Design of a New Model Predictive Controller for Networked Control Systems Subjected to Variable Network Time Delays

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Abstract—In this paper, a novel methodology is proposed to improve performance of the Networked Control System (NCS) in the face of random time-delays, using Model Predictive Controller (MPC) approach. For this purpose, a new state-feedback MPC structure is developed to cope with random network time-delays when the system is subjected to uncertainties with state and control constraints. The main idea is to reduce the disturbing effect of random network time-delays on regulatory performance of the NCS using a new robust formulation in MPC design. A terminal penalty constraint has been added to the finite horizon objective function to guarantee the stability of the system stability. Finally, applicability of the presented method is evaluated in a real pilot plant within a NCS configuration, being realized by an industrial Ethernet and Foundation Fieldbus technology. It is demonstrated that the proposed online methodology is effective to provide a better performance, having faster response, smaller overshoot and stronger robustness compared to the conventional MPC method with less aggressive control actions.

Index Terms—Network control system, Network-induced delay, model predictive controller, Random time delay

I. INTRODUCTION

A networked control system is utilized for the communication among spatially distributed system components, such as sensors, actuators, and controllers. Low cost, reduced weight and less power requirement, flexibility and ease of maintenance are some of the advantages of NCS that make it useful for most industrial applications. However, limited network communication bandwidth poses significant constraints on operation of NCS, producing network time delays and data dropouts, leading to instability of the closed-loop control system [1]. A significant research effort has been devoted to this topic in the last decade. A large number of the obtained research results aim at the time-delay compensation issue for networked control system. In [2], a robust control design method was proposed to stabilize networked control systems with short time-varying delays. A stochastic optimal controller has been designed in [3] for an NCS with network-induced time delay longer than one sampling period. A linear switched system with time-varying delay has been investigated in [4]. An output-feedback controller has been presented for the suspension systems with input delay in [5].

The progressive idea of model predictive control (MPC), also known as receding horizon control, has received much attention since 1960s mainly due to its ability to handle both constraints and time-varying behaviors. In [6,7], a complete review of MPC has been presented to discuss the recent results. The main MPC method is based on solving an open-loop optimization problem in a finite horizon under some imposed constraints at each sampling instant, but implementing only the first calculated control action.

It is a well-known fact that parameter uncertainties and time-delays cannot be avoided in many industrial processes, leading to probable performance degradation and instability [8]. Therefore, considerable research effort has been devoted to study the robustness problem of constrained uncertain systems with state delays [9, 10]. For this purpose, numerous MPC techniques have been developed to address uncertain and time-delay systems. To mention a few, an MPC methodology was proposed in [11] for uncertain systems with a priori known time delay. Authors in [9] introduced an improved time-delay dependent robust MPC, assuming a known delay. The work in [8] puts forward an MPC method for time-varying state-delay systems with uncertainty and constrained control input. However, since the stability is guaranteed under the fixed constant weighting matrix at all time, the proposed method was very limited and the conservatism may be generated [12]. An auto regression model has been presented in [13] to predict time-delay in order to be compensated by an improved GPC algorithm. In [14], the authors decrease the time-delay of forward channel by adding a compensation control structure using a predictive time-delay compensation algorithm. [15] has proposed a minimum prediction step and predictive control vector to compensate for time-delay. [16] uses a predictive control through round-trip time-delay for vehicular sensor networks. The least mean square algorithm has been used for system modeling to overcome the time delay and data loss. [17] utilizes a set of predictive control sequences to compensate for...
induced time-delay and data loss of control channel. However, detecting the induced time-delay and data loss, remains as a key problem. Aiming at the network with random time-varying delays, a network generalized predictive control algorithm has been proposed based on the state-space model to effectively accomplish the desired tracking performance [18].

Uncertainties due to random time delays in NCS have not been fully addressed in the previous works for real-time control applications. This paper presents a new formulation based on MPC approach to effectively reduce the effect of random time-delays in NCS. The main contributions of this research work can be summarized as follows:

(1) Presenting a new model for NCS being exposed to random variable time-delay.

(2) Formulation of a new state-feedback MPC for optimization problem with a finite time horizon to regulate the output in the presence of random time delay and uncertainties with constraint.

(3) Validating the effectiveness of the proposed algorithm via a practical pilot plant.

The paper is organized as follows. Section 2 formulates the problem to be solved. An MPC method is proposed in Section 3 for time-delay systems with uncertainties and constraints. Section 4 illustrates the effectiveness of the proposed method by applying to a practical benchmark plant. The paper is then concluded in Section 5.

