Relaxed Stabilization Conditions via Sum of Squares Approach for the Nonlinear Polynomial Model

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Abstract

In this paper, stabilization conditions and controller design for a class of nonlinear systems are proposed. The proposed method is based on the nonlinear feedback, quadratic Lyapunov function and heuristic slack matrices definition. These slack matrices in null products are derived using the properties of the system dynamics. Based on the Lyapunov stability theorem and Sum of Squares (SOS) decomposition techniques, the conditions are derived in terms of SOS. This approach has two main advantages. First, using the polynomial model, the proposed method uses the polynomial state space matrices in the model description. Therefore, it does not need any existing modeling methods such as the Takagi Sugeno (T-S) fuzzy model which can be a source of conservativeness in the control design conditions, because the membership function information cannot be used completely in the derivation of the controller design conditions. Second, using slack matrices, one can find the matrices that leads to applicable controller design which this means it provides extra degrees of freedom. To show the effectiveness of the proposed method, a PMSM is considered in the numerical simulation.

Keywords: Nonlinear feedback control; Permanent Magnet Synchronous Motor (PMSM); Polynomial model; Quadratic Lyapunov function; Sum of Squares (SOS).

1. Introduction

The synchronous machine is one of the most familiar machine categories which is generally used in the high power range [1]. The major plus of utilizing synchronous machines is their high efficiency, robustness and good controllability [2]. According to their non-linear dynamics, the problem of control of them is well identified to be demanding [3]. To prevail this challenging problem, different control approaches such as neural network control [4], [5], fuzzy logic control [6], [7] artificial intelligence [8], back stepping control [9], sliding mode control [10], [11] and adaptive control [12], [13] are studied.

In the recent years, many researchers have done some studies towards the nonlinear systems control. This studies mostly are based on the Lyapunov function and storage function techniques [14], [15]. These approaches are known as a difficult problem in many studies. To overcome these difficulties the numerical solutions have been proposed. One of the most studied approach is Linear Matrix Inequalities (LMI) method. In some cases, the problem of finding the Lyapunov function using LMI is an infeasible problem. However, it doesn't state that the desired system is unstable. It just informs that the LMI conditions cannot prove the stability. In this situation, there is another approach that may help to analyze the system. Sum of Squares (SOSs) decomposition [16] is a new approach which proposes a new direction to challenge these difficulties. Through this approach, stability analysis and control design of nonlinear systems can be performed efficiently via Lyapunov function. Actually, SOS approach takes advantages of polynomial matrix inequalities to design the controller and analyze the stability of the system. Feasible solutions for the controller design considering constraints can be calculated numerically [17]. Lately, many researchers pay attention to stability analysis and control synthesis of nonlinear systems via SOS approach [18]–[22]. In [23], using some constraints on the Lyapunov function, a static output controller is designed using the SOS approach. Generally speaking, the constraints make Lyapunov function to be only a function of states whose corresponding rows in the control matrix are zeroes, and its inverse have a specific form. By considering these constraints, the control design conditions evade the non-convexity of the static output feedback design. Recently, in [24], a static feedback controller has been suggested which implements an iterative SOS method, which increases

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conservativeness due to the iterative procedure. In [19] with applying a restricted region for the Lyapunov matrix, the feedback controller design conditions are solved using non-iterative algorithm. Though, this approach increase the conservativeness. In [18], the global stability using the feedback controller is discussed, in spite of many cases which the global stability is inaccessible.

In this paper, we show that the existence of a nonlinear static state feedback control law can be proposed in terms of the polynomial matrix inequalities. Furthermore, in spite of many researches, a *non-iterative* algorithm based on the SOS decomposition is suggested to solve the above-mentioned polynomial matrix inequalities so as to attain an appropriate controller gain. The proposed approach uses some heuristic slack matrices based on the dynamic of the system to relax the control design conditions. These slack matrices can provide degrees of freedom in designing the controller and it causes that the controller can be selected with different structure and degrees according to the application.

The rest of this paper is organized as follows. In Section 2, notations and preliminaries are proposed. Section 3 presents SOS-based Lyapunov stability conditions for the polynomial control system. In Section 4, the permanent magnet synchronous machine will be studied and the simulation using the proposed approach will be considered. Finally, in Section 5, a conclusion is drawn.

