Designing Fuzzy Polynomial Gain Scheduled Three-Loop Autopilot For The Pitch Channel Of A Flying Vehicle

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Received: 2015/9/15 Accepted: 2015/11/22

Abstract

This paper presents a gain scheduled autopilot for pitch channel of a flying vehicle. The selected method is based on polynomial fuzzy systems. The method does not involve linearization about operating point. First the polynomial fuzzy model of pitch channel of the flight body is derived. Next, using polynomial fuzzy system methodology the controller is design such that the outputs of the nonlinear plant drive to follow those of a stable reference model. Because of avoiding actuator saturation, some constraints derived that guarantees the amplitude of control signals be less than a specific threshold. It is considered that the controller has a known structure like three-loop autopilot. In other words the three-loop fuzzy polynomial autopilot is design to satisfy stability and performance of the closed loop system over a wide range of parameter variation. Stability and performance conditions derived in terms of sum of square will solve numerically via SOSTOOLS.

Keywords: Gain scheduling; pitch channel; three-loop autopilot; Sum of squares; fuzzy polynomial modeling

Introduction

Flying vehicle autopilot design is an interesting problem for the researchers. It has a challenging nature duo to the fact that the closed loop stability and performance has to be satisfied over a wide range of flight conditions. Gain-scheduling techniques have been used extensively for the design of flying vehicle autopilot [1-3]. Classical gain scheduling is typically based on designing a set of linear controllers for a set of flight operating point [4, 5] . An interpolation method forms the entire controller according to the scheduling variables. In the modern methodologies such as linear parameter varying (LPV) framework the control problem are formulated as linear matrix inequalities (LMI) optimization problems which are then solved using semi-definite programming [6].

Fuzzy model based (FMB) control [7] presents a systematic method for control of nonlinear systems. One of the important types of the fuzzy systems used in FMB control approach is Takagi and Sugeno (T-S) fuzzy model [8]. Designing controller based on T-S fuzzy model was proposed in [9-11]. Model based fuzzy control can be seen as a form of gain-scheduling and have received some attention during the last years [10]. Designing fuzzy gain scheduled autopilot based on T-S fuzzy model of the flying vehicles that satisfies zero steady state error in the presence of disturbances and modeling errors was proposed in [12, 13]. Some robust control methodologies such as H_∞ control has been used to design gain scheduled controller [14] based on fuzzy model of flying vehicles.

Recently an extended class of T-S fuzzy systems known as polynomial fuzzy systems [15] has been developed. A polynomial fuzzy system is such as the T-S fuzzy system with this difference that in the consequent part of the fuzzy rules presence of polynomials on system and input matrices is accepted.

Also designing controller based on T-S fuzzy systems is parameterized in terms of a linear matrix inequality; polynomial fuzzy system approach leads to a sum of square problem. Sum of square formulation has been introduced as generalization of linear matrix inequalities [16] and has many applications in systems analysis and design [17].

Polynomial fuzzy model based control (PFMB) systems [15] and [18] is an extension of the fuzzy model based control systems. Researchers' attentions on polynomial fuzzy model based control have divided in two categories; stabilization and performance. Compared to the stabilization control problem[19, 20], there are a few works addressed the tracking control problem that a controller is employed to drive the system states of the nonlinear plant to follow a stable reference model [21].

In this paper we tailor the method proposed in [21] to design controller based on three-loop structure. Whereas the three-loop controller is a dynamical control structure the method presented in [21] cannot use directly. Moreover in [21] authors does not consider limitation on amplitude of control signals. Three-loop controller structure [22] is one of the most famous classical structures to implement controller for flying vehicles. The problem is to control the pitch-axis of flying vehicle to track reference normal acceleration Numerical value of controller gains will calculate using third-party MATLAB toolbox SOSTOOLS.

The paper is organized as follows. In Section 2, preliminaries, polynomial fuzzy model and polynomial fuzzy controller are introduced. In Section 3, Polynomial Fuzzy Modeling of the Flight Vehicle is

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described. Controller design and numerical simulations are done in Section 4. In Section 5 a brief mention to the fuzzy gain scheduling design based on TS fuzzy model has done. Finally, conclusion is presented in Section 6.

2. Preliminaries

Definition 2.1 [16]: A multivariate polynomial $f(x), x \in \mathbb{R}^n$, is a sum of squares if there exist polynomials $f_i(x), i = 1,\dots,m$ such that

$$f(x) = \sum_{i=1}^{m} f_i^{\,2}(x) \qquad (1)$$

If a decomposition of $f(x)$ in the form of (1) can be obtained, it is clear that $f(x) \geq 0$ for all $x \in \mathbb{R}^n$. The problem of finding the right hand side of Eq. (1) can be formulated in terms of the existence of a positive semi-definite matrix Q such that the following proposition holds:

Proposition 2.1 [16]: Let $f(x)$ is a polynomial in $x \in \mathbb{R}^n$ of degree $2d$. Let $Z(x)$ is a column vector whose entries are all monomials in x with degree smaller than d. A monomial in $x(t)$ is a function of the form $x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$, where $\alpha_1, \alpha_2, \dots, \alpha_n$, are nonnegative integers. Therefore, $f(x)$ is said to be an SOS if and only if there exists a positive semi definite matrix Q such that

$$f(x) = Z^T(x) Q Z(x) \qquad (2)$$

2.1Polynomial Fuzzy Model

Consider a fuzzy system with p rules that its **i**-th rule defined as [15]:

IF $f_1(y(t))$ is M_1^i AND ... AND $f_\psi(y(t))$ is M_ψ^i THEN

$$\begin{aligned}\dot{x}(t) &= A_i(x(t))\hat{x}(x(t)) + B_i(x(t))u(t)\\ y(t) &= C\hat{x}\big(x(t)\big)\end{aligned} \qquad (3)$$

where M_α^i for $\alpha = 1,2,\dots,\psi$ are the fuzzy terms corresponding to the function $f_\alpha(x(t))$; $x(t) \in \mathbb{R}^n$ denotes the system state vector; $y(t) \in \mathbb{R}^l$ denotes the output vector; $A_i(x(t)) \in \mathbb{R}^{n\times N}$, $B_i(x(t)) \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{l\times N}$ are the known polynomial system, input and output matrices, respectively. In (3) $\hat{x}(x(t)) \in \mathbb{R}^N$ denotes the vector of monomials in $x(t)$ and $u(t) \in \mathbb{R}^m$ denotes the input vector. It is assumed that $\hat{x}(x(t)) = 0$ if and only if $x(t) = 0$.

The system dynamics and output of the fuzzy system is inferred as:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{p} \omega_i(y(t))\,(A_i\big(x(t)\big)\hat{x}\big(x(t)\big) + B_i\big(x(t)\big)u(t))\\ y(t) &= C\hat{x}\big(x(t)\big)\end{aligned} \qquad (4)$$

where

$$\omega_i(y(t)) = \frac{\prod_{l=1}^{\psi} \mu_{M_l^i}\,(f_l(y(t)))}{\sum_{k=1}^{p}\prod_{l=1}^{\psi} \mu_{M_l^k}\,(f_l(y(t)))} \qquad (5)$$

$$\sum_{i=1}^{p} \omega_i(y(t)) = 1\,, \omega_i(y(t)) \geq 1 \quad \forall\, i$$

where $\mu_{M_\alpha^i}(f_l(y(t)))$ is the membership function corresponding to the fuzzy term M_α^i.