II. PROBLEM FORMULATION

Consider the generic NCS model representation in Figure 1 in which the controller is located far away from the plant and the communication link among sensor, controller and actuator is maintained through a network. The communication channels between sensor-to-controller (S-C) and controller-to-actuator (C-A) are assumed to be non-ideal. Therefore, this assumption will lead to random induced network delays in both forward (i.e., $T_{sc}$) and feedback (i.e. $T_{ca}$) channels.

Generally, the induced network delays occur mainly due to network transmission distances linking the constituting elements and network traffic congestion. Obviously, longer transmission distances could lead to a longer transmission time-delays for the same network conditions in terms of network bandwidth, protocols and etc. On the other hand, the limited bandwidth of the network communication channels could produce significant network traffic congestion, leading to random network time-delay characteristics. Therefore, even for a short transmission distance, the induced time-delay may be very uncertain, especially when the network is shared with several control loops or existence of other data exchange tasks [19].

The following state-space plant model representation has been considered:

\[
\begin{align*}
\dot{x}(t) &= A_C x(t) + B_C u(t - \tau) + w'(t) \quad (1) \\
y(t) &= C_C x(t) \quad (2)
\end{align*}
\]

Where $X \in \mathbb{R}^m$, $Y \in \mathbb{R}^l$, and $U \in \mathbb{R}^m$ represent, respectively, the state variables, measured outputs, and manipulated variables. $A_C$, $B_C$ , and $C_C$ denote system matrices with appropriate dimensions. $W$ indicates disturbance which is assumed to be Gaussian variable vector. Furthermore, it is assumed that the system is subjected to state and control constraints as follows:

\[
\begin{align*}
u &\in U \\
x &\in X \\
\text{Where } U \subset \mathbb{R}^m \text{ and } X \subset \mathbb{R}^n \text{ are close and convex sets.}
\end{align*}
\]

Assuming that $T_{sc} \in R^+$ represents the update period of the controller, the discrete-time equivalent model of (1) and (2) can be written as:

\[
\begin{align*}
x(k + 1) &= A x(k) + B u(k) + w(k) \\
y(k) &= C x(k)
\end{align*}
\]

Where $A = e^{A_C T_{sc}}$, $B = \int_0^{T_{sc}} e^{A_C (T_{sc} - \tau)} B_C d \tau$ and $C = C_C$.

To develop the proposed MPC formulation, the following assumptions are made.

Assumption 2.1: The pair $(A, B)$ is completely controllable, and the pair $(A, C)$ is completely observable.

Assumption 2.2: The sum of the upper bounds of the network time-delays in the forward channel and the feedback channel is not greater than $N_t$, where $N_t$ is a positive integer.

Assumption 2.3: The uncertainty vector is bounded and lies in the following compact convex polyhedron for all admissible realization of $(x,u)$:

\[
w \in W
\]

Assumption 2.4: The data transmitted through a network are time-stamped. The sensor is time-driven while the controller and the actuator are event-driven.

Remark 1: The time stamp of the data transmitted through a network is very important for NCSs. Because, the proposed MPC is based on time.

![Fig. 1. Network control system representation.](image-url)
Lemma:

The random network time-delay is formulated as a bounded uncertainty term in the following model representation:

\[
x(k + 1) = Ax(k) + Bu(k) + W(\tau)\Delta u(k)
\]

(8)

\[
W(\tau)A^{-1}(e^{At}(T_t - \tau) - e^{At}T_t)B_c
\]

(9)

Noting that the lemma has already been proved in [20].

Remark: As the random network time-delay (\(W(\tau)\)) and actuator limitations in the regulatory form (\(\Delta u(k)\)) have limited bounds, so the \(W(\tau)\Delta u(k)\) term in the model will remain bounded.

III. DESIGN OF CONTROLLER

To design the proposed MPC control law to regulate the system output in the presence of any probable induced random network time-delay around the desired reference, the model representation in (8) is further modified using a new formulation by incorporating the following terms:

\[
e(t) = y(k) - r(k)
\]

(10)

\[
\Delta u(k) = u(k) - u(k - 1)
\]

(11)

Therefore, the model formulation could be rewritten as:

\[
\Delta y(k+1) = C* \Delta x(k) + C*B \Delta u(k)
\]

(12)

\[
e(k+1) = C* \Delta A(k) + C*B \Delta u(k) - (k+1)
\]

(13)

Accordingly, defining a new state vector as:

\[
\begin{bmatrix}
e(k) \\
\Delta u(k - 1)
\end{bmatrix}
\]

(14)

A new state-space model representation can be formulated using (14), leading to:

\[
\Delta z(k+1) = A_m \Delta z(k) + B_m \Delta u(k)
\]

(15)

Where

\[
A_m = \begin{bmatrix} CA & 0 \\ 0 & 0 \end{bmatrix}
\]

(16-a)

\[
B_m = \begin{bmatrix} CB \\ I \end{bmatrix}
\]

(16-b)

It is obvious that the new model representation has incorporated deviations in output error and control effort so as to be minimized through an optimization problem. As a result, the presence of the error term in the new model formulation will reduce the effect of the induced network time-delay.

