Gain-Scheduled Controller Design for a LPV Model of a Turboshaft Driving Variable Pitch Propeller

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Abstract

This paper has proposed a gain-scheduled controller with stability proof and guaranteed cost for a turboshaft driving a variable pitch propeller. In order to overcome the complexity of the nonlinear model, a linear parameter varying (LPV) model is proposed for the first time which is in affine form. Proposed model is established based on a family of local linear models and is suitable for LPV gain scheduling methods. Thus a gain scheduled design procedure is proposed which considers parameter dependent Lyapunov matrices to ensure stability and a quadratic cost function for guaranteed performance of the closed loop system. Proposed procedure also has the advantage of considering an upper bound for change rate of the scheduling signal which decreases conservativeness. Controller design problem and calculating its gain matrices is formulated in a set of Linear Matrix Inequalities which easily can be solved using LMILAB toolbox. Simulation results showed the effectiveness and practicality of the proposed procedure.

Keyword: Gas Turbine Engine, Linear Parameter Varying System, Gain-scheduled Controller, Linear Matrix Inequality

1. Introduction

This paper considered LPV gain scheduling control of a class of gas turbine engine. Gas turbine engine is a family of jet engines with different name related to their task , such as turbojet, turbofan, turboprop and turboshaft. In turboshaft the generated thrust of jet engine is converted to mechanical energy in propeller. This system is employed to generate torque in helicopters, boats, hovercraft and tanks. Fundamentals of design and analysis of control systems for gas turbine is covered in [1]. In order to achieve maximum efficiency, system should be kept close to its limits [2]. A wide variety of control method have been used for control of this system such as multivariable robust control [3], sliding mode [4], adaptive method [5] and neural network [6]. The main obstacle to design a control system is complex dynamic of the nonlinear model. A nonlinear model of turboshaft driving Variable Pitch Propeller based on physical equations is presented in [7]. In order to achieve a simpler model, a family of linear models is presented using global linearization method in some operating points. In [8] classical gain scheduling method is investigated for this family of linear models. For stability proof a single Lyapunov matrix is computed via linear matrix inequalities for closed loop system in all operating points. They just analyzed the stability and did not give a solution for optimal controller.

Gain scheduling is one of the most popular method for designing controller for nonlinear systems and it is widely known that they have a better performance than the robust ones [9]. A comprehensive review on this method can be found in [10] and [11]. One popular method in LPV controller analysis and design is using parameter dependent Lyapunov function for stability proof (see for example [12, 13] and reference there in).This can improve the performance of the controller in comparison to single Lyapunov matrix. However, it should be considered that the dependency should not yield in a controller which uses derivative of scheduling parameters [13]. On the other hand considering the maximum change rate of scheduling signal can decrease conservativeness. A LPV gain scheduled controller design method which considers upper bound of derivative of scheduling parameters is presented in [14]. This method guarantees the stability and performance;

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however, it converts the design problem into nonconvex matrix inequalities.

In this paper a gain-scheduled controller is designed which ensures stability and guaranteed performance. Therefore a affine LPV model based on family of linear models is proposed for turboshaft plant. Scheduling signal is exogenous and assumed to be available. Proposed model interpolates a family of local LTI models .In every operating point an equivalent value is assigned to scheduling signal. Proposed LPV model changes between local LTI models as scheduling signal changes between its equivalent values. The model has the merit and advantages of using vast variety linear and LPV methods for stability and performance analysis and design. A gain scheduled controller design procedure similar to [14] is presented. Controller design problem and calculating its variable gain matrices is formulated in a set of LMIs. Proposed controller for turboshaft plant in comparison with previous works has the advantages as follow: a) considering a quadratic cost function for the performance of the system, b) using a parameter dependent Lyapunov function, c) considering maximum rate value of scheduling signal changes.

This paper is organized as follows: in section (2) turboshaft model is overviewed and the LPV model is presented. In section 3 the design procedure is presented. Simulation results and conclusion are presented in sections (4) and (5) respectively.