Remark 1: The traditional T-S fuzzy model is a special case of the system (4) if $A_i(x(t))$ and $B_i(x(t))$ are constant matrices for all i and $\hat{x}(x(t)) = x(t)$.

2.2 Polynomial Fuzzy Controller

Consider a stable reference model as:

$$\begin{aligned}\dot{x}_r(t) &= A_r\hat{x}_r(x_r(t)) + B_r r(t)\\ y_r(t) &= C\hat{x}_r(x_r(t))\end{aligned} \qquad (6)$$

where $x_r(t) \in \mathbb{R}^N$ is the state vector of the reference model, $\hat{x}_r(x_r(t)) \in \mathbb{R}^N$ is a vector of monomials in $x_r(t)$, $A_r \in \mathbb{R}^{n\times N}$ and $B_r \in \mathbb{R}^{n\times m}$, are the system and input matrices, respectively. In (6), $r(t) \in \mathbb{R}^m$ is the reference input vector and $y_r(t) \in \mathbb{R}^m$ is the output vector of the reference model.

Consider the state tracking error as:

$$\hat{e}(t) = \hat{x}(x(t)) - \hat{x}_r(x_r(t)) \qquad (7)$$

Therefore the output error can be defined as:

$$e_y(t) = y(t) - y_r(t) = C\hat{e}(t) \qquad (8)$$

The output feedback polynomial fuzzy controller is defined as

$$u(t) = \sum_{j=1}^{p} \omega_j(y(t))\, F_j(h(t)) e_y(t) + G_j(h(t)) y_r(t) \qquad (9)$$

where $h(t) = (y(t), y_r(t))$. In (9), $G_j(h(t)) \in \mathbb{R}^{m\times N}$ and $F_j(h(t)) \in \mathbb{R}^{m\times N}$ are the polynomial feedback gains to be determined. Define $X(\tilde{x}) \in \mathbb{R}^{N\times N}$

$$X(\tilde{x})^T = X(\tilde{x}) = \begin{bmatrix} X_{11} & \mathbf{0} \\ \mathbf{0} & X_{22}(\tilde{x}) \end{bmatrix} > 0 \qquad (10)$$

where $\tilde{x} = [x_{j_1}, x_{j_2}, \dots, x_{j_q}, x_{r_{k1}}, x_{r_{k2}}, \dots, x_{r_{k_s}}]^T$ and $j = [j_1, j_2, \dots, j_q]$ denotes the row indices that the entries of the entire row of $B_i(x)$ for all i are zeros. Similarly, $k = [k_1, k_2, \dots, k_s]$ denotes the row indices that the entries of the entire row of $B_r(x_r)$ are all zeros.

The polynomial feedback gains are defined as:

$$\begin{aligned}F_j(h) &= [M_j(h) \quad 0]X^{-1} = M_j(h){X_{11}}^{-1}\\ G_j(h) &= [N_j(h) \quad 0]X^{-1} = N_j(h){X_{11}}^{-1}\end{aligned} \qquad (11)$$

where $M_j(h) \in \mathbb{R}^{m\times l}$ and $N_j(h) \in \mathbb{R}^{m\times l}$.

Theorem 1: The output-feedback polynomial fuzzy controller (9) is able to drive the system states of the system (4) to follow those of the stable reference model (6) if there exists pre-defined SOS scalar polynomial functions $\varepsilon_1(\tilde{x})$ and $\varepsilon_2(x, x_r, w)$ and decision polynomial matrices $X(\tilde{x}) = X(\tilde{x})^T \in \mathbb{R}^{N\times N}$ in the form of (10), feedback gains $M_j(h) \in \mathbb{R}^{m\times l}$ and $N_j(h) \in \mathbb{R}^{m\times l}, j = 1,2,\dots,p$ such that

$$v^T\big(X(\tilde{x}) - \varepsilon_1(\tilde{x})\big)v \quad \text{is SOS}$$

$$-\rho^T \sum_{i=1}^{p}\sum_{j=1}^{p} \bar{\omega}_i{}^2\bar{\omega}_j{}^2 \Xi_{ij}(x,x_r) + \varepsilon_2(x,x_r,\omega))\rho \quad \text{is SOS}$$

where

$$\tilde{\Xi}_{ij}(x,x_r) = \begin{bmatrix} \Phi_{ij}^{(1)}(x,x_r) + \Phi_{ij}^{(1)}(x,x_r)^T + I & * \\ \Phi_{ij}^{(2)}(x,x_r)^T & -\sigma_1^2 I \\ \Phi_{ij}^{(3)}(x,x_r)^T & 0 \quad - \end{bmatrix} \quad (12)$$

and $\omega = [\bar{\omega}_1{}^2, \bar{\omega}_2{}^2, \dots, \bar{\omega}_p{}^2]$, $\rho \in \mathbb{R}^{2N+m}$ and $, v \in \mathbb{R}^N$ are arbitrary vectors independent of x and x_r. In (12) σ_1 and σ_2 are pre-defined scalars to be determined and

$$\Phi_{ij}^{(1)}(x,x_r) = \Gamma^{-1}\tilde{A}_i(x)\Gamma X(\tilde{x}) + \Gamma^{-1}\tilde{B}_i(x)[M_j(h)\ \ \mathbf{0}]$$

$$\Phi_{ij}^{(2)}(x,x_r) = \Gamma^{-1}\big(\tilde{A}_i(x) - \tilde{A}_r\big)\Gamma X(\tilde{x}) + \Gamma^{-1}\tilde{B}_i(x)[N_j(h)\ \ \mathbf{0}],$$

$$\Phi_{ij}^{(3)}(x,x_r) = \Gamma^{-1}\tilde{B}_r$$

where $\Gamma \in \mathbb{R}^{l\times l}$ and

$$\Gamma = [C^T(CC^T)^{-1} \quad ortc\,(C^T)] \quad (13)$$

Proof: see [21]. ∎

Using Corollary 1 can reduce the computational demand to find a feasible solution that satisfies the conditions of Theorem 1.

Corollary 1: [21] The output-feedback polynomial fuzzy controller (11) can drive the system states of the nonlinear plant in the form of (4) to follow those of the stable reference model (6) if there exists pre-defined SOS scalar polynomial functions $\varepsilon_1(\tilde{x})$ and $\varepsilon_2(x,x_r,w)$ and decision variables, i.e., polynomial matrices $X(\tilde{x}) = X(\tilde{x})^T \in \mathbb{R}^{N\times N}$ in the form of (10) $M_j(h) \in \mathbb{R}^{m\times l}$, $N_j(h) \in \mathbb{R}^{m\times l}, j = 1,2,\dots,p$ such that

$$v^T\big(X(\tilde{x}) - \varepsilon_1(\tilde{x})\big)v \quad \text{is SOS}$$

$$-\rho^T\Big(\Xi_{ij}(x,x_r) + \Xi_{ji}(x,x_r) + \varepsilon_2(x,x_r,w)\Big)\rho \quad \text{is SOS}$$

where $\rho \in \mathbb{R}^{2N+m}$ and $v \in \mathbb{R}^N$ are arbitrary vectors independent of x and x_r.

