Performance of Cyclostationary Detector of SC-FDMA under Synchronization Impairment

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***Abstract*—In a framework where synchronization issues are taken into account, we examine the performance of a cyclostationary detection scheme proposed for interleaved single carrier frequency division multiple access (SC-FDMA). The detection method under investigation here exploits the quasi-periodicity property of interleaved SC-FDMA signal. Inserting synchronization pilot signals in the data stream might deteriorate this quasi-periodicity property. Although the detection method is afflicted by pilot signals and loses performance, we show that it still outperforms energy detection and cyclic prefix based detection. We also investigate the performance of the detector when the system is hit by frequency offset. We derive the distribution of the detection metric in the presence of frequency offset. When frequency offset is small enough, we prove that it does not change the metric distribution. We also show that for arbitrary practical frequency offsets, the performance degradation is still negligible.**

Keywords: Cognitive radio, SC-FDMA, Detection, Synchronization, Frequency offset, Spectrum Sensing.

I. Introduction

Cognitive radio (CR) is a promising solution to overcome the spectrum scarcity in wireless communications. In this method, the spectrum is shared between licensed and unlicensed users. The former group, which are also known as primary users (PU), are given priority to occupy the available spectrum. The latter group or secondary users (SU), however, can transmit only if the spectrum is not used by the primary users. Hence, secondary users need to sense the spectrum to find the spectrum holes available for transmission and to detect the primary user when it starts transmission [1]. By appropriate sensing of the spectrum, SU is able to vacant the spectrum and avoid interfering with the primary user. Several methods have been proposed for spectrum sensing in cognitive radio networks. As one of the basic yet efficient methods, we can name energy detection (ED) [1]. This method is applicable to any signal and does not require information regarding the transmission scheme. On the other hand, ED has poor performance in low signal to noise ratio (SNR) regimes. There are also other methods such as matched filters which require complete knowledge regarding the primary user signalling to obtain their best performance [2]. Methods known as waveform detection exploit information regarding the pilot signals transmitted between the data stream to detect the presence of the signal [3]. Cyclostationary detection techniques use the periodicity in the signal or one of its statistics for detection [4] [5]. The detection method under investigation in this paper is a cyclostationary detection technique proposed in [6] for spectrum sensing of interleaved single carrier frequency division multiple access (SC-FDMA) signals. SC-FDMA is a generalized multi-carrier technique adopted as uplink air interface in fourth generation cellular systems [7]. This method illustrates low peak to average power ratio (PAPR) while obtains appropriate performance in multi-path channels [8] [9].

Considering the vast use of SC-FDMA method, it is important to develop appropriate spectrum sensing methods for this transmission scheme. In [6], a cyclostationary detection method is proposed for interleaved SC-FDMA signals. This method exploits the quasi-periodicity property of interleaved SC-FDMA signals to develop a detection metric. It is shown that the proposed detection scheme outperforms energy detection and the cyclic prefix (CP) based cyclostationary detection method.

In this paper, we investigate the detection performance of the method proposed in [6] when synchronization issues are considered. Since inserting reference signals might destroy the periodicity property of interleaved SC-FDMA signals, it is of importance to examine the detection performance in such scenario. We evaluate the performance when demodulation reference signal (DM-RS) proposed in long term evolution (LTE) advanced are transmitted within the data stream. As we show through simulations, our detection method in a system with pilot signal obtains the same probability of detection in a pilot free system at 1.5dB higher SNRs. Yet, the detection performance of our technique even with pilot signals is still significantly superior than that of ED and CP-based detection. We also derive the detection metric and its distribution in the presence of frequency offset. We show that when frequency offset is small enough, the distribution of the metric approaches that of a synchronized system. Even in the presence of practical frequency offsets, the loss in the detection performance is around 0.2 dB.

The rest of the paper is organized as follows. Section II describes the system model and the cyclostationary detection technique. In section III, we derive the distribution of the detection metric in the presence of frequency

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offset. A discussion on the performance of the system in the presence of demodulation reference signal is also presented in this section. In section IV, the simulation results are presented. Section V concludes the paper.