2. Notations and Preliminaries

2.1. Notations

In the rest of the paper, the following notations are examined. A monomial in $x(t) = [x_1(t), \dots, x_n(t)]$ is a function of the form $x_1^{d_1}(t) \dots x_n^{d_n}(t)$, where $d_i, i = 1, \dots, n$ are nonnegative integers. The degree of the monomial is defined as $d = \sum_{i=1}^{n} d_i$. A polynomial $p(x(t))$ is defined as a finite linear combination of monomials with real coefficients. A polynomial $p(x(t))$ is considered to be SOS if it can be represented as $p(x(t)) = \sum_{j=1}^{m} q_j(x(t))^2$, where $q_j(x(t))$ is a polynomial and m is a positive integer. Thus, $p(x(t)) \succ 0$, if it is an SOS. The expressions of $M \succ 0$, $M \succeq 0$, $M \prec 0$ and $M \preceq 0$ demonstrate the positive-, semipositive-, negative-, seminegative-definite matrices M, respectively.

2.2. Semidefinite and SOS programming

Convex optimization have some different types and one of them is semidefinite programming (SDP). The goal of semidefinite programming is to minimize a linear objective function over the intersection of the cone formed by positive semidefinite matrices with an affine space. A sum-of-squares problem is an optimization problem with a linear objective function and specific polynomial constraints on the decision variables, which satisfies the sum-of-squares property. If the polynomial constraints are affine in decision variables, the SOS optimization problem can be indicated as a semidefinite-programming one [18].

2.3. Sum of squares

The common SOS problem is to study the non-negativity of a polynomial $f(y)$, stated by powers of y and its related coefficients. The idea is to change the non-negativity by the corresponding condition of being SOS polynomials and attempt to explore for such decomposition. The essential ideas of the SOS decomposition are now briefly discussed in the following.

- Sum of Squares polynomials

The major point in the SOS approach is exploring for an expression of a polynomial as the sum of squares of simpler polynomials.

Definition 1: The set of Sum of Squares polynomials in the variables "x", stand for $\sum_x$, is the set defined by

$$\sum_x = \{p \in R_x | p = \sum_{i=1}^{t} f_i^2 \ , f_i \in R_x\} \quad (1)$$

with $t \in Z^+$ [24].

- SOS matrices

In the following proposition, the SOS programming can study the positiveness of matrices with polynomial elements.

Proposition 1 [18]: Let $L(x)$ be an $N \times N$ symmetric matrix of degree $2d$ in $x \in R^n$. Moreover, let $z(x)$ be a column vector whose entries are all monomials in y with degree no greater than d, and consider the following conditions:

a) $L(x) \succeq 0, \forall x \in R^n$

b) $v^T L(x) v$ is SOS, where $v \in R^n$

c) There exists a positive semidefinite matrix Q such that

$$v^T L(x) v = \left(v \otimes z(x)\right)^T Q \left(v \otimes z(x)\right)$$

Where $\otimes$ denoted the Kronecker product.

Then $(a) \rightarrow (b)$ and $(b) \leftrightarrow (c)$.

2.4. System description

Consider the following polynomial state space representation:

$$\dot{x}(t) = A(x(t))\hat{x}(x(t)) + B(x(t))u(t) \quad (2)$$

where $x = [x_1^T \ x_2^T \ \dots \ x_n^T]^T \in \mathcal{R}^{n \times 1}$ and $u \in \mathcal{R}^{m \times 1}$ are the state and control input

vector, $\hat{x} = [\hat{x}_1^T \ \hat{x}_2^T \ \dots \ \hat{x}_n^T]^T \in \mathcal{R}^{n\times 1}$ is a vector of monomials in x, $A(x)$ and $B(x)$ are the polynomial system and polynomial input matrices, respectively. Since $\hat{x}$ is a vector of monomials in x, therefore, there exists a matrix $T \in \mathcal{R}^{n\times n}$ which is polynomial in x, such that