2.3 Constraint on control input and plant output magnitudes

In many applications, control signal magnitude saturation is one of the main sources of performance limitation. On the other hands, during transient response the system output has not to take any values duo to the sensor saturation. Therefore one has to considered the plant input and output magnitudes limitation in controller synthesis.

Assume that $V(t) = e^T(t)\,P(\tilde{x})e(t)$ is Lyapunov function and

$$e^T(0)Pe(0) < 1 \quad (14)$$

Using Schur complement we can write (14) as:

$$\begin{bmatrix} 1 & e^T(0) \\ e(0) & X(\tilde{x}) \end{bmatrix} > 0 \quad (15)$$

where $X = P^{-1}$. The constraint on the control input defined as:

$$\|u(t)\|_2 \le \beta \quad (16)$$

From (9) and (16):

$$u(t)^T u(t) = \sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)\omega_j(y)\{e_y{}^T F_i(h)^T F_j(h) e_y + y_r{}^T G_i(h)^T G_j(h) y_r + e_y{}^T F_i(h)^T G_j(h) y_r + y_r{}^T G_i(h)^T F_j(h) e_y\} \le \beta^2 \quad (17)$$

Therefore,

$$\frac{1}{\beta^2}\sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)\omega_j(y) [e_y^T \quad y_r^T]\begin{bmatrix} F_i(h)^T F_j(h) & F_i(h)^T G_j(h) \\ G_i(h)^T F_j(h) & G_i(h)^T G_j(h) \end{bmatrix}\begin{bmatrix} e_y \\ y_r \end{bmatrix} \le 1 \quad (18)$$

Eq. (18) can be written as:

$$\frac{1}{\beta^2}\sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)\omega_j(y) [e_y^T \quad y_r^T]\begin{bmatrix} F_i(h)^T \\ G_i(h)^T \end{bmatrix}[F_j(h) \quad G_j(h)]\begin{bmatrix} e_y \\ y_r \end{bmatrix} \le 1 \quad (19)$$

Defining $R_i = [F_i(h) \quad G_i(h)]$ and since $e^T(t)X^{-1}(\tilde{x})e(t) < e^T(0)X^{-1}(\tilde{x})e(0) \le 1$ for $t \ge 0$, if

$$\frac{1}{\beta^2}\sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)\omega_j(y)\,[e_y^T \quad y_r^T]R_i{}^T R_j\begin{bmatrix} e_y \\ y_r \end{bmatrix} \le e^T(t)X^{-1}(\tilde{x})e(t) \quad (20)$$

then (19) holds. In (20) $\bar{X}^{-1}$ Defined as $\bar{X}^{-1} = \begin{bmatrix} X^{-1}(\tilde{x}) & 0 \\ 0 & 0 \end{bmatrix}$. It can be seen from (20) that

$$\sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)(y)\omega_j(y) [e_y^T \quad y_r^T](\frac{1}{\beta^2}R_i{}^T R_j - \bar{X}^{-1})\begin{bmatrix} e_y \\ y_r \end{bmatrix} \le 0 \quad (21)$$

From the left hand side of the last equation:

$p \quad p$

(22)

$$\begin{bmatrix} e_y^T & y_r^T \end{bmatrix}\left(\frac{1}{\beta^2}R_i{}^TR_j + \frac{1}{\beta^2}R_j{}^TR_i - 2\bar{X}^{-1}\right)\begin{bmatrix} e_y \\ y_r \end{bmatrix} = \frac{1}{2}\sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)\omega_j(y)\begin{bmatrix} e_y^T & y_r^T \end{bmatrix}\left(\frac{1}{\beta^2}(R_i{}^TR_j + R_j{}^TR_i) - \frac{1}{\beta^2}\left((R_i{}^T - R_j{}^T)(R_i - R_j) - 2\bar{X}^{-1}\right)\begin{bmatrix} e_y \\ y_r(t) \end{bmatrix} \le \frac{1}{2}\sum_{j=1}^{p}\sum_{j=1}^{p}\omega_i(y)\omega_j(y)\begin{bmatrix} e_y^T & y_r^T \end{bmatrix}\left(\frac{1}{\beta^2}(R_i{}^TR_j + R_j{}^TR_i) - 2\bar{X}^{-1}\right)\begin{bmatrix} e_y \\ y_r(t) \end{bmatrix} = \sum_{i=1}^{p}\omega_i(y)\begin{bmatrix} e_y^T & y_r^T \end{bmatrix}\left(\frac{1}{\beta^2}R_i{}^TR_i - \bar{X}^{-1}\right)\begin{bmatrix} e_y \\ y_r(t) \end{bmatrix}$$

It can be seen from the last equation If

$$(\frac{1}{\beta^2}R_i{}^TR_i - \bar{X}^{-1}(\tilde{x})) \le 0 \qquad (23)$$

then (21) holds. Multiplying both side of (23) by $\bar{X}(\tilde{x})$ gives

$$(\frac{1}{\beta^2}\bar{X}^T(\tilde{x})R_i{}^TR_i\bar{X}(\tilde{x}) - \bar{X}(\tilde{x})) \le 0 \qquad (24)$$

Substituting $R_i = [F_i(h) \quad G_i(h)]$ in to (24) and applying Schur complement to the consequence results:

$$\begin{bmatrix} X(\tilde{x}) & 0 & M_i^T \\ 0 & 0 & N_i^T \\ M_i & N_i & \beta^2 I \end{bmatrix} \ge 0 \qquad (25)$$

Theorem 2: Assume that the initial condition $e(0)$ is known. The constraint $\|u(t)\|_2 \le \beta$ is enforced at all times $t \ge 0$, if

$$\begin{bmatrix} 1 & e^T(0) \\ e(0) & X(\tilde{x}) \end{bmatrix} \ge 0 \qquad (26)$$

$$\begin{bmatrix} X(\tilde{x}) & 0 & M_i^T \\ 0 & 0 & N_i^T \\ M_i & N_i & \beta^2 I \end{bmatrix} \ge 0 \quad \text{for } i = 1,2,\dots,p \qquad (27)$$

hold, where $X = P^{-1}$, $M_i = F_iX$ and $N_i = G_iX$.