II. System Model and Spectrum Sensing Method

A. System Model

In an SC-FDMA system with M users, we denote the data block of the u-th user by $\mathbf{a}^u = [a_0^u, ..., a_{N-1}^u]^T$. The data symbols, a_i^u which are from a quaternary phase shift keying (QPSK) constellation with amplitude χ are assumed to be independent and identically distributed (i.i.d). The data block $\mathbf{a}^u$ is converted into frequency domain by an N-point FFT to generate block $\mathbf{A}^u = [A_0^u, ..., A_{N-1}^u]^T$. We assign N sub-carriers of a total number of MN sub-carriers into these N frequency components A_i^u [10]. In this assignment, we might assign contiguous sub-carriers to one user which is known as localized SC-FDMA. Alternatively, we can assign equi-distanced sub-carriers into a user which is known as interleaved SC-FDMA. The sub-carrier mapping process can be denoted by multiplying the frequency domain vector, $\mathbf{A}^u$, by a mapping matrix $\mathbf{P}^u$. The (m, n)-th element of this matrix for interleaved and localized sub-carrier mapping can be expressed as

$$\mathbf{P}^u(m, n) = \delta(m - n - (u-1)N), \tag{1}$$

and

$$\mathbf{P}^u(m, n) = \delta(m - nM - u), \tag{2}$$

respectively where $0 \leq n \leq N-1$, $0 \leq m \leq MN-1$ and $\delta(.)$ is the Dirac delta function. After mapping, an MN-point inverse fast Fourier transform (IFFT) converts the signal into time domain as the SC-FDMA output signal as

$$\mathbf{b}^u = \check{\mathbf{F}}_{MN}\mathbf{P}^u\mathbf{F}_N\mathbf{a}^u, \tag{3}$$

where $\mathbf{F}_P$ and $\check{\mathbf{F}}_P$ denote the matrices of P-point FFT and IFFT, respectively. The presentation of frequency domain received signal is as follows

$$\mathbf{R} = \sum_{u=1}^{M} \mathbf{H}^u\mathbf{P}^u\mathbf{F}_N\mathbf{a}^u + \mathbf{Z}, \tag{4}$$

where $\mathbf{Z} = \mathbf{F}_{MN}\mathbf{z}$, $\mathbf{z}$ is the additive white Gaussian noise vector in time domain and $\mathbf{H}^u$ is the diagonal frequency domain channel matrix. In the receiver, the signal is first demapped and then equalized in the frequency domain. The details of the above equations can be found in [8].

B. Detection Scheme

In [6], a spectrum sensing method for detection of interleaved SC-FDMA signals has been proposed. This method is a cyclo-stationary detection scheme which exploits the quasi-periodicity property of the interleaved SC-FDMA signal and generates an appropriate metric for detection. The quasi-periodicity property of interleaved SC-FDMA signal can be verified when the output signal is expressed in terms of the original data symbols as

$$b_m^v = \frac{1}{M} e^{j\frac{2\pi m v}{MN}} a^v_{\lfloor \frac{m}{M} \rfloor}, \tag{5}$$

where $m = 0, ..., MN-1$ and $\lfloor . \rfloor$ shows the greatest integer number smaller than the argument.

The spectrum sensing method used to detect the v-th user is accomplished in the following steps

- Suppressing the sub-carriers allocated to other users
- Calculating the metric
- Comparing the metric with the threshold

The details of these steps are presented in [6]. In what follows we explain them briefly. The first step can be performed by frequency domain zero padding where the received signal in the frequency domain is multiplied by the following matrix

$$Q_{i,j}^v = \begin{cases} 1 & i = lM + v, j = l + 1 \\ 0 & \text{otherwise} \end{cases},$$

where $l = 0, ..., N-1$ and $v = 1, ..., M$.
The metric is calculated over a window with length W as

$$\widetilde{J} = \sum_{m=1}^{W} \widetilde{r}_m^v \widetilde{r}_{m+N}^{v^*} e^{j\frac{2\pi v}{M}},$$

where $\widetilde{\mathbf{r}}^v$ is as follows

$$\widetilde{\mathbf{r}}^v = \check{\mathbf{F}}_{MN}\mathbf{Q}^v\mathbf{F}_{MN}\mathbf{r}. \tag{6}$$

