Robust Optimal Control of Uncertain Nonlinear Switched System using Approximate Dynamic Programmin

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***Abstract*— This paper presents robust optimal control of an uncertain nonlinear switched system with forced subsystems. The uncertainties include external disturbance and parametric uncertainties. Switching signal and control input are designed to minimize a given cost function. Approximate dynamic programming (ADP) has been efficiently applied to certain switched systems as an optimal control strategy. Since approximate dynamic programming method is model based, there would seem to be some difficulties to apply approximate dynamic programming to uncertain switched system. To overcome these mentioned problems, this paper presents an appropriate model. In order to apply proposed control approach, robust time-delay controller is added with ADP control. At first uncertainties are compensated by robust time-delay controller. Then the switching signal and the control input are design by approximate dynamic programming that provides a feedback solution for unspecified initial conditions. The discussing boundedness of states and simulation results verify the effectiveness of the proposed control approach.**

***Index Terms*—Approximate dynamic programming, Robust time-delay controller, Uncertain switched system.**

I. Introduction

A switched system is a type of hybrid system, that includes of a group of continuous-time or discrete-time subsystems, a switching rule that organizes the switching between them, and the states that show the active subsystem.

It is well known that a broad range of engineering problems such as in robotics, in industrial systems, and in power systems can be modeled as a switched system [1-2]. Due to their significance in the engineering applications, the optimal control of switched systems has attracted much attention from many researchers in the control field [3-5].

In some papers, the mode sequence is selected as priori [6-8]. Some other Authors designed the mode sequence in order to achieve optimal control.

Optimal control of nonlinear switched system by using a constructive parallel algorithm is presented in [9]. By using an improved conjugate gradient algorithm and a discrete filled function method, an improved bi-level algorithm is proposed to solve optimal control of nonlinear switched system in [10].

Authors in [11] developed optimal control of nonlinear switched system by applying optimistic planning (OP) algorithms that can solve general optimal control with discrete inputs such as switches.

The optimal control is studied in [12], [13] by using dynamic programming and genetic algorithm, respectively. Switching time sequence is designed in [14] by evaluating the cost function for randomly selected switching sequence.

All the methods in cited papers are presented for a specific initial condition. In the recent researches, approximate dynamic programming (ADP) has been used to provide comprehensive solutions to achieve optimal control [15–17]. ADP is commonly applied utilizing two neural networks (NN) [18]. The authors of [19–22] also investigated the ADP-based approaches to optimal switching.

The approximate dynamic programming control performance is a desired control in switched systems with no uncertainties. Since approximate dynamic programming method is model based, there are difficulties to apply approximate dynamic programming to uncertain switched systems.

This paper introduces a nonlinear discrete model with lumped uncertainty of uncertain nonlinear switched system. The difference between the model and an actual system are considered as a lumped uncertainty. A two term control law is proposed; in this, the first term is an approximate dynamic programming and the second term is a robust time-delay estimator to compensate the uncertainties.

The rest of this paper is organized as follows: the modeling of the uncertain nonlinear switched system with forced subsystems is presented in section2. Section 3 develops the robust optimal control of uncertain nonlinear switched system using approximate dynamic programming and time delay controller. Discussing boundedness of states is presented in section4. Section 5 illustrates simulation results. Finally section 6 concludes the paper.

II. The Modeling of Uncertain Nonlinear Switched System with Forced Subsystems

Consider a class of uncertain nonlinear switched systems described by

$$\frac{d^n x}{dt^n} = f_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1} x}{dt^{n-1}}) + b_i u + d \quad (1)$$

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where i, x, u, b_i، d and f_i are the active subsystem, the state, the control input, the ith subsystem input coefficient, the random disturbance and the ith subsystem nonlinear function includes state and its derivatives.
Using nominal terms in (1), the following can be derived:

$$\frac{d^n x}{dt^n} = \hat{f}_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1}x}{dt^{n-1}}) + \hat{b}_i u + \varphi_i \quad (2)$$

where $\hat{f}_i$ and $\hat{b}_i$ are the nominal terms for the real terms f_i and b_i, respectively.