2. Plant model

Turboshaft system driving variable pitch propeller consists of jet engine and propeller subsystems (figure 1) . Input air is pressurized in compressor then combined with fuel and ignited in combustion chamber. Generated hot gases make thrust in low turbine. This thrust is carried to the propeller through gearbox. A high turbine also is used to provide needed torque for compressor. In order to make a quick change in output torque, the angle of propeller is varying. Propeller angle and fuel flow are considered as the input control variables. Low spool and high spool speed are the state variables. The nonlinear model of the plant is:

$$\begin{aligned} \dot{x}(t) &= f(x(t),u(t),v(t)) \\ y(t) &= g(x(t),u(t),v(t)) \end{aligned} \tag{1}$$

in this model, $x(t)=[x_1(t),x_2(t)]^T$ is state vector, $x_1(t)$ is high spool and $x_2(t)$ is low spool speed. Vector $u(t)=[u_1(t),u_2(t)]^T$ is the input where $u_1(t)$ and $u_2(t)$ are fuel flow and propeller angle respectively. Disturbance signal $v(t)$ neglected in this paper. $f(.)$ and $g(.)$ are nonlinear functions. The nonlinear model uses an initial guess for mass flow and turbin pressure ratio, also includes a Newton iteration to compute engin dynamics)For more details see [7]). This complexity in plant model restricts range of control methods that can be used to guaranee stability and performance of the system. To overcome the complexity of nonlinear model, in next section the parameter varying model is presented.

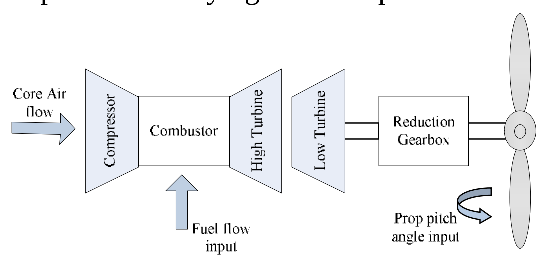

Figure 1. Schematic of turboshaft driving variable pitch propeller

2.1. Linear Parameter Varying Model

Linearizing the nonlinear model (1) in five operating point is given in [8]. The LPV model of the plant based on the family of local linear models is considered as follows:

$$\begin{aligned} \dot{x}(t) &= A(a(t))x(t)+B(a(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

Parameter a is time varying and denotes scheduling signal. In this paper state space matrices are in the affine form, i.e.

$$\begin{aligned} A(a) &= A_0+\sum_{i=1}^{p} q_i(a)A_i \\ B(a) &= B_0+\sum_{i=1}^{p} q_i(a)B_i \end{aligned} \tag{3}$$

A_i and B_i are the state space matrices of linear models, $q(.)=[q_1(.),q_2(.),...,q_p(.)]^T$ is vector of scheduling parameters satisfying

$$\begin{aligned}&\underline{q}_i \le q_i(a) \le \overline{q}_i\\ &\left|\dot{q}(a)\right| \le r_i\end{aligned} \tag{4}$$

Where $\underline{q}_i, \overline{q}_i$ and r_i are known constants. For simplicity in writing the parameter t is dropped.

Most important step in constituting a LPV model is choosing appropriate scheduling parameters. LPV model is considered to be a linear interpolation of the family of local linear models. When scheduling signal changes from operation point i to operation point j, q_i goes to zero and q_j goes to one (figure 2). Thus the scheduling parameters are defined as:

$$q_i(a) \triangleq \begin{cases} \dfrac{1}{a_i - a_{i-1}}a & a_{i-1} \le a < a_i \\ \dfrac{-1}{a_{i+1}-a_i}a & a_i \le a < a_{i+1} \\ 0 & o.w. \end{cases} \tag{5}$$

a_i is the value of scheduling signal in operating point i.

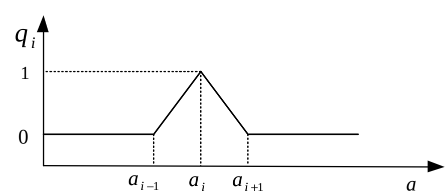

Figure 2. Scheduling parameters for turboshaft LPV model

3. Gain scheduling controller design

In this section gain scheduled controller design procedure is presented. The objective is that the plant output variables track the reference signal r. Thus a PI scheduled controller is considered :

$$u = F_p(a)e + F_I(a)\int e\ dt \tag{6}$$

where $e \triangleq y - r$ is output tracking error. Controller gain matrices have the following form:

$$\begin{aligned}F_P(a) &\triangleq F_{P0} + \sum_{i=1}^{p} q_i(a)F_{Pi}\\ F_I(a) &\triangleq F_{I0} + \sum_{i=1}^{p} q_i(a)F_{Ii}\end{aligned} \tag{7}$$