Theorem 3: Assume that the initial condition $e(0)$ is known. The constraint $\|y(t)\|_2 \le \gamma$ is enforced at all times $t \ge 0$, if

$$\begin{bmatrix} 1 & e^T(0) \\ e(0) & X(\tilde{x}) \end{bmatrix} \ge 0 \qquad (28)$$

$$\begin{bmatrix} X(\tilde{x}) & XC^T \\ XC & \gamma^2 I \end{bmatrix} > 0 \qquad (29)$$

Proof: The proof is the same procedure as in Theorem 2. ■

3. Polynomial Fuzzy Modeling of the Flight Vehicle

Consider the longitudinal dynamics of the airframe shown in Figure 1 around its center of mass as [23]:

$$\dot{\alpha}(t) = K_\alpha M(t)C_n[\alpha(t),\delta(t),M(t)]\cos(\alpha(t)) + q(t)$$
$$\dot{q}(t) = K_q M^2(t)C_m[\alpha(t),\delta(t),M(t)] \qquad (30)$$

where α denotes the angle of attack, q denotes the pitch rate, δ denotes the deflection angle, M denotes velocity in terms of the Mach number. The command input in system (30) is the elevator defection angle and the output is the vertical acceleration that can be given by

$$\eta(t) = K_z M^2 C_n[\alpha(t),\delta(t),M(t)] \qquad (31)$$

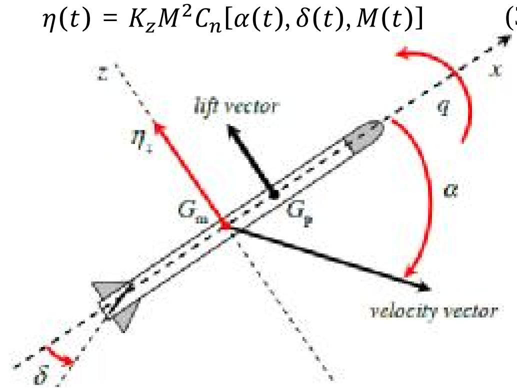

Figure 1. Flying vehicle pitch view

In (30) and (31) C_n and C_m are aerodynamic coefficients that are given by

$$C_n[\alpha,\delta,M] = C_{n\alpha}[\alpha,M]\alpha + C_{n\delta} * \delta \qquad (32)$$
$$C_m[\alpha,\delta,M] = C_{m\alpha}[\alpha,M]\alpha + C_{m\delta} * \delta \qquad (33)$$
$$C_{n\alpha}[\alpha,M] = (\frac{180}{\pi})^3 a_n\alpha^2 + (\frac{180}{\pi})^2 b_n|\alpha| + \frac{180}{\pi}c_n\left(2 - \frac{M}{3}\right) \qquad (34)$$
$$C_{m\alpha}[\alpha,M] = (\frac{180}{\pi})^3 a_m\alpha^2 + (\frac{180}{\pi})^2 b_m|\alpha| + \frac{180}{\pi}c_m\left(-7 + \frac{8M}{3}\right) \qquad (35)$$
$$C_{n\delta} = \frac{180}{\pi}d_n \quad , C_{m\delta} = \frac{180}{\pi}d_m \qquad (36)$$

It is noted that K_α, K_q and K_z are altitude dependent variables that it is assumed are constant. The numerical values of parameters in (32)-(36) are presented in Appendix A. The nonlinear model (30) is valid for $-20 < \alpha < +20$ and $1.5 < M < 3$. The performance goals for the closed-loop system are as:

- Maintain stability over the operating range specified by $(\alpha(t), M(t))$
- Tracking reference model in $\eta(t)$ with M constant and variable, and time constant no

greater than 0.7s, maximum overshoot no greater than 10%, and settling time is 1s.

- Maximum tail deflection rate for a 1g step command in $\eta(t)$ should not exceed 25 deg/s.

According to equation (30) and the state vector is $x = [\alpha \quad q]^T$, $\hat{x}_r(x) = x_r$ and $\hat{x}(x) = x$, the polynomial matrix of the system can be written as:

$$A_p = \begin{bmatrix} K_a M C_{na}\cos(\alpha) & 1 \\ K_q M^2 C_{ma} & 0 \end{bmatrix}$$
$$B_p = \begin{bmatrix} K_a M C_{nd}\cos(\alpha) \\ K_q M^2 C_{md} \end{bmatrix} \quad (37)$$

For the simplicity, one can replace non-polynomial term $|\alpha|$ with:

$$|\alpha| = 2.0863\alpha^2 + 0.10922 \quad (38)$$

The fuzzy form of the system could be written using sector nonlinearity [24]. It is considered premise variables of fuzzy controller rules as $f_1(z) = M$ and $f_2(z) = \cos(\alpha)$. It can be seen that $f_1(z) \in [1.5 \quad 3]$ and $f_2(z) \in [0.94 \quad 1]$. Based on sector nonlinearity the i'th rule of fuzzy model of system (30) can be written as:

$$\text{IF } z_1 \text{ is } M_1^i \text{ AND } z_2 \text{ is } M_2^i \text{ THEN}$$
$$\dot{x}(t) = A_i(x(t))\hat{x}(x(t)) + B_i(x(t))u(t) \quad (39)$$

The membership function of fuzzy sets M_j^i for i = j = 2 can be written as:

$$\mu_{M_1^1}(z_1) = \frac{f_1(z) - f_{1_{min}}}{f_{1_{max}} - f_{1_{min}}} = \frac{M - 1.5}{3 - 1.5}$$
$$\mu_{M_1^2}(z_1) = \frac{f_{1_{max}} - f_1(z)}{f_{1_{max}} - f_{1_{min}}} = \frac{3 - M}{3 - 1.5}$$
$$\mu_{M_2^1}(z_1) = \frac{f_2(z) - f_{2_{min}}}{f_{2_{max}} - f_{2_{min}}} = \frac{\cos(\alpha) - 0.9396}{1 - 0.9396} \quad (40)$$
$$\mu_{M_2^2}(z_1) = \frac{f_{2_{max}} - f_2(z)}{f_{2_{max}} - f_{2_{min}}} = \frac{1 - cos(\alpha)}{1 - 0.9396}$$

and

$$A_1 = \begin{bmatrix} K_a M_{max} C_{na}|_{M_{max}}\cos(\alpha)_{max} & 1 \\ K_q {M_{max}}^2 C_{ma}|_{M_{max}} & 0 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} K_a M_{max} C_{nd}\cos(\alpha)_{max} \\ K_q {M_{max}}^2 C_{md} \end{bmatrix}$$
$$C_1 = \begin{bmatrix} K_z {M_{max}}^2 C_{na}|_{M_{max}} & 0 \end{bmatrix};$$
$$D_1 = K_z {M_{max}}^2 C_{nd}$$
$$A_2 = \begin{bmatrix} K_a M_{max} C_{na}|_{M_{max}}\cos(\alpha)_{min} & 1 \\ K_q {M_{max}}^2 C_{ma}|_{M_{max}} & 0 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} K_a M_{max} C_{nd}\cos(\alpha)_{min} \\ K_q {M_{max}}^2 C_{md} \end{bmatrix}$$
$$C_2 = C_1; D_2 = D_1 \quad (41)$$
$$A_3 = \begin{bmatrix} K_a M_{min} C_{na}|_{M_{min}}\cos(\alpha)_{max} & 1 \\ K_q {M_{min}}^2 C_{ma}|_{M_{min}} & 0 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} K_a M_{min} C_{nd}\cos(\alpha)_{max} \\ K_q {M_{min}}^2 C_{md} \end{bmatrix}$$
$$C_3 = \begin{bmatrix} K_z {M_{min}}^2 C_{na}|_{M_{min}} & 0 \end{bmatrix}$$
$$D_3 = \begin{bmatrix} K_z {M_{min}}^2 C_{nd} \end{bmatrix}$$
$$A_4 = \begin{bmatrix} K_a M_{min} C_{na}|_{M_{min}}\cos(\alpha)_{min} & 1 \\ K_q {M_{min}}^2 C_{ma}|_{M_{min}} & 0 \end{bmatrix}$$
$$B_4 = \begin{bmatrix} K_a M_{min} C_{nd}\cos(\alpha)_{min} \\ K_q {M_{min}}^2 C_{md} \end{bmatrix}$$
$$C_4 = C_3; D_4 = D_3$$