Here, the lumped uncertainty φ_i is expressed as follows:

$$\varphi_i = f_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1}x}{dt^{n-1}}) - \hat{f}_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1}x}{dt^{n-1}}) + \left(b_i - \hat{b}_i\right)u + d \quad (3)$$

The Lumped uncertainty φ_i includes the parametric uncertainty and the external disturbance.

From (2), we derive the state-space model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{a}\hat{f}_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1}x}{dt^{n-1}}) + \mathbf{b}_i u + \mathbf{g}\varphi_i \quad (4)$$

where $\mathbf{x}$, u, $\mathbf{A}$, $\mathbf{a}$, $\mathbf{b}_i$ and $\mathbf{g}$ are state vector, control input, the state coefficient matrix, nonlinear function coefficient matrix, the input gain matrix in the ith subsystem and uncertainty coefficient matrix, respectively.

$$\mathbf{x} = \begin{bmatrix} x \\ \frac{dx}{dt} \\ \vdots \\ \frac{d^{n-1}x}{dt^{n-1}} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$
$$\mathbf{a} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{b}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \hat{b}_i \end{bmatrix} \quad \mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (5)$$

From (4), one can obtain a discrete switched system using a sampling period sampling period σ which is a small positive constant. Substituting $k\sigma$ into t for approximating $\dot{\mathbf{x}}$ as $\dot{\mathbf{x}} = (\mathbf{x}(t+\sigma) - \mathbf{x}(t))/\sigma$ in (4), we obtain a discrete switched system

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{a}_k \hat{f}_{ik} + \mathbf{b}_{i,k} u_k + \mathbf{g}_k \varphi_{i,k} \quad (6)$$

where $\mathbf{x}_k = \mathbf{x}(k\sigma)$, $\mathbf{A}_k = \mathbf{I} + \sigma\mathbf{A}$, $\mathbf{a}_k = \sigma\mathbf{a}$, $\mathbf{b}_{i,k} = \sigma\mathbf{b}_i$, $u_k = u(\sigma k)$, $\mathbf{g}_k = \sigma\mathbf{g}$, $\hat{f}_{ik} = \sigma\hat{f}_i(\mathbf{x}_k)$ and $\varphi_{i,k} = \varphi_i(k\sigma)$.

III. Robust Optimal Control of Uncertain Nonlinear Switched System

To apply the robust optimal control of uncertain nonlinear switched system, a two-term control law is proposed. The first term is an approximate dynamic programming controller and the second term is a robust time-delay controller.

The uncertainties are compensated by robust time-delay controller. The switching signal and the control input are design by approximate dynamic programming that provides a feedback solution for unspecified initial conditions.

The system (6) is thus presented as

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{a}_k \hat{f}_{ik} + \mathbf{b}_{i,k} u_{1,k} + \mathbf{b}_{i,k} u_{2,k} + \mathbf{g}_k \varphi_{i,k} \quad (7)$$

where $u_{1,k}$ and $u_{2,k}$ are the first and the second terms of the control input.

A. Robust Time Delay Control

In order to apply approximate dynamic programming to uncertain nonlinear switched system, the uncertainties are compensated by robust time delay control. The basic idea behind overcoming the uncertainty problem is the use of a procedure that successfully estimates the uncertainty in the robust impedance control of a hydraulic suspension system [23], the control of flexible-joint robots [24] and the optimal control for a robot manipulators [25,26].
To create the dynamics of the tracking error well-defined such that the switched system can track the desired trajectory, we make the following assumptions.