$$\dot{\hat{x}}(x) = T(x)\dot{x} \tag{3}$$

where matrix T is calculated by

$$T(x) = \frac{\partial \hat{x}(x)}{\partial x} \tag{4}$$

Based on the nonlinear plant (2), a nonlinear static state feedback controller is utilized as:

$$u(t) = K(x(t))\hat{x}(x(t)) \tag{5}$$

where $K(x(t))$ is a polynomial matrix in x. By replacing the control law (5) in the open-loop system (2), the closed-loop polynomial system is achieved:

$$\dot{x}(t) = [A(x(t)) + B(x(t))K(x(t))]\hat{x}(x(t)) \tag{6}$$

The objective is to derive stabilization conditions in terms of polynomial matrix inequalities which can be solved efficiently by SOS techniques.

3. Relaxed Stability Analysis Conditions

In this section, the more relaxed stability conditions using some slack matrices will be derived using SOS approaches, will be proposed.

Consider the following null product which will be used for the later purposes in stability analysis:

$$\{[\dot{x}(t) - [A(x(t)) + B(x(t))K(x(t))]\hat{x}(x(t))]^T \times [(M_1(x(t))\dot{x}(t) + M_2(x(t))\hat{x}(t)] + *\} = 0 \tag{7}$$

where $M_1(x(t)) \in R^{n\times n}$ and $M_2(x(t)) \in R^{n\times N}$ polynomial slack matrices. Furthermore * denotes the transpose of the former term.

In order to achieve the control design conditions for the closed-loop system, we use the following quadratic Lyapunov function:

$$V(x) = \hat{x}^T(x)P(\tilde{x})\hat{x}(x) \tag{8}$$

where $\tilde{x} = \left[x_{k_1}, x_{k_2}, \dots, x_{k_q}\right]$ and k_i are defined as the following remark.

Remark 1: To assist the controller design using SOS, the row indices that the entries of the entire row of $B(x(t))$ are all zero are denoted by $K = \{k_1, k_2, \dots, k_q\}$.

Lemma 1: The closed-loop system (6) is exponentially stable with the given decay rate $\rho > 0$ if it fulfills the following inequality:

$$\frac{dV(t)}{dt} + 2\rho V(t) \le 0 \tag{9}$$

Theorem 1: Consider the polynomial state feedback controller (5) and the polynomial model (2). The closed-loop system is exponentially stable with a known $\rho > 0$, if the following SOS constraints are satisfied:

$$\left(\tilde{P}(x) - \epsilon_1(x)I\right)\hat{x} \in \Sigma_x \tag{10}$$

$$\hat{x}^T(-H + \epsilon_2(x)I)\hat{x} \in \Sigma_x \tag{11}$$

where

$$H = \begin{pmatrix} H(11) & H(12) \\ * & H(22) \end{pmatrix}$$

which the entries are

$$H(11) = 2\rho\tilde{P}(x) + \sum_{k\in K}\frac{\partial \tilde{P}(x)}{\partial x_k}A_k(x)\hat{x}(x) - [A(x)N(x_{\bar{k}}) + B(x)L + *]$$

$$H(12) = \alpha(x_k)\tilde{P}(x) + T^{-1}(x)N(x_k)\alpha(x_k) - N^T A^T(x) - L^T(x)B^T(x)$$

$$H(22) = [T^{-1}(x)N(x_{\bar{k}})\alpha(x_k) + *]$$

Where $M_1^{-1}(x) = T^{-1}(x)M_2^{-1}(x)\alpha(x_k)$, $\tilde{P}(x) = M_2^{-T}PM_2^{-1}$, $L = KM_2^{-1}$, $M_2^{-1} = N(x_{\bar{k}})$. Furthermore, $\bar{K}$ is complement of K, $\alpha(x_k)$ is an SOS polynomial matrix with the suitable dimension and $A_k(x)$ denotes the k^{th} row of $A(x)$. In this condition, the controller gain will be calculated via $K = LN^{-1}$.

