**-0.1639**	**-0.6662**	**5.607**
7	**-0.05442**	**-1.908**	**0.7183**	**17.37**
8	**-0.02939**	**23.76**	**-0.6017**	**-11.84**

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Table 5 . Input MF Parameters of S-TSK For Inputs of The First Degree

Input Variable Name→ MF Number↓	x_1	x_2	x_3	Parameter name
$\sim_{1,i}$	**10**	**0.0785**	**0.0050**	Mean
	25.4797	**0.3684**	**3.0798**	σ
$\sim_{2,i}$	**70**	**1.1196**	**7.2467**	Mean
	25.4796	**0.6285**	**3.0652**	σ

Table 6 . Input MF Parameters of S-TSK For Inputs of The Second Degree

Input Variable Name→ MF Number↓	z_1	z_2	z_3	Parameter name
$\sim_{1,i}$	**100**	**-0.0604**	**-0.0016**	Mean
	2038.4	**0.7120**	**22.2570**	σ
$\sim_{2,i}$	**4900**	**1.3110**	**52.4182**	Mean
	2038.4	**0.5392**	**22.2563**	σ

Table 7 . Consequent Coefficients of S-TSK (1 to 32)

Rule Number	Output Parameter Name						
	a_{10}	a_{20}	a_{30}	a_{11}	a_{22}	a_{33}	a_{00}
1	-0.262	-2.439	45.817	0.112	-11.101	-29.908	9.280
2	-2.669	-53.170	-0.104	-2.290	-447.336	-0.785	0.033
3	19.732	-7.376	-3091.648	-6.469	-16.530	1685.651	-0.145
4	52.915	96.925	-203.965	116.934	127.924	-466.655	6.016
5	0.923	-3.089	-10.719	0.414	3.584	-9.088	0.524
6	0.043	12.809	-0.496	-4.372	-87.188	-0.268	0.007
7	-0.792	-1.908	-469.196	-16.178	-3.104	430.451	-0.052
8	3.500	21.455	-37.394	273.634	29.583	-97.489	0.304
9	2.939	345.300	-21.864	0.296	114.997	3.023	-4.759
10	-3.027	136.590	-5.722	0.453	-761.014	-3.046	-1.054
11	-130.039	7.804	262.565	-14.872	13.564	108.005	-5.513
12	159.118	-40.791	-229.417	-23.152	-144.313	101.924	-67.650
13	-0.437	79.412	0.323	0.438	24.356	1.102	-0.271
14	0.658	-124.484	-0.317	1.031	-89.807	-4.897	-0.033
15	-5.947	4.590	15.164	-30.767	7.591	-250.870	-0.316
16	-44.346	-14.787	15.911	-60.181	-29.945	423.941	-5.144
17	-2.184	-31.752	383.234	0.727	-88.752	-212.267	32.753
18	-4.799	558.125	30.443	-14.119	875.734	99.397	1.859
19	0.294	34.557	-16.206	-0.056	-12.067	5.148	24.435
20	-0.001	-222.111	0.998	0.328	-230.889	32.856	1.697
21	3.283	-10.054	68.889	2.564	-19.047	-52.698	1.853
22	-0.280	123.358	5.478	-34.461	200.339	15.362	0.104
23	2.423	8.164	-2.653	0.292	8.901	1.842	1.379
24	0.397	-88.284	-0.055	1.773	-145.991	1.718	0.102
25	15.156	-10.148	-40.035	1.801	28.891	-24.403	1.014
26	-19.026	-239.916	51.193	2.798	-688.352	-8.345	14.333
27	-0.611	10.000	-25.062	-0.045	110.469	0.807	-6.022
28	0.599	149.681	11.842	-0.066	-227.664	1.878	3.402
29	0.447	12.523	-3.333	3.750	33.085	31.551	0.049
30	6.730	-62.439	-0.943	7.689	-148.043	-54.691	0.992
31	-1.236	6.060	-1.328	-0.345	48.199	0.205	-0.359
32	0.722	19.664	0.962	-0.168	-68.318	3.855	0.203

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Table 8. Consequent Coefficients of S-TSK (33 to 64)

Rule Number	Output Parameter Name						
	a_{10}	a_{20}	a_{30}	a_{11}	a_{22}	a_{33}	a_{00}
33	1.329	-7.754	10.559	0.058	12.336	-18.437	0.295
34	1.576	53.499	1.274	-2.474	-185.329	4.893	0.047
35	-10.194	-4.623	-940.451	-5.442	-6.728	1037.151	-0.296
36	-21.002	46.774	-85.293	118.035	64.520	-251.125	-0.435
37	-2.355	-24.146	8.893	0.081	7.379	-18.128	-0.037
38	0.231	83.296	0.997	-1.014	-199.778	2.945	0.003
39	-3.612	-5.008	-903.569	-2.827	-7.313	907.892	-0.074
40	6.893	54.791	-71.909	54.254	75.790	-174.604	0.245
41	-1.254	180.377	0.091	0.454	49.971	16.564	-0.166
42	6.843	-222.820	-1.053	0.227	-199.810	-20.918	0.178
43	-4.587	11.914	6.664	-20.534	19.159	-752.358	-0.421
44	-356.120	-30.400	36.573	-10.420	-64.437	999.724	-12.617
45	1.157	206.850	0.784	0.137	91.867	14.099	0.019
46	2.731	-284.354	-1.114	0.147	-95.934	-16.117	0.053
47	-7.395	8.867	2.363	-7.823	17.278	-786.000	-0.181
48	-104.876	-27.018	66.935	-8.202	-47.335	827.163	-2.143
49	5.140	-24.675	121.010	0.445	-42.421	-129.222	1.053
50	1.671	266.317	10.853	-14.038	435.258	32.401	0.098
51	3.253	10.837	-6.359	-0.159	6.179	6.493	0.770
52	1.907	-205.067	-0.151	0.174	-349.443	2.169	0.105
53	-8.144	-24.836	116.050	0.449	-44.040	-110.077	-0.127
54	-1.959	305.268	9.018	-6.635	502.363	21.503	-0.050
55	-6.286	11.946	-4.633	0.064	7.439	3.513	-0.099
56	0.277	-202.544	-0.267	0.226	-306.428	-0.305	0.008
57	-1.747	36.018	-2.261	2.503	87.109	86.748	-0.011
58	50.005	-135.823	-2.815	1.120	-322.627	-118.600	1.908
59	-5.847	19.847	-0.813	0.041	130.138	1.345	-0.345
60	3.471	71.287	0.824	-0.083	-70.766	1.779	0.200
61	-0.075	15.933	-0.968	0.951	65.416	95.284	0.003
62	13.356	-108.808	-8.091	1.016	-241.179	-99.744	0.279
63	-0.329	12.619	0.217	-0.040	112.155	-1.579	-0.013
64	0.238	52.948	0.263	-0.027	-44.571	2.494	0.008

5. Conclusion and Recommendation

In this paper, a method is introduced for training class of second order TSK systems (call Square TSK systems in this paper) by the famous ANFIS algorithm. The proposed method (called Neural Square Fuzzy Inference System or ANSFIS) is applied to a set of data acquired from the PV panel. The performance of the trained system is measured by the RMSE and compared with a traditional first order TSK system trained under the same circumstances by ANFIS. Results show that the method ANSFIS has a good approach and can train the second order system so as it gives a noticeably much better performance than the first order one. However training a real second or higher order TSK system might give even better results so it worth studying methods for training high order TSK systems easily.

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