Assumption 1: The desired trajectory $x_{\mathbf{d}}$ must be smooth in the sense that x and its derivatives up to a necessary order are available and all uniformly bounded.
The smoothness of the desired trajectory can be guaranteed by proper trajectory planning.
As a necessary condition to design a robust controller, the matching condition, outlined below, must be satisfied:
Matching condition: the uncertainty must enter the system through the same channel as the control input. Then, the uncertainty is said to satisfy the matching condition [27] or equivalently, it is said to be matched. We ensure the matching condition since in the system (6), the lumped uncertainty $\varphi_{i,k}$ enters the system by the same channel as the control input u.
As a necessary condition to design a robust control, the external disturbance d in (1) must be bounded.

Assumption 2: The external disturbance d is bounded as:

$$\|d\| \le d_{\max} \quad (8)$$

where $d_{\max}$ is a positive constant.

Assumption 3: The function $f_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1}x}{dt^{n-1}})$ in (1) is globally Lipschitz or there is a positive definite lyapunov function $V(\mathbf{x})$ for the $\dot{x} = f_i(x, \frac{dx}{dt}, ..., \frac{d^{n-1}x}{dt^{n-1}})$ where the $\dot{V}(\mathbf{x})$ is negative.

A two term control law is proposed in equation (7). The Performance of the proposed control is improved if the lumped uncertainty $\varphi_{i,k}$ is compensated. The uncertainty is perfectly compensated if

$$\mathbf{b}_{i,k} u_{2,k} = -\mathbf{g}_k \varphi_{i,k} \quad (9)$$

Since $\varphi_{i,k}$ is not known, the control law (9) cannot be defined. To estimate the uncertainty, we obtain from (8)

$$\mathbf{g}_k \varphi_{i,k} = \mathbf{x}_{k+1} - \mathbf{A}_k \mathbf{x}_k - \mathbf{a}_k \hat{f}_{ik} - \mathbf{b}_{i,k} u_{1,k} - \mathbf{b}_{i,k} u_{2,k} \quad (10)$$

Since $\mathbf{x}_{k+1}$ is not available in the kth step, $\mathbf{g}_k \varphi_{i,k}$ cannot be calculated. Instead, the previous value of $\mathbf{g}_k \varphi_{i,k}$ is used as

$$\mathbf{g}_{k-1}\varphi_{i,k-1} = \mathbf{x}_k - \mathbf{A}_{k-1}\mathbf{x}_{k-1} - \mathbf{a}_{k-1}\hat{f}_{ik-1} - \mathbf{b}_{i,k-1}u_{1,k-1} - \mathbf{b}_{i,k-1}u_{2,k-1} \quad (11)$$

The term $\mathbf{g}_{k-1}\varphi_{i,k-1}$ can be calculated since all the terms in the right hand side of (11) are known and available. The proposed robust control law is thus defined as:

$$\mathbf{b}_{i,k}u_{2,k} = -\mathbf{g}_{k-1}\varphi_{i,k-1} \quad (12)$$

we express the second term in the control law by substituting (11) into (12) to yield

$$\mathbf{b}_{i,k}u_{2,k} = -\mathbf{x}_k + \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{a}_{k-1}\hat{f}_{ik-1} + \mathbf{b}_{i,k-1}u_{1,k-1} + \mathbf{b}_{i,k-1}u_{2,k-1} \quad (13)$$

B. Approximate Dynamic programming

The approximate dynamic programming (ADP) controller has been efficiently used as an optimal controller in certain nonlinear switched system. Substituting (12) into (8) yields

$$\mathbf{x}_{k+1} = \mathbf{A}_k\mathbf{x}_k + \mathbf{a}_k\hat{f}_{ik} + \mathbf{b}_{i,k}u_{1,k} + \mathbf{g}_k\varphi_{i,k} - \mathbf{g}_{k-1}\varphi_{i,k-1} \quad (14)$$