Proof. Considering Lemma 1 and the Lyapunov function (8), one can obtain:

$$[\dot{\hat{x}}^T(x)P(\tilde{x})\hat{x}(x) + *] + \hat{x}^T(x)\dot{P}(\tilde{x})\hat{x}(x) + \hat{x}^T(x)2\rho P(\tilde{x})\hat{x}(x) < 0$$

Recalling the following properties

$$\dot{P}(\tilde{x}) = \sum_{k\in K}\frac{\partial P(\tilde{x})}{\partial x_k}A_k(x)\hat{x}(x)$$

$$\dot{\hat{x}}(x) = \frac{\partial \hat{x}(x)}{\partial x}\dot{x} = T(x)\dot{x} \tag{7}$$

Equation (12) can be rewritten as:

$$[\hat{x}^T T^T(x)P(\tilde{x})\hat{x}(x) + *] + \hat{x}^T(x)[2\rho P(\tilde{x})\hat{x}(x) + \sum_{k\in K}\frac{\partial P(\tilde{x})}{\partial x_k}A_k(x)\hat{x}(x)]\hat{x}(x) < 0 \tag{13}$$

Adding the null term (7) yields:

$$[\hat{x}^T T^T(x)P(\tilde{x})\hat{x}(x) + *] + \hat{x}^T(x)[2\rho P(\tilde{x}) + \sum_{k\in K}\frac{\partial P(\tilde{x})}{\partial x_k}A_k(x)\hat{x}(x)]\hat{x}(x) - [\dot{x}^T M_1^T(x)\dot{x} + \dot{x}^T M_2(x)\hat{x}(x) - \hat{x}^T(x)N^T(x)M_1(x)\dot{x} - \hat{x}^T(x)N^T(x)M_2(x)\hat{x}(x) - \hat{x}^T(x)K^T(x)B^T(x)M_1(x)\dot{x} - \hat{x}^T(x)K^T(x)B^T(x)M_2(x)\hat{x}(x)] + *] < 0 \tag{14}$$

Defining the vector $\xi = [\hat{x}(x) \ \dot{x}]^T$, one can have

$$\frac{dV(t)}{dt} + 2\rho V(t) \le \xi^T\bar{H}_{ij}\xi < 0 \tag{15}$$

where

$$\bar{H} = \begin{pmatrix} \bar{H}(11) & \bar{H}(12) \\ * & \bar{H}(22) \end{pmatrix}$$

which

$$\bar{H}(11) = 2\rho P(\dot{x}) + \sum_{k\in K} \frac{\partial P(\dot{x})}{\partial \omega_k} A_k(x)\dot{x}(x) - [A^T(x)M_2(x) + M_2^T(x)B(x)K(x) + *]$$

,

$$\bar{H}(12) = P(\dot{x})T(x) + M_2^T(x) - A^T(x)M_1(x) - K^T(x)B^T(x)M_1(x)$$

and

$$\bar{H}(22) = [M_1(x) + *].$$

Pre- and post-multiplying both sides of the $\bar{H}$ by

$$\begin{pmatrix} M_2^{-1}(x) & 0 \\ 0 & M_1^{-1}(x) \end{pmatrix}$$

yields:

$$\bar{\bar{H}} = \begin{pmatrix} \bar{H}(11) & \bar{H}(12) \\ * & \bar{\bar{H}}(22) \end{pmatrix}$$

which the entries are

$$\bar{\bar{H}}(11) = 2\rho M_2^{-T}(x)P(\dot{x})M_2^{-1}(x) + \sum_{i\in K} M_1^{-T}(x)\frac{\partial P(\dot{x})}{\partial \omega_i} M_2^{-1}(x)A_i(x)\dot{x}(x) - [A(x)M_2^{-1}(x) + B(x)K(x)M_2^{-1}(x) + *]$$

$$\bar{\bar{H}}(12) = M_2^{-T}(x)P(\dot{x})T(x)M_1^{-1}(x) + M_1^{-1}(x) - M_2^{-T}(x)A^T(x) - M_2^{-T}(x)K^T(x)B^T(x)$$

$$\bar{\bar{H}}(22) = [M_1^{-1}(x) + *]$$

where $M_1^{-1}(x) = T^{-1}(x)M_2^{-1}(x)\alpha(x_k)$, $\hat{P} = M_2^{-T}PM_2^{-1}$, $L = KM_2^{-1}$, $M_2^{-1} = N(x_k)$.

Furthermore, $\bar{K}$ is complement of K. In this condition, the controller gain will be calculated via $K = LN^{-1}$.