The proposed MPC objective is formulated based on a quadratic cost function to be minimized using quadratic programming in finite horizon as follows:

\[
J(z_k, u_k) = V_f(z_N) + \sum_{t=0}^{N-1} J(z_t, u_t)
\]

(17-a)

\[
V_{MPC} = \min_{u(k)} J(z_k, u_k)
\]

\[
y_{\min} \leq y(k + i | k) \leq y_{\max} \quad i = 1, ..., N
\]

\[
u_{\min} \leq u(k + i | k) \leq u_{\max} \quad i = 1, ..., N
\]

\[
z(k) = z(k)
\]

\[
z(k + i + 1 | k) = Az(k + i | k) + Bu(k + i | k) > 0
\]

\[
u(k+i) = K^*z(k+i) \quad N_u < i < N_y
\]

Where

\[
I(z_i, u_i) \triangleq \|z_i\|^2 + \|u_i\|^2
\]

(17-b)

\[
V_f(z_N) \triangleq \|z_N\|^2
\]

(17-c)

\[
I(z_i, u_i) \text{ denotes quadratic stage cost function and } V_f(z_N) \text{ is terminal penalty function. Where } Q, R \text{ represent positive definite weighting matrices for states and control signal, respectively, and } P \text{ is terminal penalty weighted matrix.}
\]

Solving the optimization problem (17), leads to \(\Delta u\) as follows:

\[
\Delta u = [\Delta u(k | k), \Delta u(k + 1 | k), ..., \Delta u(k + N | k)]^T
\]

(18)

Then, \(\Delta u(k | k)\) will be transmitted from network to actuator.

To eliminate the effects of random time-delays, being longer than sample time, the generated \(\Delta u\) is organized in an information packet which can then be transferred to an actuator. Therefore, if a long time-delay occurs, the actuator can use \(u(k+1 | k) \ldots u(k+N | k)\) until arrived of a new information packet. Of course, it is noted that the control horizon should be set longer than the maximum probable time-delay. It is obviously expected that by applying the developed method, the network time-delay in the backward channel will be limited.

A. Stability analysis

To ensure stability of the introduced finite horizon objective function, a terminal constraint is added. As discussed in [21], the terminal constraint can be determined using the following:

\[
K_{L_Q} = -(R + B_m'PB_m)B_m'PA_m
\]

(19-a)

\[
P = (A_m + B_mK_{L_Q})'P(A_m + B_mK_{L_Q}) + K_{L_Q}'RK_{L_Q} + Q
\]

(19-b)

It is noted that \(P\) influences the system settling cost taken from present time to the infinity under the control assumption (19). Although, the system contains induced network time-delay, the discussed terminal penalty could be manipulated to guarantee the system stability.

IV. BENCHMARK PLANT CASE STUDY

Controlling the level of a coupled tank pilot is used as the benchmark process plant to evaluate performance of the developed MPC method in real-time implementation under a networked control configuration.

Fig. 2 illustrates the experimental pilot plant being utilized as
the test bed. As shown, it is a coupled tank plant in which the level of the first tank and the flow rate of water flowing into the cascaded two tanks can be managed. The pilot plant is appropriately equipped based on SMAR foundation Fieldbus structure with one Fieldbus H1 control valve. The communication media, whose role is to connect the control room and the pilot, is provided with Ethernet cable as OPC servers. The system platform has meticulously been elaborated in [22-23].

![Fig. 2. Experimental pilot plant representation](image)

As depicted in Fig. 2, the warm water at the condition of under saturation is manipulated by a pump which is then directly guided towards the vaporizing tank to enter the tank. The electric heater heats the water inside the tank and keeps it at the saturation condition and produces vapor. This saturated vapor is released in the open air and the saturated water is returned to the storage tank. The pressure of the saturated water is higher than the atmospheric pressure and therefore, it undergoes pressure reduction when enters the storage tank and hence get flashed, causing the production of some further amount of vapor gets, being released in the open air. The warm water available in the storage tank exchanges heat with the surrounding environment and gets colder.