By including dynamic of the integral part into plant model, PI controller design problem becomes static controller design and the resulted augmented model is:

$$A_{aug} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, B_{aug} = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_{aug} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \tag{8}$$

then scheduled controller is:

$$u = F(a)e \tag{9}$$

where:

$$F(a) \triangleq F_0 + \sum_{i=1}^{p} q_i(a)F_i \tag{10}$$

and $F_i \triangleq [F_{Pi} \quad F_{Ii}], i = 0,...,p$. For stability proof a parameter dependent Lyapunov matrix is considered:

$$P(a) \triangleq P_0 + \sum_{i=1}^{p-1} q_i(a)P_i \tag{11}$$

In order to obtain a guranteed performance, a quadratic cost function is considered:

$$J = \int_0^\infty (x^TQx + u^TRu)dt \tag{12}$$

where Q and R are constant matrices with appropriate dimensions. Substituting equation (9) in (2) leads to closed loop system:

$$\dot{x} = (A(a) - B(a)CF(a))x \tag{13}$$

Definition 1. If there exist an input control signal u^* and scalar J^* such that the closed loop system is stable and $J < J^*$ then J^* is the guaranteed cost.

To design the controller, a theorem similar to [14] is used. Define

$$\underline{q}_m \le \quad q_m \triangleq \sum_{i=1}^{p} q_i \quad \le \overline{q}_m \tag{14}$$

Theoreme 1. Closed loop LPV system (13) is stable with guaranteed cost if there exist symmetric and positive definite matrices $P_0, P_1, ..., P_p$ such that the system of matrix inequality (15) is satisfied.

$$\begin{cases} 2M_0 + (M_i + M_j)\underline{q}_m + (M_{ij} + M_{ji})\underline{q}_m^2 < 0 \\ 2M_0 + (M_i + M_j)\overline{q}_m + (M_{ij} + M_{ji})\overline{q}_m^2 < 0 \\ M_{ij} + M_{ji} \ge 0; i = 1,2,..., p;\ j = i, i+1,..., p \end{cases} \tag{15}$$

where

$$M_0 = \begin{bmatrix} W_{110} & W_{120} \\ W_{120}^T & W_{220} \end{bmatrix}, M_i = \begin{bmatrix} W_{11i} & W_{12i} \\ W_{12i}^T & W_{22i} \end{bmatrix}, M_{ij} = \begin{bmatrix} W_{11ij} & W_{12ij} \\ W_{12ij}^T & W_{22ij} \end{bmatrix}$$

and

$$W_{110} = N_1 + N_1^T$$

$$W_{11i} = 0$$

$$W_{11ij} = 0$$

$$W_{120} = P_0 + N_2^T - N_1A_{c0}$$

$$W_{12i} = P_i - N_1 A_{ci}$$
$$W_{12ij} = -N_1 A_{cij}$$
$$W_{220} = \sum_{k=1}^{p} r_k P_k - N_2 A_{c0} - A_{c0}^T N_2^T + Q$$
$$W_{22i} = -N_2 A_{ci} - A_{ci}^T N_2^T + C^T F_0^T R F_i C + C^T F_i^T R F_0 C$$
$$W_{22ij} = -N_2 A_{cij} - A_{cij}^T N_2^T + C^T F_i^T R F_j C$$
$$A_{c0} = A_0 + B_0 F_0 C$$
$$A_{ci} = A_i + B_0 F_i C + B_i F_0 C$$
$$A_{cij} = B_i F_j C$$

For a proof see [14]. Theorem 1 formulates the design problem in a set of bilinear matrix inequalities. To convert (15) into a set of LMIs, a linearization method similar to [15] is used.

$$\begin{aligned}\lin(C^T F_i^T R F_j C) \cong\, & C^T Z_i^T R F_j C + C^T F_i^T R Z_j C \\ & - C^T Z_i^T R Z_j C\end{aligned} \tag{16}$$

where in each iteration $Z_i = F_i$. This linearization needs an initial guess for gain matrices, thus in the following a theorem is proposed to get an initial controller.

Lemma 1.[12] consider a quadratic function of vector q:

$$f(q_1,...,q_p) = a_0 + \sum_{i=1}^{p} a_i q_i + \sum_{i<j}^{p} b_{ij} q_i q_j + \sum_{i=1}^{p} c_i q_i^2 \tag{17}$$

assume that $f(.)$ be multiconvex that is $\partial^2 f(q)/\partial q_i^2 = 2c_i \geq 0, i = 1,...,p$, then $f(.)$ is negative for all values of q_i if it is negative for its corners i.e. $q_i \in \{\underline{q}_i, \overline{q}_i\}$.