This completes the proof. ■

4. Simulation Study: Permanent Magnet Synchronous Motor

A surface-mounted PMSM can be represented by the following nonlinear equation [25]:

$$\begin{aligned} \dot{\omega} &= k_1 i_{qs} - k_2\omega - k_3 T_L \\ \dot{i}_{qs} &= -k_4 i_{qs} - k_5\omega + k_6 V_{qs} - \omega i_{ds} \\ \dot{i}_{ds} &= -k_4 i_{ds} + k_6 V_{ds} + \omega i_{qs} \end{aligned} \quad (16)$$

where the parameters and their definitions are given in [25]. Suppose that time derivative of load torque can be neglected (i.e. $\dot{T}_L = 0$) [26], [27]. By considering $(\beta = k_1 i_{qs} - k_2\omega - k_3 T_l)$ and employing a procedure as discussed in [26], the following new system representation is obtained:

$$\begin{aligned} \dot{\omega} &= \beta \\ \dot{i}_{qs} &= -k_1 k_5\omega - k_2\beta - k_1 k_4 i_{qs} - k_1\omega i_{ds} + k_1 k_6 V_{qs} \\ \dot{i}_{ds} &= \omega i_{qs} - k_4 i_{ds} + k_6 V_{ds} \end{aligned} \quad (17)$$

The main advantage of dynamic (17) compared to (16) is to eliminate the unknown load torque T_L. In this paper, it is assumed that β, ω, i_{ds} and i_{qs} are known. If β is not available, one may calculate it using the formulation $(\beta = k_1 i_{qs} - k_2\omega - k_3 T_l)$. Therefore, T_l should be estimated [28]. The electrical rotor angular speed ω must be kept at the stable value ω_d. Suppose that $\dot{\omega_d} = 0$ [25], [26]. Dynamic equations (17) can be transformed in such way that the equilibrium point will be in the origin and can be rearranged in form of non-linear state space equation as follows:

$$\begin{bmatrix} \dot{z} \\ \dot{\beta} \\ \dot{i}_{ds} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k_1 k_5 & -k_2 & -k_1 z \\ 0 & 0 & -k_4 \end{bmatrix} \begin{bmatrix} z \\ \beta \\ i_{ds} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 k_6 V_{qs} - k_1 k_4 i_{qs} - k_1 k_5\omega_d - k_1\omega_d i_{ds} \\ k_6 V_{ds} + (z + \omega_d) i_{qs} \end{bmatrix} \quad (18)$$

There is one non-linear term in (18) (i.e.$-k_1 z$). Therefore, the usual approaches like LMI cannot deal with this problem. Therefore, one of the available solutions is to use the T-S modelling which has the conservativeness. However, using the polynomial modeling we can deal with this situation.

In continue of this section, the objective is to design the controller for the PMSM using the polynomial modeling and the SOS approach.

To simulate the system and its controller, the parameters $k_1, \dots, k_6$ are needed. These parameters are as follows: $k_1 = (\frac{3P^2\lambda}{8})/J$, $k_2 = B/J$, $k_3 = (\frac{P}{2})/J$, $k_4 = R_s/L_s$, $k_5 = \lambda/L_s$, $k_6 = 1/L_s$, where $P = 12$, $R_s = 0.99$, $L_s = 5.82$, $\lambda = 0.079153$, $J - 0.00120754$, $B - 0.0003$.

Using Theorem 1, the Lyapunov matrix which is obtained using the SOS solver will be as follows

$$V(x) = 0.00288x_1^2 + 0.00018x_1x_2 + 0.00058x_1x_3 + 0.0058x_2^2 + 0.000006x_2x_3 + 0.0039x_3^2$$

Comparison to the controllers utilized in [25] and [26], implementing the controller based on Theorem 1 of this paper is more applicable. Also, comparison with [18] showed that the stability region or the proposed approach in this paper is much more than one proposed in [18].