In the third step, we utilize Neyman-Pearson strategy for detection as

$$|\widetilde{J}| \underset{H_1}{\overset{H_0}{\lessgtr}} \eta,$$

in which hypothesis H_0 and H_1 denote the absence and presence of the primary user, respectively, and η is the threshold. The threshold is derived as

$$\eta = \sqrt{2}\sigma_0 \text{erfc}^{-1}(P_{fa}) + m_0. \tag{7}$$

for a specific probability of false alarm, P_{fa}. In (7), m_0 and σ_0^2 are the mean and variance of the metric for the first hypothesis (absence of PU) which can be expressed as [6]

$$m_0 = W\sigma_{z'}^2\left(\frac{M-1}{M}\right), \tag{8}$$

$$\sigma_0^2 = W\sigma_{z'}^4\big(M-2+\frac{2}{M}\big). \tag{9}$$

It is worth mentioning that the equal number of sub-carriers for different users is not a necessary condition for the detection method to work. In fact, as long as the quasi-periodicity of the SC-FDMA signal holds, we can utilize this method for signal detection. Thus, the proposed detection scheme can also be used when the users have different number of sub-carriers as long as the sub-carrier mapping is interleaved which results in a quasi-periodic signal.

III. Performance of Detector under Synchronization Impairment

In this section, we assume a practical system with frequency instabilities. To overcome this problem, known reference or pilot signals are transmitted to the receiver to estimate the frequency offset. In the following two sub-sections, we analyze the performance of the proposed spectrum sensing method in a synchronization framework. First, we analyze the effect of the uncompensated frequency offset on the metric distribution and performance of the detector. Moreover, we consider a framework when pilot signals are inserted within the data stream.

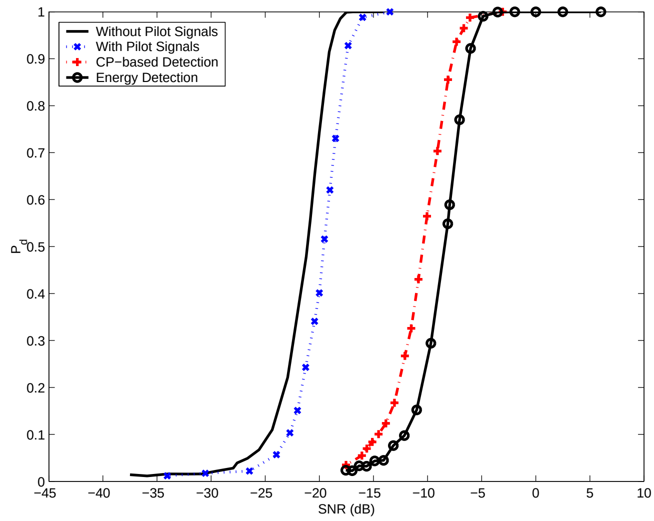

Fig. 1: Comparison of detection performance of our method with and without pilots and that of energy detection and CP detection.

A. Effect of Frequency Offset on Detection Performance

If the user of interest, v-th user, is impaired by a frequency offset with normalized of $\varepsilon = \delta f T$, the SC-FDMA symbol which was presented in (5) will change to

$$b_m = \frac{1}{M} a_{\lfloor \frac{m}{M} \rfloor} e^{j\frac{2\pi m(v+\varepsilon)}{MN}}. \tag{10}$$

Let us denote the frequency domain representation of the signal $\mathbf{b}$ by $\mathbf{B}$. In the first step of the detection algorithm, we suppress sub-carriers allocated to other users by the following filtering process.

$$D_n = \begin{cases} B_n & n = pM+v \\ 0 & \text{otherwise} \end{cases}. \tag{11}$$

The output signal of the filter in time domain, $\mathbf{d}$, is generated by taking an MN-point inverse FFT of $\mathbf{D}$. Thus, d_m can be expressed as

$$d_m = \frac{1}{MN}\sum_{p=0}^{N-1} B_{v+pM} e^{j\frac{2\pi(v+pM)m}{MN}}. \tag{12}$$

Since B_{v+pM} can written as

$$B_{v+pM} = \frac{1}{M}\sum_{q=0}^{MN-1} a^v_{\lfloor \frac{q}{M} \rfloor} e^{-j\frac{2\pi pq}{N}} e^{j\frac{2\pi\varepsilon q}{MN}},$$

the signal d_m will be simplified to

$$d_m = \frac{1}{M^2} a^v_{\lfloor \frac{m}{M} \rfloor} e^{j\frac{2\pi(v+\varepsilon)m}{MN}} \frac{1-e^{j2\pi\varepsilon}}{1-e^{j\frac{2\pi\varepsilon}{M}}}.$$