Based on lemma 1 and Riccati inequality [16], following theorem is presented for frozen time values of scheduling parameters.

Theorem 2. Closed loop LPV system (13) is stable with optimum cost for frozen values of q_i, if there exist symmetric and positive definite matrices $P_0, P_1, ..., P_p$ such that the matrix inequality (18) and (19) for $i = 1,..., p$ are satisfied. Then the optimum controller is as (20).

$$\begin{bmatrix} A_i^T P_i + P_i A_i & -P_i B_i \\ -B_i^T P_i & 0 \end{bmatrix} \geq 0 \tag{18}$$

$$\begin{bmatrix} (A_0 + A_i)^T (P_0 + P_i) + (P_0 + P_i)(A_0 + A_i) + Q & * \\ -(B_0 + B_i)^T (P_0 + P_i) & -R \end{bmatrix} < 0 \tag{19}$$

$$F(q) = R^{-1} B_0^T P_0 + R^{-1} \sum_{i=1}^{p} (B_0^T P_i + B_i^T P_0 + B_i^T P_i) q_i \tag{20}$$

Proof. Write Riccati inequality for parameter dependent values of matrices:

$$\begin{aligned}
&\begin{bmatrix} A(q)^T P(q) + P(q)A(q) + Q & * \\ -B(q)^T P(q) & -R \end{bmatrix} = \\
&\quad \begin{bmatrix} A_0^T P_0 + P_0 A_0 + Q & * \\ -B_0^T P_0 & -R \end{bmatrix} \\
&\quad + \sum_i \begin{bmatrix} A_i^T P_0 + P_0 A_i + A_0^T P_i + P_i A_0 & * \\ -B_i^T P_0 - B_0^T P_i & 0 \end{bmatrix} q_i \\
&\quad + \sum_{i<j} \begin{bmatrix} A_i^T P_j + P_j A_i & * \\ -B_i^T P_j & 0 \end{bmatrix} q_i q_j \\
&\quad + \sum_i \begin{bmatrix} A_i^T P_i + P_i A_i & * \\ -B_i^T P_i & 0 \end{bmatrix} q_i^2 \\
&\quad < 0
\end{aligned} \tag{21}$$

By using lemma 1 this inequality holds if (18) hold and it is satisfied for corner values of $q \in \{\underline{q}_i, \overline{q}_i\}, i = 1,..., p$. From (5) one have:

$$\underline{q}_i = 0,\ \overline{q}_i = 1 \tag{22}$$

Substituting (22) in (21) leads to inequality (19). Then optimum controller is:

$$F(q) = R^{-1} B(q) P(q) \tag{23}$$

Substituting (11) and using (22) leads to (20) and proof ends.

Note that this theorem is correct for time frozen values of scheduling parameters and for time varying case there is no guarantee for stability. However, using this controller as an initial guess for theorem 1, guarantee can be obtained.

4. Simulation results

The problem of finding a controller resulted in a set of Linear matrix inequalities. This inequalities are solved using LMILAB toolbox. Constants are assigned as $r = 10$, $a_0 = 0.3361$, $a_1 = 0.6473$, $a_2 = 0.8818$, $a_4 = 1.3810$, $Q = 10^{-4} I_2$ and $R = 10^{-6} I_2$, where I_n is $n \times n$ unity matrix. Plant matrices are:

$$A_0 = \begin{bmatrix} -0.38 & -8\times10^{-4} \\ 0.26 & -0.34 \end{bmatrix},\ B_0 = \begin{bmatrix} 0.7 & 0 \\ 0.1 & -2.4\times10^{3} \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -0.47 & 0.0328 \\ 0.06 & -0.3 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.4 & 0 \\ 0.07 & -0.0086 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1.52 & 0.0618 \\ 0.19 & -0.76 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.87 & 0 \\ 0.2 & -0.0206 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -2.45 & 0 \\ 0.94 & -1.76 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.44 & 0 \\ 0.68 & -0.0516 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -4.62 & 8\times10^{-4} \\ 3.24 & -1.96 \end{bmatrix},\ B_4 = \begin{bmatrix} 0.7 & 0 \\ 0.53 & -0.8476 \end{bmatrix}$$