In order to apply the ADP, a nominal model in the form of discrete switched system is suggested from (14) as follows

$$\mathbf{x}_{k+1} = \mathbf{A}_k\mathbf{x}_k + \mathbf{a}_k\hat{f}_{ik} + \mathbf{b}_{i,k}u_{1,k} \quad (15)$$

Once the NNs' weights are trained using Algorithm2 in [22], one may use them for online optimal control/scheduling of the system. This is done in real-time through feeding the current state $\mathbf{x}_k$ and time k to equation in [22]:

$$i_k^*(\mathbf{x}_k) = \arg\min_{i\in M}(\sigma(\mathbf{x}_k)^T V_k^i R V_k^{iT}\sigma(\mathbf{x}_k) + W_{k+1}^T\phi(\mathbf{x}_{k+1})), \quad \forall k \in \mathrm{K} \quad (16)$$

$$u_k^* = u_k^{i_k^*(\mathbf{x}_k),*} \cong V_k^{i_k^*(\mathbf{x}_k)T}\sigma(\mathbf{x}_k) \quad (17)$$

where

$$V_k^T\xi(\mathbf{x}_k) \cong u_k^*(\mathbf{x}_k) \quad (18)$$

$$W_k^T\phi(\mathbf{x}_k) \cong J_k^*(\mathbf{x}_k) \quad (19)$$

$$J = \psi(\mathbf{x}_N) + \sum_{k=0}^{N-1}\left(Q(\mathbf{x}_k) + u_k^T R u_k\right) \quad (20)$$

Calculate the optimal mode $i_k^*(\mathbf{x}_k)$ and the optimal control input u_k^* by using (16) and (17). Where Vector Valued functions $\xi:\mathbb{R}^n \to \mathbb{R}^p$ and $\phi:\mathbb{R}^n \to \mathbb{R}^q$ represent the selected smooth basis functions, where p and q are the respective number of (linearity independent) neurons. Matrices $V_k \in \mathbb{R}^p$ and $W_k \in \mathbb{R}^q$ are the unknown weights of the actor and the critic networks at time step k, respectively. Hence the optimal solution can be found online in a feedback form.

IV. Boundedness of States

The final control law is obtained by using (13), (16) and (17). The states and control input are bounded in predetermined domain by using approximate dynamic programming. Under bounded state and control input, assumption 1-2 and matching conditions, the lumped uncertainty is bounded.

Under assumption 3, there are two possibilities as follows:

1. Since switched system (14) is globally Lipschitz in $(\mathbf{g}_k\varphi_{i,k} - \mathbf{g}_{k-1}\varphi_{i,k-1}, u)$ under first condition in assumption 3 and theorem (global existence and uniqueness) presented by [28], the solution of the switched system (14) exists for all times. So solutions of the switched system (14) do not present finite escape time [29]. As a result, solution of the switched system (14) is bounded due to known initial and final times.

2. According theorem given in chapter 5 in [30] and under second condition in assumption 3, solution of the switched system (14) is bounded.

According to the reasoning given above, the discrete nonlinear switched system (14) provides a bounded output $\mathbf{x}_{k+1}$ under the bounded input $\mathbf{g}_k\varphi_{i,k} - \mathbf{g}_{k-1}\varphi_{i,k-1}$.

The robust time-delay control law (13) has the main role in compensating the uncertainty. If there exists a much difference between the nominal model (15) and the actual system (6), the closed-loop system (6) is subject to a large uncertainty. The residual uncertainty in the closed-loop system (14) is reduced from a large value of $\mathbf{g}_k\varphi_{i,k}$ to a small value of $\mathbf{g}_k\varphi_{i,k} - \mathbf{g}_{k-1}\varphi_{i,k-1}$ due to the use of robust time-delay control law (13). As a result, the performance of the control system is improved by reducing the residual uncertainty. The residual uncertainty $\mathbf{g}_k\varphi_{i,k} - \mathbf{g}_{k-1}\varphi_{i,k-1}$ will be very small when the uncertainty is smooth and the sampling time is very short.