4. Controller Design

The structure intended for the controller is shown in Figure 2. This configuration is known as the three-loop controller [22]. According to Figure 2, we encounter dynamic feedback controller design problem; whereas the design procedure described in Section 2.2 has developed to design the static output feedback controller. To overcome this difficulty we split the controller to the two parts; the controller gains and the controller structure. The fixed parts of the controller and the sensor block are augmented to the plant model. It can be seen that the resulting augmented systems has 3 inputs and 2 outputs.

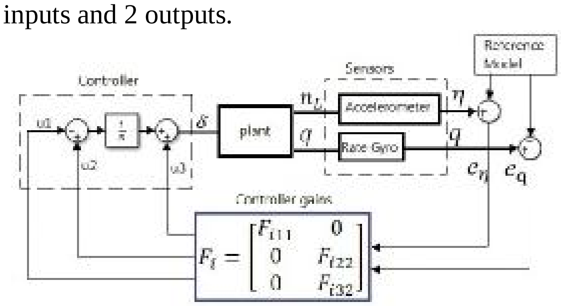

Figure 2. Closed loop diagram of the pitch channel

Assume that the sensor block shown in Figure 2 has the following dynamical model

$$\begin{cases} \dot{x_s} = A_s x_s + B_s y \\ y_s = C_f x_s + D_s y \end{cases} \quad (42)$$
$$y = [n_L \quad q],\ y_s = [\eta \quad q],$$

where x_s is the state vector, y is the physical output of the plant and y_s is the output of the sensor block.

We assume the state space models of the sensor block as:

$$A_s = \begin{bmatrix} -600 & 0 \\ 0 & 0 \end{bmatrix}$$
$$B_s = \begin{bmatrix} 600 & 0 \\ 0 & 1 \end{bmatrix}$$
$$C_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (43)$$
$$D_s = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Similarly, the controller block shown in Figure 2 can be written as:

$$\begin{cases} \dot{x_c} = A_c x_c + B_c u \\ u_c = C_c x_c + D_c u \end{cases}$$
$$u = [u_1 \quad u_2 \quad u_3]^T$$
$$A_c = 0 \quad (44)$$
$$B_c = [-1 \quad 1 \quad 0]$$
$$C_c = 1$$
$$D_c = [0 \quad 0 \quad 1]$$

where x_c is the state of the controller and $u_c = \delta$ is the control signal. Using (39), (41) and (42), (44) the augmented system matrices can be driven as:

$$A = \begin{bmatrix} A_{pi} & B_{pi} C_c \\ \mathbf{0}_{1\times 3} & A_c \end{bmatrix}, B = \begin{bmatrix} B_{pi} D_c \\ B_c \end{bmatrix},$$
$$C = [C_{pi} \quad D_{pi} C_c], D = D_{pi} D_c \quad (45)$$

where

$$A_{pi} = \begin{bmatrix} A_i & \mathbf{0}_{2\times 1} \\ B_s C_i & A_s \end{bmatrix}, B_{pi} = \begin{bmatrix} B_i \\ B_s D_i \end{bmatrix},$$
$$C_{pi} = \begin{bmatrix} D_s C_i & C_s \\ [\mathbf{0}_{1\times 1} \quad 1] & \mathbf{0}_{1\times 1} \end{bmatrix}, D_{pi} \quad (46)$$
$$= \begin{bmatrix} D_s D_i \\ \mathbf{0}_{1\times 1} \end{bmatrix}$$

Using (13), Γ calculates as:

$$\Gamma = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (47)$$

On the other hands a stable reference model in the form of (6) can be chosen as a linear time invariant system with four states that its system and input matrices are:

$$A_r = \begin{bmatrix} -600 & -877 & -877 & -200(\\ 0 & -16.35 & -16.35 & -43. \\ -0.096 & -0.08 & 0 & (\\ 0 & 0.9698 & -0.03023 & -0.5 \end{bmatrix}$$
$$B_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.096 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C \quad (48)$$
$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$D = 0$$

All the rows of the matrix B are non-zero but first, second and fourth rows of B_r are all zeros. So $\tilde{x}$ in Eq. (10) chooses as $\tilde{x} = \left[x_{r_1}, x_{r_2}, x_{r_4}\right]^T$. $X(\tilde{x})$ chooses as a constant matrix and $M_j(h)$ and $N_j(h)$ for all j as polynomial matrices with degree of 2. Selecting $\sigma_1 = 25$, $\sigma_2 = 25$, $\varepsilon_1 = 0.001$ and $\varepsilon_2 = 0.001$ and employing Corollary 1 to determine the controller gains leads to a feasible solution and the controller gains obtain as:

$$F_i = \begin{bmatrix} F_i(1,1) & 0 \\ 0 & F_i(2,2) \\ 0 & F_i(3,2) \end{bmatrix} \quad (49)$$
$$G_i = \mathbf{0}$$
$$\text{for } j = 1, \dots, 4$$

F1(1,1)= 0.08984

F1(2,2)= $7.344.$

F1(3,2)= $3.724q^2 + 3.724\eta^2 + 118.8$

F2(1,1)= 0.2747

F2(2,2)= 21.6

F2(3,2)= $1.924q^2 + 1.924\eta^2 + 61.46$

F3(1,1)= 0.2847

F3(2,2)= 22.6

F3(3,2)= $1.924q^2 + 1.934\eta^2 + 61.36$

F4(1,1)= 0.2747

F4(2,2)= 20.6

F4(3,2)= $1.934q^2 + 1.824\eta^2 + 62.36$

Using controller gains (49) it can be constructed controller based on (9). In consequence simulations are done for several flight conditions. Because in this paper the flight condition is relates to the Mach, we select $M \in \{1.5, 2.3, 3\}$.

Figure 3 to Figure 5 show the behavior of the closed loop system for Mach of 3.