The desired electrical rotor angular speed is set as $\omega_d = 100$. Figs. 1-6 indicate the states' evolution and control input of the closed-loop PMSM using degrees of 0 and 2 for N and L matrices, respectively. Fig. 1 shows that the PMSM successfully converges to its desired equilibrium point. Also, Fig. 2 demonstrates that the rotor angular acceleration β varies between $[0\ \ 125]\ rad/s^2$, which is completely acceptable and applicable compared to[25] and [26] which β is obtained in order of 10^4. Fig. 3 and Fig. 4 denotes the PMSM d-axes current i_{ds} and the PMSM d-axes current i_{qs}, respectively. Furthermore, Fig. 5 and Fig. 6 are the PMSM voltage v_{qs} and v_{ds}, respectively.

Since one of the advantages of the proposed method is the ability to change the degree of the slack matrices.

Therefore, in the second part of the simulation, the degree of 2 and 4 are chosen for the N and L matrices, respectively. Using this degree, one can have a tradeoff between the speed of the convergence of the states and the amplitude of the control signals. Figs. 7-9 demonstrate that the states of the PMSM converges to their equilibrium point quicker than the former simulation.

As a third part of the simulation study, the aging problem is considered for PMSM. Aging change λ, R_s and L_s around 5% of their initial value. With changing these parameters, the feasibility of the Theorem 1 was checked and the result was that the Theorem 1 can handle all the range of the changes in these parameters.

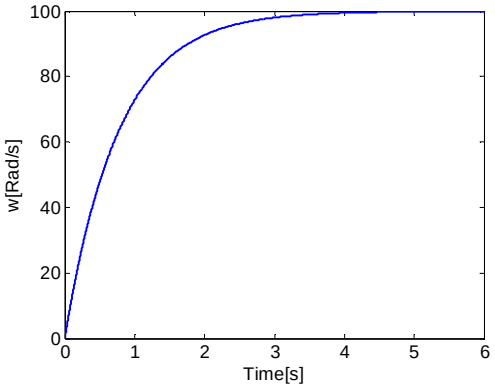

Figure 1. PMSM electrical rotor angular speed ω

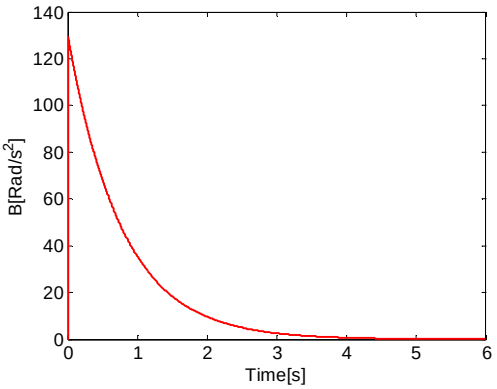

Figure 2. PMSM electrical rotor angular acceleration β

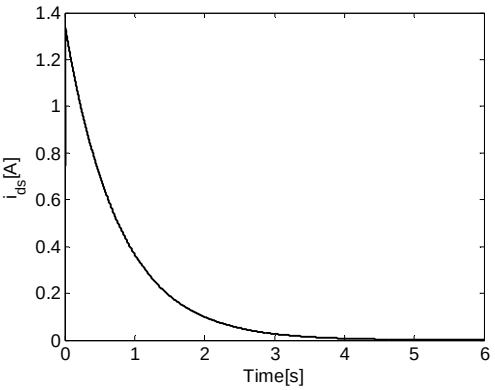

Figure 3. PMSM d-axes current i_{ds}.

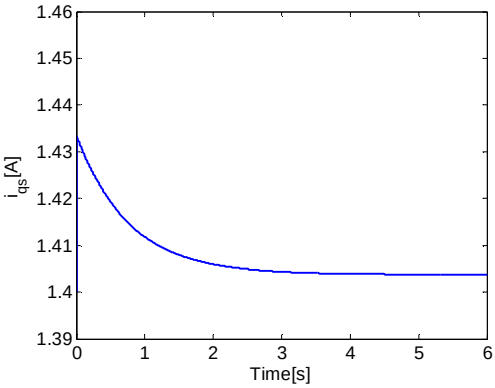

Figure 4. PMSM q-axes current i_{qs}.