Thus, the two hypothesis are modified to

$$H_0 : \tilde{r}^v_m = \frac{1}{M} e^{j\frac{2\pi m v}{MN}} z'_{\lfloor \frac{m}{M} \rfloor},$$
$$H_1 : \tilde{r}^v_m = \frac{1}{M} e^{j\frac{2\pi m v}{MN}} \left[\frac{e^{j\frac{2\pi\varepsilon m}{MN}}}{M} \frac{1-e^{j2\pi\varepsilon}}{1-e^{j\frac{2\pi\varepsilon}{M}}} a^v_{\lfloor \frac{m}{M} \rfloor} + z'_{\lfloor \frac{m}{M} \rfloor} \right].$$

The distribution of $\tilde{J}$ under both hypotheses can be approximated by Gaussian. The mean and variance of $\tilde{J}$ under hypothesis H_0 are

$$m_0 = E\{\widetilde{J}|H_0\} = \sigma_{z'}^2 W \frac{M-1}{M},$$

$$\sigma_0^2 = W\Big(M-2+\frac{2}{M}\Big)\sigma_{z'}^4,$$

which are the same as mean and variance of the metric for a perfectly synchronized system. For hypothesis H_1, we have

$$m_1 = W\left(\frac{1}{M^4}\chi^2\frac{\sin^2(2\pi\varepsilon)}{\sin^2(\frac{2\pi\varepsilon}{M})} e^{-j\frac{2\pi\varepsilon}{M}} + \sigma_{z'}^2\right)\big(\frac{M-1}{M}\big),$$

$$E\{|\widetilde{J}|^2|H_1\} = \frac{1}{M^4}\sum_{i=1}^{i=6}\kappa_i$$

where we have

$$\kappa_1 = \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_i a_{i+N}^* a_j^* a_{j+N})$$

$$= W\frac{\chi^4}{M^4}\frac{\sin^4(\pi\varepsilon)}{\sin^4(\frac{\pi\varepsilon}{M})}\left[M-2+\frac{2}{M}+W(\frac{M-1}{M})^2\right],$$

$$\kappa_2 = \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_i a_{i+N}^* z_j^{'*} z_{j+N}^{'}) + \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_j^* a_{j+N} z_i^{'} z_{i+}^{'*}$$

$$= W^2\chi^2\frac{\sin^2(\pi\varepsilon)}{\sin^2(\frac{\pi\varepsilon}{M})}(\frac{M-1}{M})^2\sigma_{z'}^2 2\cos(\frac{2\pi\varepsilon}{M}),$$

$$\kappa_3 = \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_i a_j^* z_{i+N}^{'*} z_{j+N}^{'}),$$

$$= W\chi^2\frac{\sin^2(\pi\varepsilon)}{\sin^2(\frac{\pi\varepsilon}{M})}\left[\sum_{l=1}^{M-1} 2(\frac{M-l}{M})^2\cos(\frac{2\pi l\varepsilon}{M})\right]\sigma_{z'}^2,$$

$$\kappa_4 = \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_{i+N}^* a_{j+N} z_i^{'} z_j^{'*}) = \kappa_3,$$

$$\kappa_5 = \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_i a_{j+N} z_{i+N}^{'*} z_j^{'*}) + \sum_{i=1}^{W}\sum_{j=1}^{W} E(a_{i+n}^* a_j^* z_i^{'} z_{j+N}^{'})$$

$$= 2W\chi^2\frac{\sin^2(\pi\varepsilon)}{\sin^2(\frac{\pi\varepsilon}{M})}\left[(\frac{M-1}{M})^2\cos(\frac{2\pi\varepsilon(2i+N)}{MN})\right.$$

$$+\sum_{l=1}^{M-1}(\frac{M-l}{M})^2\cos(\frac{2\pi\varepsilon(2i+lN)}{MN})$$

$$\left.+\sum_{l=1}^{M-1}(\frac{M-l}{M})^2\cos(\frac{2\pi\varepsilon(2i+(l-2)N)}{MN})\right]\sigma_{z'}^2,$$