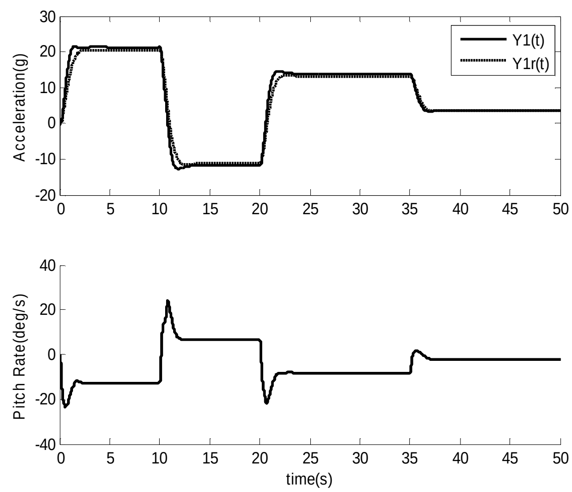

Figure 3. Output tracking for Mach 3

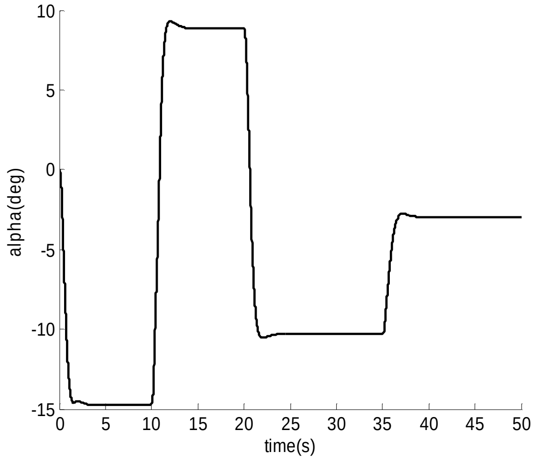

Figure 4. Angle of attack response for Mach 3

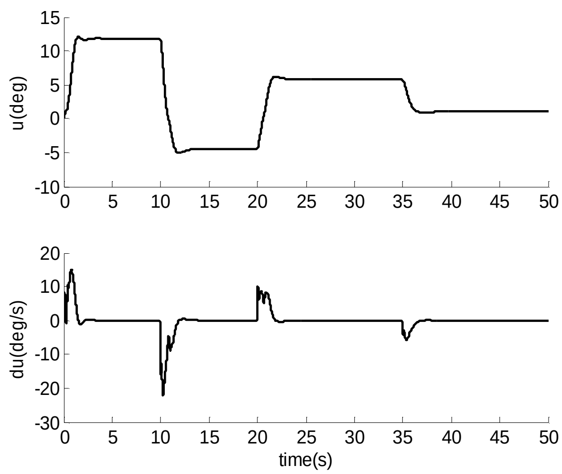

Figure 5. Control signal variations for Mach 3

It can be seen from Figure 3 to Figure 5 that the system output of the fly vehicle is able to follow stable reference model.

In Figure 6 to Figure 8 the results for Mach of 2.3 have been shown.

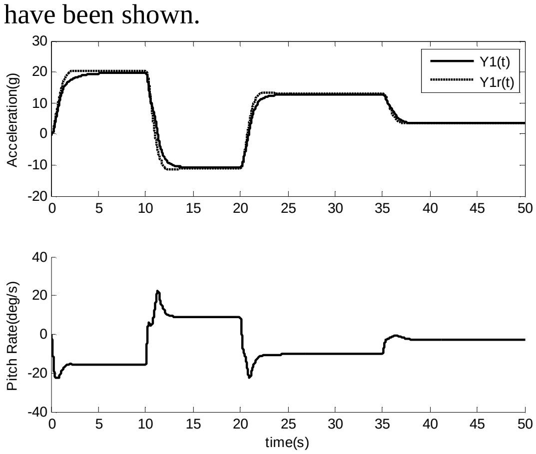

Figure 6. Output tracking for Mach 2.3

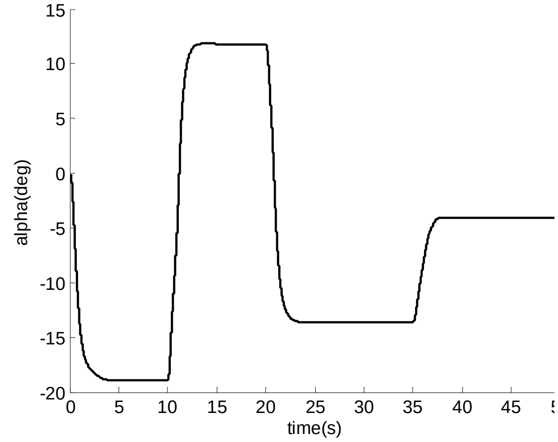

Figure 7. Angle of attack response for Mach 2.3

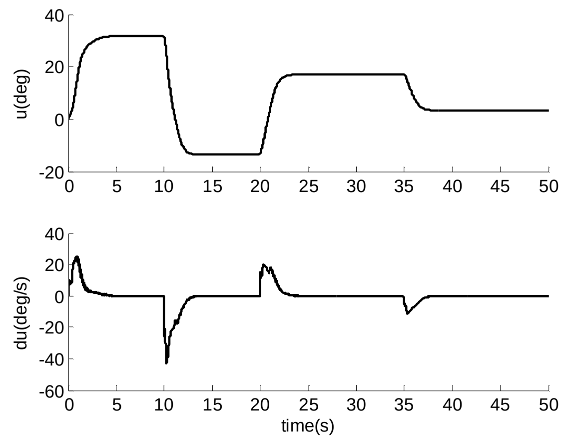

Figure 8. Control signal variations for Mach 2.3

It can be seen from Figure 6 to Figure 8 that the tracking of the acceleration reference in the second operating condition is done very well.

In Figure 9 to Figure 11 simulation results for Mach of 1.5 have been shown.