Computed controller gain matrices are:

$$F_{P0} = \begin{bmatrix} 1042.7 & -361.6 \\ 2.86 & -3.305 \end{bmatrix},\ F_{I0} = \begin{bmatrix} 176.0 & -84.148 \\ 1.055 & -0.168 \end{bmatrix}$$

$$F_{P1} = \begin{bmatrix} 562.64 & 75.57 \\ 2.24 & -21.49 \end{bmatrix},\ F_{I1} = \begin{bmatrix} 1127.1 & 150.56 \\ 4.3683 & -42.94 \end{bmatrix}$$

$$F_{P2} = \begin{bmatrix} 788.2 & 284.46 \\ 1.95 & -36.46 \end{bmatrix},\ F_{I2} = \begin{bmatrix} 1620.0 & 553.93 \\ 3.64 & -72.847 \end{bmatrix}$$

$$F_{P3} = \begin{bmatrix} 428.668 & 718.79 \\ -2.507 & -52.80 \end{bmatrix},\ F_{I3} = \begin{bmatrix} 859.05 & 1437.0 \\ -5.25 & -105.5 \end{bmatrix}$$

$$F_{P4} = \begin{bmatrix} 572.04 & 585.91 \\ -142.74 & -748.1 \end{bmatrix},\ F_{I4} = \begin{bmatrix} 1009.0 & 1117.1 \\ -285.90 & -1.49 \end{bmatrix}$$

In order to evaluate proposed controller, in first case, scheduling signal is considered to move linearly between five operating values (figure 3). Simulation results are shown in figure 4 and 5. High turbine and low turbine spool speeds properly tracked reference signal (figure 4). Fuel flow and propeller angle signals are shown in figure 5. One advantage of the proposed design procedure is ensurig stability and guranteed cost in the case of high rate change in scheduling signal. Therefore, In order to evaluate the stability and performance of the closed loop system, in second case, a 0.05 Hz sinusoidal signal changing between three operation point is considered (figure 6). The simulation results are shown in figures 7 and 8. It can be clearly seen that stability is still maintained in the case of increasing the change rate.

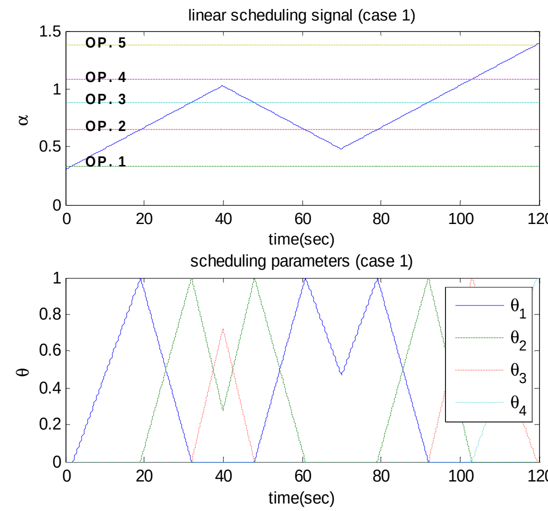

Figure 3. Scheduling signal and parameters (case 1)

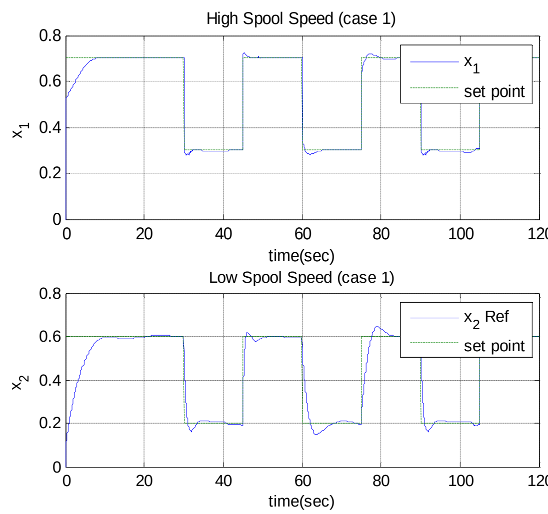

Figure 4. High and Low spool speed of turboshaft (case 1)