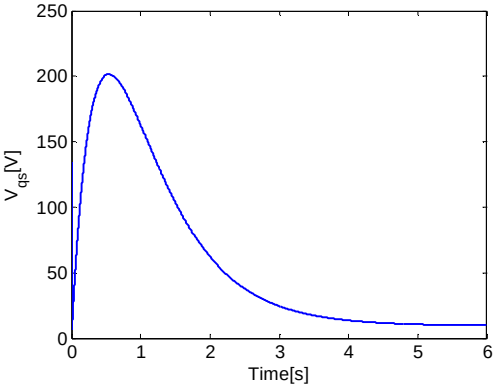

Figure 5. PMSM voltage v_{qs}

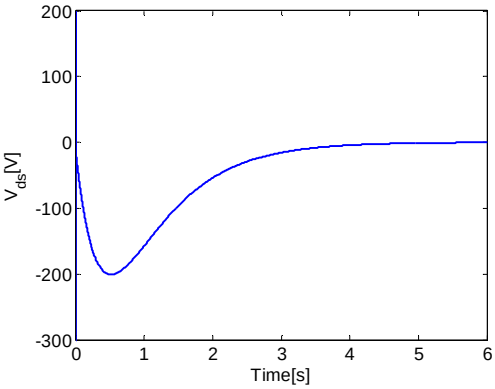

Figure 6. PMSM voltage v_{ds}

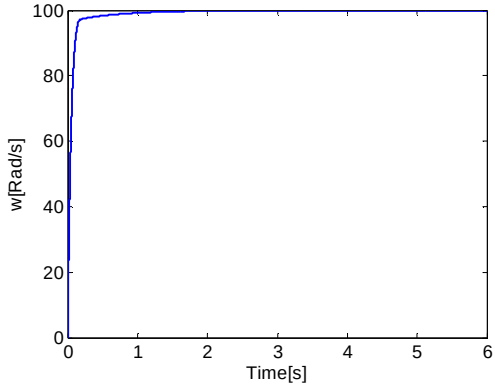

Figure 7. PMSM electrical rotor angular speed ω with degrees of 2 and 4 for the N and L matrices

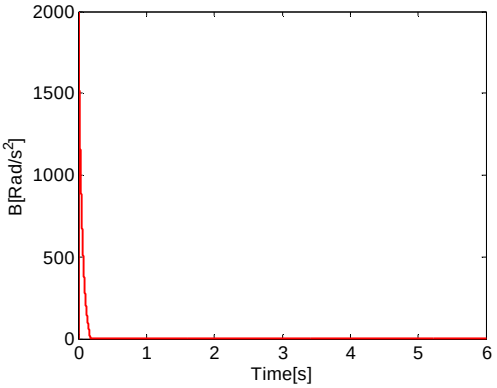

Figure 8. PMSM electrical rotor angular acceleration β with degrees of 2 and 4 for the N and L matrices

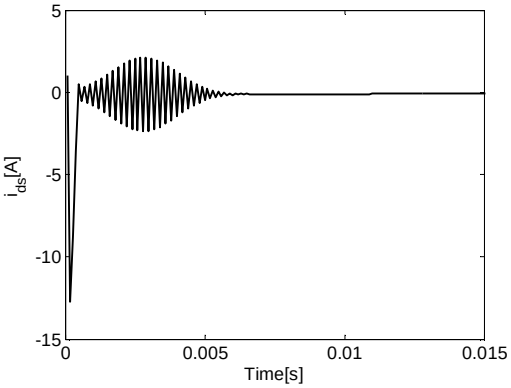

Figure 9. PMSM d-axes current i_{ds} with degrees of 2 and 4 for the N and L matrices

5. Conclusions

In this paper, a new relaxed stabilization condition in terms of SOS is proposed. The proposed approach uses some null matrix along the nonlinear modeling and quadratic Lyapunov function to relax the stabilization conditions. The stabilization conditions obtained in the term of SOS and solved efficiently using the SOSTOOLS. Then the proposed method applied for a PMSM. The result shows that the selection of the null matrices in the proposed method can handle the complexity and precision of the designed controller. More development of the current study is ongoing, such as calculation of the stability region which is forgotten in many control design researches. Furthermore, the study of disturbance using the robust control approaches and also limiting the amplitude of the control input are the interesting topics which can be considered for this problem.

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