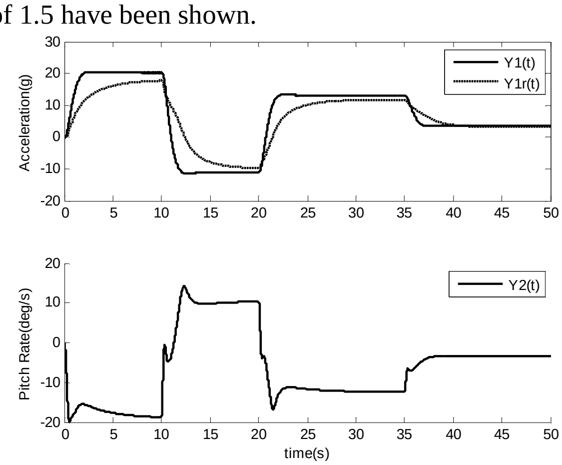

Figure 9. Output tracking for Mach 1.5

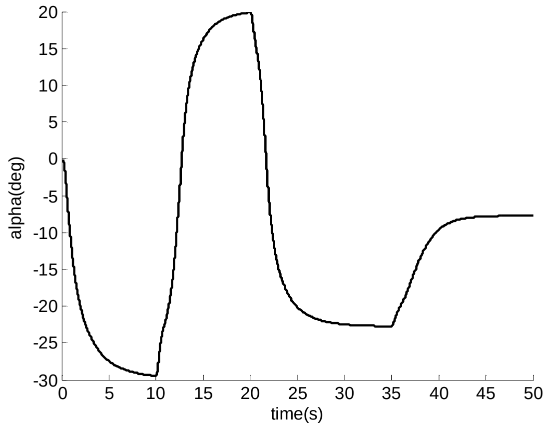

Figure 10. Angle of attack response for Mach 1.5

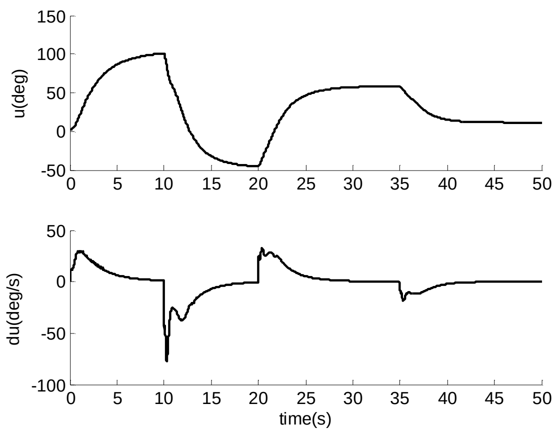

Figure 11. Control signal variations for Mach 1.5

It can be seen from Figure 9 to Figure 11 that the system is stabilized via the SOS designed controller and follow stable reference model.

In addition to the analysis of the closed loop system in frozen operating points, we consider a scenario that the velocity of the flying vehicle changes as in

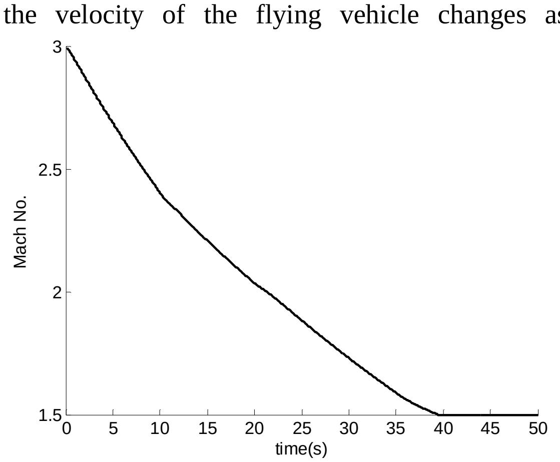

Figure 12.

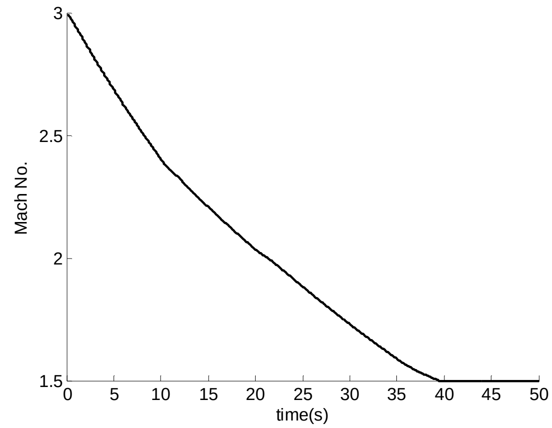

Figure 12. Dynamic speed (mach)

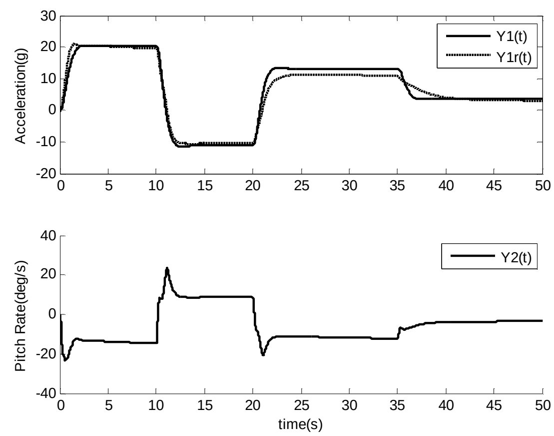

Figure 13. Output tracking in variable operating point scenario

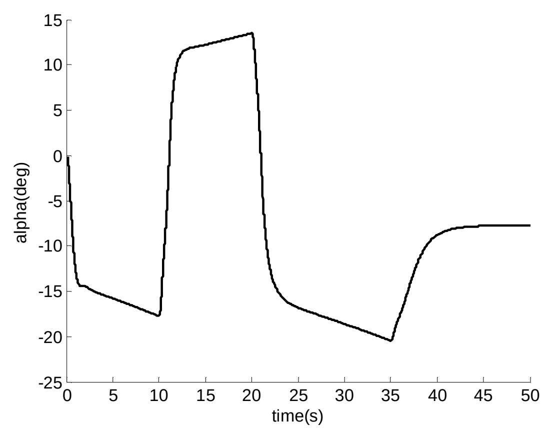

Figure 14. Angle of attack response in variable operating point scenario.

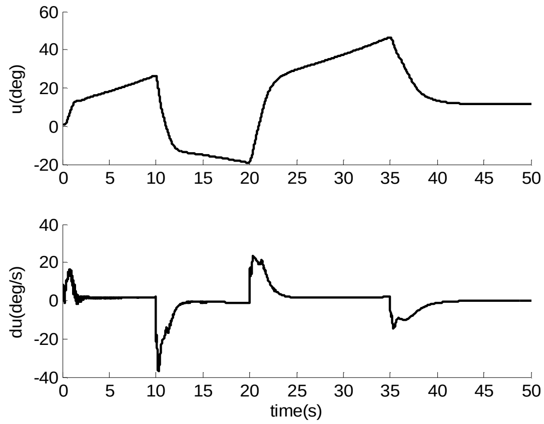

Figure 15. Control signal variations in variable operating point scenario

It can be seen from Figure 13 to Figure 15 that the stability and performance of the closed loop system remains acceptable.

From the practical point of view, a closed loop system is encounter with the limitations of magnitude of control input. Therefore duo to avoidance of actuator saturation it is important to consider control signal amplitude limitations in controller design procedure.

Whereas, $u_c = \delta = u_3 + x_c$ applying constraint on magnitude of control signal leads to applying input constraint on (u_3) and output constraint on x_c simultaneously. Therefore Theorem 2 and Theorem 3 are employed to design the controller. For this purpose we choose $\beta = 6$ and $\gamma = 40$. Simulation shown in Figure 16 and Figure 17 for variable operating point scenario implicates the effectiveness of the results of Theorem 2 and Theorem 3.