V. The Simulation Results

The performance of the proposed method is evaluated through the following two simulations.

Consider one example of the scalar switching system with two modes given in [22] that satisfies second condition in assumption 3 with lyapunov function $V(\mathbf{x}) = x^2$.

The nominal model is

$$\dot{x} = \begin{cases} f_1(x) + g_1(x)u \equiv -x + u \\ f_2(x) + g_2(x)u \equiv -x^3 + u \end{cases} \quad (21)$$

The actual model is

$$\dot{x} = \begin{cases} f_1(x) + g_1(x)u \equiv -0.8x + 0.8u + d \\ f_2(x) + g_2(x)u \equiv -0.8x^3 + 0.8u + d \end{cases} \quad (22)$$

The selected cost function is $J = 50(x(tf)+2)^2 + \int_0^{tf} 0.5u(t)^2 dt$ where $t_f = 1\ s$, hence, the objective is directing the final state toward value -2. In this example the following basis functions are used as follows:

$$\phi(x) = \begin{bmatrix}1 & x & x^2 & x^3 & x^4 & x^5 & x^6\end{bmatrix}^T, \quad \xi(x) = \begin{bmatrix}1 & x & x^2 & x^3 & x^4 & x^5\end{bmatrix}^T \quad (23)$$

The horizon is discretized to time steps, i.e., $\sigma = 0.005\ s$ .The NNs are utilized for controlling initial condition $x(0) = 2$, once the networks are trained.

The uncertainty may include the external disturbances and parametric uncertainty. To consider the parametric uncertainty, all parameters of the nominal model used in the control law are given as %25 larger than the real ones. The external disturbance is a random signal with the mean=0 and standard deviation=2 with a period of 0.25 second as shown in Fig. 1.

The uncertainty is unknown; however, this example considers a bounded uncertainty to check the performance of the proposed control system.

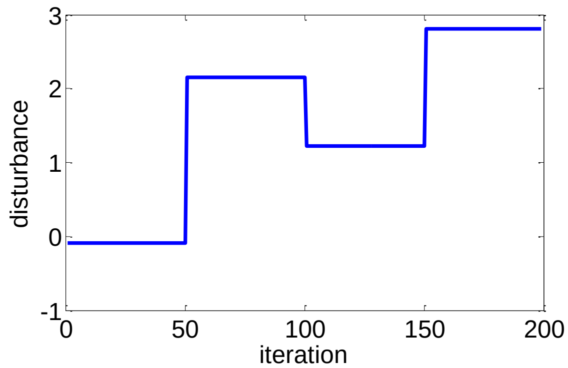

Fig.1. Random disturbance

Simulation1. The final law includes (13), (16) and (17) for optimal controlling of uncertain nonlinear switched system (22) with nominal model (21) is simulated. The results, including the histories of the state, the active mode, and the first control input, second control input, cost function are shown in Fig.2, Fig.3, Fig.4, Fig.5 and Fig.6, respectively.