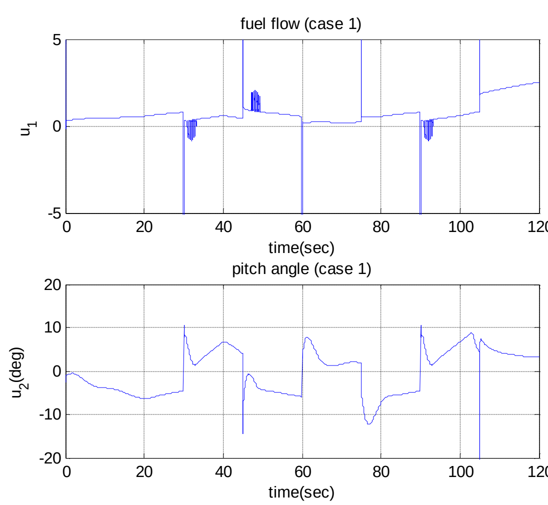

Figure 5. Input fuel flow and propeller angle (case 1)

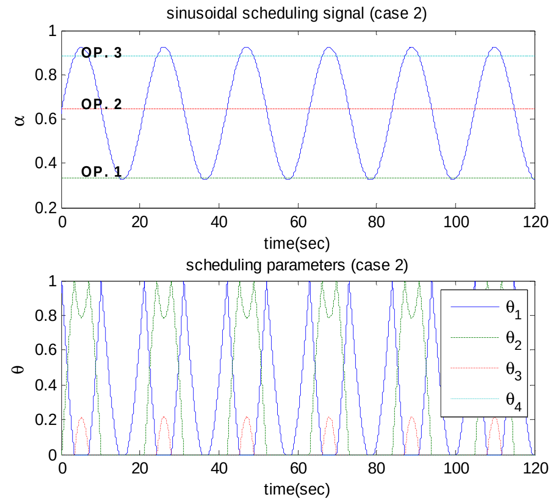

Figure 6. Scheduling signal and parameters (case 2)

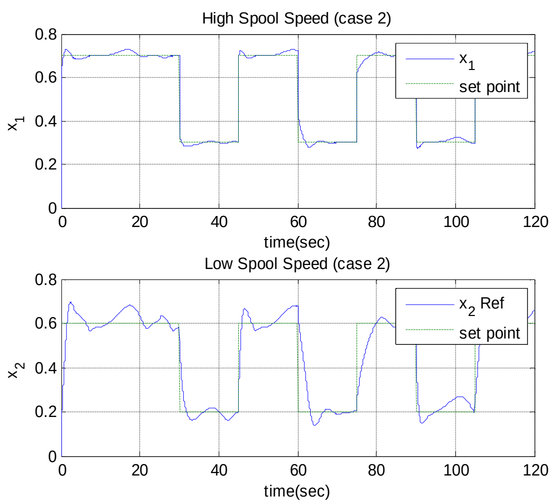

Figure 7. High and Low spool speed of turboshaft (case 2)

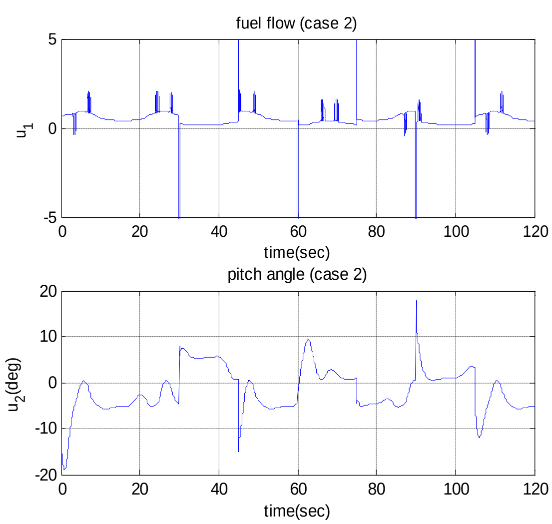

Figure 8. Input fuel flow and propeller angle (case 2)

5. Conclusion

Main goal of this paper is to design controller for turboshaft engine driving variable pitch propeller which could ensure stability and performance. Thus a LPV model in affine form is proposed for the MIMO plant. Since proposed model established based on a collection of linear models in different operating points, it can be close to the nonlinear model and at the same time enjoys a vast variety linear and LPV analysis and design methods. Then, a LPV gain scheduled controller design procedure is proposed which guarantees stability and performance of the closed loop system for output tracking problem. Benefits of the proposed procedure are mentioned and simulation is conducted to prove practicality of the procedure. It has been shown that output properly tracked reference signals and also in the case of quickly change between operating points, closed loop system remained stable.

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