The P&ID block diagram of the pilot plant has already been presented in [22]. A cascade control system, including an outer level loop and an inner flow loop is implemented to maintain the water level in tank as the main control objective. The process includes a vessel and a storage tank. The water is pumped from the storage tank (T-100) into the vessel (V-100) via a pump (P-100), as shown in the P&ID diagram, where it is heated and evaporated by an electric element (H-100). In order to control the level, a cascade-loop is used while the vapor pressure is controlled by a simple-loop. Both controllers are configured in the fieldbus platform. One important control issue concerns with the vaporizing level of the tank. The changes occurred in this parameter, not only changes the fluid level in the tank, but also affects the vapor pressure.

The main control objective of this pilot plant is to control the level of vaporizing tank which could be manipulated by a Fieldbus type control valve. This type of valve has an inner control loop which controls the exact position of the valve according to a given set point. In this case study, there exists some constraints which must be considered in the optimization. For instance, actuator must be fully open or closed. Therefore, the control signal is selected in range of [0,100] spam. However, the tank level is bounded which causes limitation in output signal.

MATLAB software package is connected to the pilot plant software platform via an OPC software for practical control implementation. For this purpose, the proposed MPC structure is first constructed in MATLAB software environment. The data are measured by the installed sensor using Fieldbus and then transferred to MATLAB software. Then, the control signal is generated in MATLAB and is transferred to the actuator via Fieldbus. In this case study, the feed forward-backward channel transformation network exists. Therefore, it inherits the possibility of having induced time-delays in both forward and backward paths.

To enable artificial simulation of random time-delays in the performed experimental tests, a “random time-delay block” has been introduced in MATLAB software environment. The random time delay varies between 0 to 3 seconds.

The corresponding pilot plant model has been derived in the previous work [24], obtaining through system identification approach. To investigate performance of the proposed MPC against conventional MPC, the controller parameters for both methods have been set equal which are presented in TABLE I.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>1</td>
</tr>
<tr>
<td>$N$</td>
<td>3</td>
</tr>
<tr>
<td>$R$</td>
<td>1</td>
</tr>
<tr>
<td>$P$</td>
<td>1</td>
</tr>
<tr>
<td>$Q$</td>
<td>1</td>
</tr>
</tbody>
</table>

The resulting experimental responses have been illustrated in Figs. 3 and 4.

![Fig. 3. Comparative output regulatory responses](image)
As shown in Figs. 3 and 4, the developed online MPC is able to present better regulatory output response with higher robustness than the conventional MPC method.

Moreover, Fig. 4 clearly demonstrates that the control moves \((Au)\) are limited and bounded for the regulatory objective in the face of random time-delay. Therefore, the introduced lemma is confirmed for practical implementation of the proposed MPC in the considered case study.

The results indicate that the conventional method leads to unstable system output due to the adverse time delay effect. While the proposed method, maintaining the closed-loop stability, is able to present a significantly improved performance due to the dedicated time delay consideration in the control design scheme.

V. CONCLUSION

In this paper, the control problem of NCS has been investigated for uncertain systems being exposed to random network time-delays and state and control constraints. To address the problem, a new state-feedback MPC formulation has been introduced by incorporating the uncertainties due to random network time-delays and effective deviated state variable manipulations. Furthermore, a terminal constraint has been added to the finite horizon objective function to ensure stability of the system stability. The generated control move is transferred to actuator in an information packet so as to eliminate the effects of random network time-delay, being longer than sample time. Furthermore, the actuator could use \(u(k+1|k) \ldots u(k+N|k)\) until arrival of a new information packet. Of course, it is noted that the control horizon should be set longer than the maximum probable time-delay. It is obviously expected that by applying the developed method, the network time-delay in the backward channel will be limited.

The proposed MPC control structure was compared with a conventional MPC approach. The simulation results show a better promising output regulatory performance with higher robustness and less aggressive control moves.

The induced network time delay puts NCS at the risk of the closed-loop instability and hence the conventional MPC is often forced to consider a possible compromised scheme to compensate for the destabilizing effect of the induced time-delay. However, the conventional MPC has been modified in the proposed methodology by changing states and incorporating the designed terminal penalty scheme, based on the induced networked time delay, to retrieve the closed-loop stability and presenting a better control performance.

The future research works could focus on long induced network time-delay compensation for nonlinear plant and design of a controller to reduce the effects due to both time-delay and packet loss in NCS based on the introduced idea of robust MPC.

REFERENCES