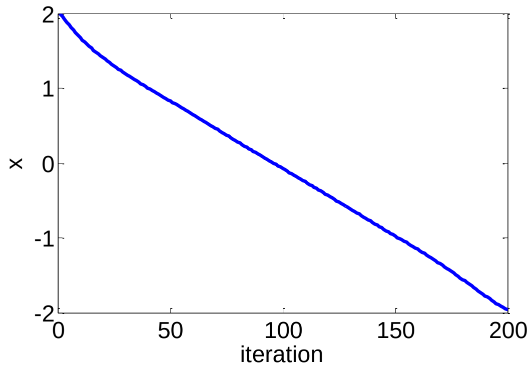

Fig. 2. Performance of proposed control

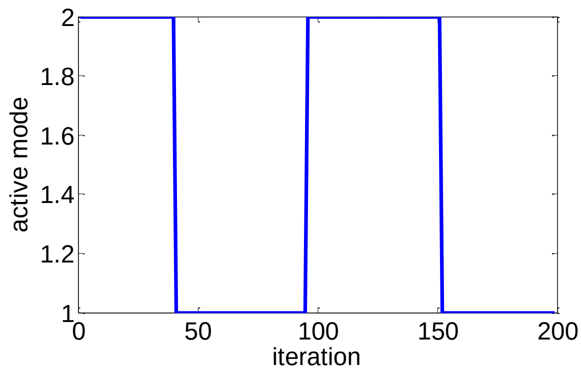

Fig. 3. Active subsystem in first simulation

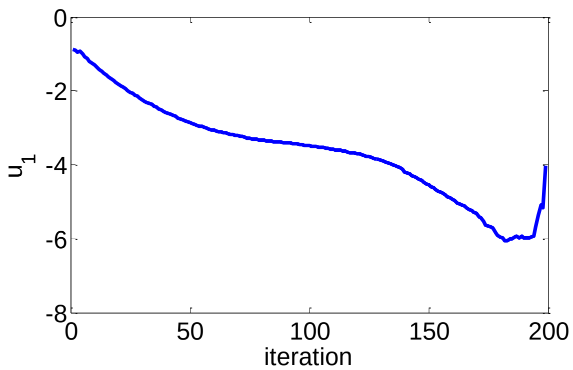

Fig. 4. First control input of proposed control

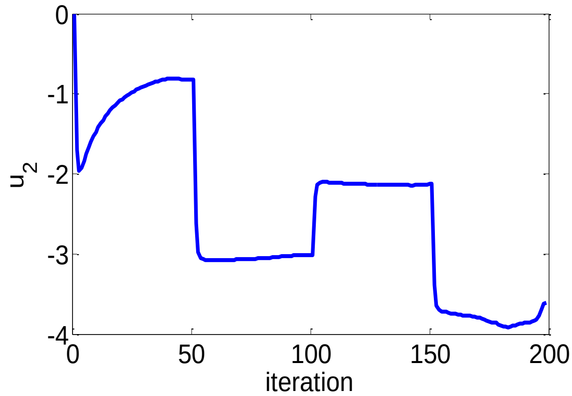

Fig. 5. second control input of proposed control

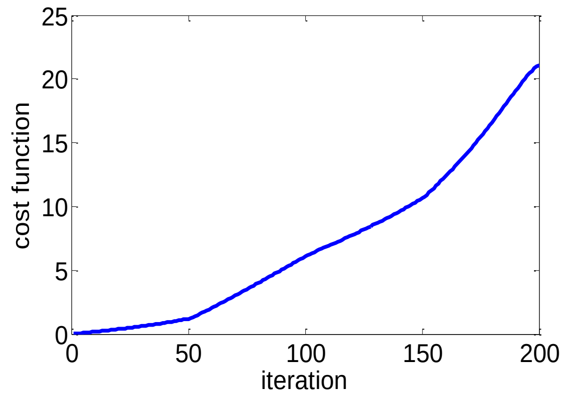

Fig. 6. Cost function of proposed control in first simulation

The state history shows that the controller has successfully driven the initial state to close to the desired terminal point in the given time. The final error is 0.0432. Second control input Jumps as shown in fig.5 that it is due to compensate random disturbance with sudden changes as in fig.1.

Simulation2. The effect of the robust time-delay controller in compensating the uncertainty is evaluated in this simulation. For this purpose, the time-delay controller is removed. The final law includes (16) and (17) without compensating uncertainties for optimal controlling of uncertain nonlinear switched system (22) with nominal model (21) is simulated. The results, including the histories of the state, the active mode, and the control input, cost function are shown in Fig.7, Fig.8, Fig.9 and Fig.10, respectively. The state history shows that final error is 0.2593.

Compared to Simulation 1, the final error and final cost function are increased than the ones in simulaton1.