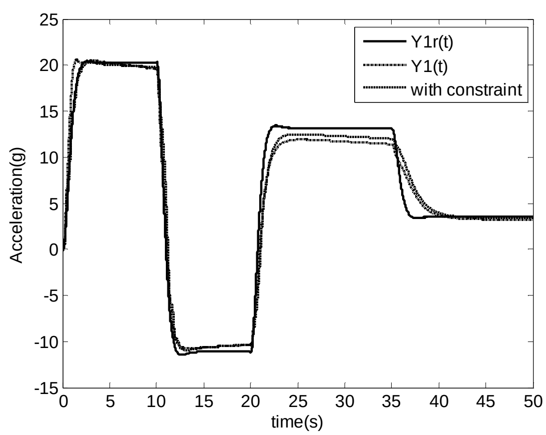

Figure 16. Output tracking with constraint on control signal

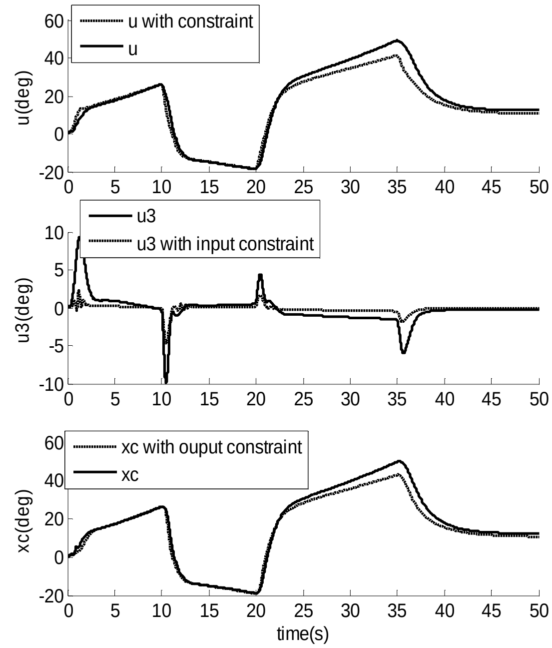

Figure 17. Constraint on control signal

Figure 16 shows the response of the closed loop system. Although the performance of the closed loop system has no effective change, decrement in the magnitude of the control signal is seen obviously.

5. Fuzzy gain scheduling based on TS model

For comparison purposes, we construct a T-S fuzzy model to represent the nonlinear plant (30). Based on the sector nonlinearity concept, the T-S fuzzy model that exactly represents plant (30) derives as:

$$\dot{x} = \sum_{i=1}^{8} \omega_i(z)[A_i x + B_i u] \qquad (50)$$

where $z = [M \ \cos\alpha \ \alpha^2]^T$. It can be seen from (50) that T-S fuzzy model of the plant has 8 rules with the system and input matrices that are shown in Appendix B. The operating range of $f_3(z) = \alpha^2$ is considered as $f_3(z) \in [0 \quad 400]$. It can be seen that the fuzzy polynomial model based control approach demonstrates an enhanced feedback compensation capability with less number of rules.

6. Conclusion

In this paper a fuzzy gain scheduling autopilot designed for a flying vehicle. The design autopilot performed based on polynomial fuzzy system theory. From theoretical points of view, polynomial fuzzy method is less conservative than T-S fuzzy system. The used method guaranteed the stability and performance of the closed loop system over a wide range of operation. Some SOS conditions that applying constraint on the magnitude of control signal derived for the prescribed control method. The method tailored for a famous classical autopilot structure known as three-loop autopilot. Using third-party MATLAB toolbox, SOSTOOLS, a feasible solution that satisfied the stability and performance have been obtained. Simulation confirmed the ability of controller to derive the system states to follow those of the stable reference model.

Appendix A

Table 1: details of pitch axis Flight vehicle model

k_α =	$(0.7)P_0S/mv_s$	
k_q =	$(0.7)P_0Sd/I_y$	
k_z =	$(0.7)P_0S/m$	
A_x =	$(0.7)P_0SC_a/m$	
P_0 =	$973.3\ lbf/ft^2$	Static pressure at 20000ft
S =	$0.44\ ft^2$	Surface Area
m =	$13.98\ slugs$	Mass
v_s =	$1036.4\ ft/s$	Speed of sound at 20000ft
d =	$0.75\ ft$	Diameter
I_y =	$182.5\ slug.ft^2$	Pitch moment of inertia
C_a =	-0.3	Drag coefficient
ζ =	0.7	Actuator damping ratio
a_n =	$0.000103\ deg^{-3}$	$a_m = 0.000215\ deg^{-3}$
b_n =	$-0.00945\ deg^{-2}$	$b_m = -0.0195\ deg^{-2}$
c_n =	$-0.1696\ deg^{-1}$	$c_m = 0.051\ deg^{-1}$
d_n =	$-0.034\ deg^{-1}$	$b_m = -0.206\ deg^{-1}$

Appendix B

TS fuzzy model for flight vehicle:

$$A_1 = \begin{bmatrix} -0.52 & 1 & 0 & -0.05 \\ -43.69 & 0 & 0 & -32.71 \\ 1.61\times 10^4 & 0 & -600 & 1752.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 & 0 & -5.68 \\ 0 & 0 & -3271.5 \\ 0 & 0 & 1752.20 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.55 & 1 & 0 & -0.06 \\ -43.69 & 0 & 0 & -32.71 \\ 1.61\times 10^4 & 0 & -600 & 1752.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 & -6.04 \\ 0 & 0 & -3271.5 \\ 0 & 0 & 1752.20 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -563.53 & 1 & 0 & -0.06 \\ -1.03\times 10^5 & 0 & 0 & -32.71 \\ 1.63\times 10^7 & 0 & -600 & 1752.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_3 = \begin{bmatrix} 0 & 0 & -6.04 \\ 0 & 0 & -3271.5 \\ 0 & 0 & 1752.20 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -529.49 & 1 & 0 & -0.05 \\ -1.03\times 10^5 & 0 & 0 & -32.71 \\ 1.63\times 10^7 & 0 & -600 & 1752.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_4 = \begin{bmatrix} 0 & 0 & -6.04 \\ 0 & 0 & -3271.5 \\ 0 & 0 & 1752.20 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} -0.07 & 1 & 0 & -0.01 \\ -45.17 & 0 & 0 & -130.86 \\ 4.71\times 10^4 & 0 & -600 & 7.00\times 10^3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_5 = \begin{bmatrix} 0 & 0 & -11.36 \\ 0 & 0 & -1.30\times 10^4 \\ 0 & 0 & 7.08\times 10^3 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} -0.081 & 1 & 0 & -0.01 \\ -45.17 & 0 & 0 & -130.86 \\ 4.71\times 10^4 & 0 & -600 & 7.00\times 10^3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_6 = \begin{bmatrix} 0 & 0 & -12.09 \\ 0 & 0 & -1.30\times 10^4 \\ 0 & 0 & 7.08\times 10^3 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} -1126.8 & 1 & 0 & -0.12 \\ -4.13\times 10^5 & 0 & 0 & -130.86 \\ 6.53\times 10^7 & 0 & -600 & 1752.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_7 = \begin{bmatrix} 0 & 0 & -12.09 \\ 0 & 0 & -1.30\times 10^4 \\ 0 & 0 & 7.08\times 10^3 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A_8 = \begin{bmatrix} -1058.7 & 1 & 0 & -0.11 \\ -4.13\times 10^5 & 0 & 0 & -130.86 \\ 6.53\times 10^7 & 0 & -600 & 7.00\times 10^3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B_8 = \begin{bmatrix} 0 & 0 & -11.36 \\ 0 & 0 & -1.30\times 10^4 \\ 0 & 0 & 7.08\times 10^3 \\ -1 & 1 & 0 \end{bmatrix}$$

$$C_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D_i = 0$$

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