$$\kappa_6 = \sum_{i=1}^{W}\sum_{j=1}^{W} E(z_i^{'} z_{i+N}^{'*} z_j^{'*} z_{j+N}^{'}) =$$

$$WM^4\Big(M-2+\frac{2}{M}\Big)\sigma_{z'}^4 + W^2M^2\sigma_{z'}^4(M-1)^2.$$

It is noteworthy that the detection method is not confined to the QPSK constellation. If the constellation is not QPSK, the equations have to be modified slightly by replacing χ^2 by the variance of the constellation.

Proposition 1: For small frequency offsets, the mean and variance of the metric approach their corresponding values in a synchronized scenario.

Proof: As previously mentioned, the mean and variance under the hypothesis H_0 when the system is impaired by frequency offset is the same as their corresponding values for the synchronized system. For the second hypothesis, we have

$$\lim_{\varepsilon\to 0} m_1 = W\bigg(\frac{\chi^2}{M^2}+\sigma_{z'}^2\bigg)(\frac{M-1}{M}),$$

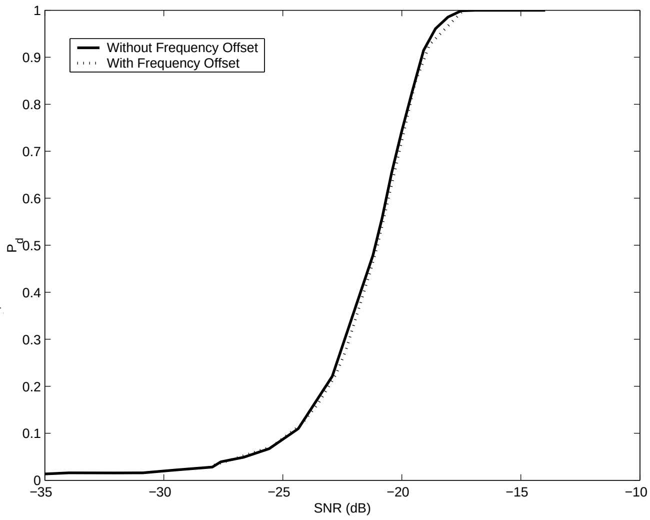

Fig. 2: Probability of detection when the frequency offset of the users are [0.095 0 0.13 0 0.07].

$$\lim_{\varepsilon\to 0} var(\tilde{J}) = \lim_{\varepsilon\to 0}\frac{\sum\kappa_i}{M^4} - m_1^2 =$$

$$W\Big[\Big(\frac{\chi^4}{M^4}+\sigma_{z'}^4\Big)\big(M-2+\frac{2}{M}\big)+\frac{2\sigma_{z'}^2\chi^2(2M^2+1)}{3M^3}\Big].$$

The above values are the same as the mean and variance of the metric under hypothesis H_1 when the system is fully synchronized. ■

B. Effect of the Reference Signals on Detection Performance

Reference signals are generally used in practical communication systems for coherent demodulation, channel estimation and synchronization purposes [8]. In what follows, we evaluate the effect of the reference signal on the performance of the detection method discussed in section II. Since SC-FDMA is used in uplink LTE-advanced, we consider the reference signals proposed in LTE-A standard. These reference signals fall into one of the following categories: demodulation reference signal (DM-RS) and sounding reference signal (SRS). The former is used for channel estimation for coherent demodulation while the latter is used for channel estimation in a wider range to provide information for channel dependent scheduling [11]. If the length of the sequence is larger than 12, the base sequence for both reference signals is a Zadoff-Chu sequence. Otherwise, it is a complex exponential [11]. A u-th root Zadoff-Chu sequence of length L is generated as follows

$$x_u(n) = e^{-j\frac{\pi un(n+1)}{L}}.$$

Zadoff-Chu sequences are appropriate from both auto/cross-correlation and PAPR point of view. It is noteworthy that DM-RS is transmitted only in physical