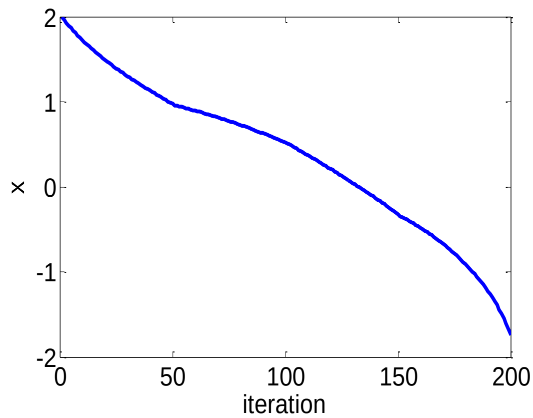

Fig. 7. Performance of ADP control

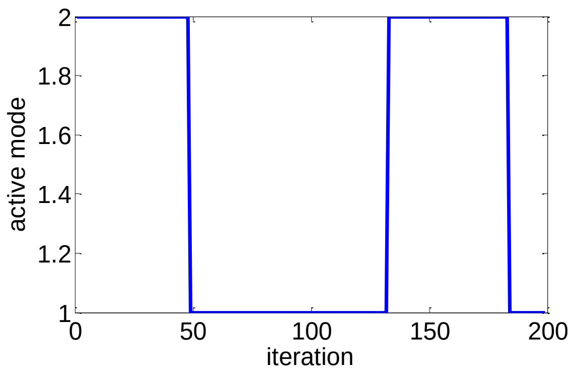

Fig. 8. Active subsystem in second simulation

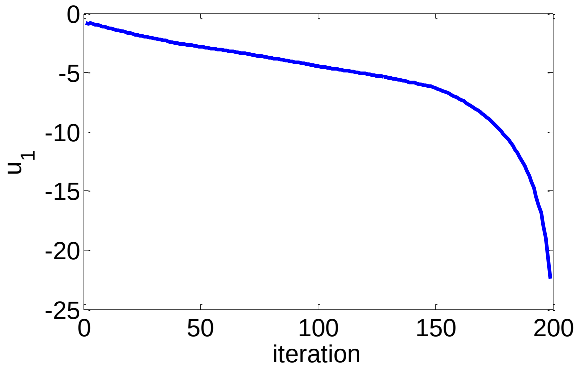

Fig. 9. Control efforts of ADP control

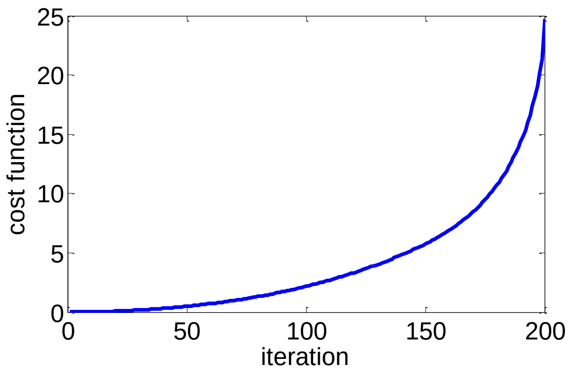

Fig. 10. Cost function of ADP control in second simulation

VI. Conclusions

This paper presented optimal control of uncertain nonlinear switched systems that includes external disturbance and parametric uncertainties. Challenges to apply approximate dynamic programming to optimal control of mentioned system are resolved by using robust time-delay controller. The model uncertainty was efficiently compensated using a discrete robust time-delay controller. Then switching signal and control input were designed by using approximate dynamic programming. The robust controller estimated and compensated the uncertainty such that the use of nominal model became efficient. The robust controller has played an important role to improve the performance of the control system by reducing the residual uncertainty in the closed-loop system. The control system can overcome a wide range of uncertainty including external disturbances, parametric uncertainty. Simulation results are shown effectiveness of the proposed control approach.

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