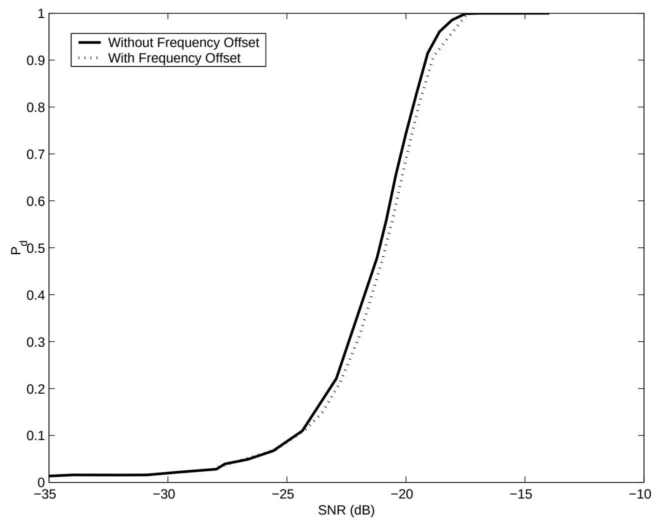

Fig. 3: Probability of detection when the frequency offset of the users are [0.17 0 0.2 0 0.09].

resource blocks whereas SRS is not necessarily transmitted within data. Since our main concern here is to examine whether the presence of the reference signal violates the quasi-periodicity of the transmitted data, we concentrate on DM-RS. We should also note that in LTE uplink transmission, to preserve low PAPR, the reference signal is not transmitted in parallel with other uplink transmissions. Instead, it is transmitted over specific symbols dedicated to this purpose. For instance, DM-RS is transmitted over the fourth symbol of the uplink slot for normal cyclic prefix (CP) and over the third symbol if we have extended cyclic prefix [11]. We recall that in LTE-A each slot consists of seven consecutive symbols. In next section, through simulations, we determine the effect of DM-RS on detection performance.

IV. Simulation Results

In this section, we evaluate the performance of the mentioned cylcostationary detection scheme in the presence of frequency instabilities. We consider a system with $M = 5$ users where the original data block length is $N = 256$. First, we assume that by using pilot signals we can correctly estimate the frequency offset and compensate it. As described in the previous section, the pilot signals are in the form of Zadoff-Chu sequences. We simulated a system in which DM-RS is transmitted over the fourth SC-FDMA block. To examine the effect of inserting these pilot signals on the performance, we evaluate the probability of detection for such system. This curve is presented in Fig. 1. As can be seen, in the presence of pilot signals the performance degradation is about 1.5 dB at most signal to noise ratios. If we compare this curve with the corresponding curves presented in [6] for energy detection and CP-based detection we can see that the discussed cyclostationary detector outperforms ED and CP detectors by 11.5 dB, 9.5 dB, respectively. In the considered CP detector the length of the detector is $\frac{1}{8}$ of the block length.
We also consider a case where the frequency offset of the users are not compensated. Let us assume the normalized frequency offset of the first user, third user and fourth user are 0.095, 0.13 and 0.07, respectively. The probability of detection of our cyclostationary detector is presented in Fig. 2 for this system. As can be seen, the performance loss of the detection method is negligible. Fig. 3 presents a similar curve when the mentioned frequency offsets are increased to 0.17, 0.2 and 0.09, respectively. In this case we have around 0.3 dB performance loss which is tolerable.

It is worth mentioning that the comparison between the proposed cyclostationary detector and that of other important detection schemes such as CP-detector and energy detector can be found in [6] when there is no frequency offset. In the presence of frequency offset, the performance of energy detector des not change as the overall received energy is not affected by the frequency offset.The performance of the CP detector slightly degrades. The main goal of the paper is to show that the performance of our proposed scheme, even under frequency uncertainties, is significantly better than that of other detection schemes even without frequency offset. In other words, we can obtain the same performance at almost 10-12 dB lower SNR.

V. Conclusion

In this paper, we evaluated the effect of frequency offset and presence of pilot signals on the performance of a cyclostationary detector designed for SC-FDMA signals. We showed that inserting pilot signals in the form of Zadoff-Chu sequences degrades the probability of detection around 1.5 dB. However, the considered detector still outperforms other detectors such as energy detector and CP detector. We also derived the distribution of the metric in the presence and absence of the primary user when the system is not perfectly synchronized. We showed that the loss in the detection performance is negligible when the system is impaired by frequency offsets in the practical range.

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