

# Analysis the Performance of Controllers for He-Ne Laser Stabilization by Combination of Frequency Locking and Power Balanced Methods for Nano-Metrology Applications

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## Abstract

In this paper, analysis the performance of PI, PI-like fuzzy, and parallel fuzzy P+ fuzzy I controllers for He-Ne lasers frequency stabilization by combination of frequency locking and power balanced methods is presented. He-Ne lasers can be attributed to an unstable system due to the influence of environmental factors on its' frequency. Therefore, the stabilization of He-Ne laser is so important in sensitive applications such as laser interferometers and nanometrology systems. The simulation results of controllers by powerful software MATLAB/SIMULINK-GUI show that parallel fuzzy P+ fuzzy I controller has better stabilization performance and integrated absolute error (IAE) than others. Also, frequency fluctuations of He-Ne laser is about  $2 \times 10^{-11}$  by parallel fuzzy P+ fuzzy I controller.

**Keywords:** Cavity Length, Controller, He-Ne Lasers, Nanometrology, Thermal Instability.

## 1. Introduction

He-Ne lasers have been noticed because of their capabilities, abilities and advantages in many fields such as industry, medical and laboratory researches in recent years. In spite of having these features due to very complexity and influenced by environmental conditions is an unstable system. The frequency of He-Ne lasers tend to fluctuate because of thermal instability and this factor changes laser cavity length. Therefore, stability of output power of He-Ne lasers is necessary in sensitive applications such as laser interferometers and nanometrology systems.

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Generally, in using of interferometry for measuring any physical quantity, changing unwanted frequency causes changing unwanted phase difference and as a result, limits the measurement accuracy.

The PI control loop and fuzzy controller is widely used in many industrial applications. The PI controller is the most popular controller among industry process that includes a proportional gain  $K_p$ , an integral time constant  $T_i$ . The main reason for using PI controller in industrial applications is due to low cost, simplicity of performance and ease of design [1-4]. Recently, a number of control structures have been researched by real-time tuning the parameters of controllers [3, 4].

Fuzzy logic controller has emerged as one of the most useful research areas in the fuzzy control theory. Some tuning methods have been published before to obtain appropriate controller parameters based on system model. Internal model control structure provides a successful response based on its estimated model [5].

Several methods have been proposed for the stabilization of He-Ne lasers. At first, Suh *et al.* presented three-mode He-Ne laser stabilizer in 1993 [6]. The high frequency stabilization method was introduced by Yeom and Yoon in 2005 [7].

This paper proposes the parallel fuzzy P+ fuzzy I controller for stabilizing He-Ne lasers and then compares the performance of this controller to PI, and PI-like fuzzy controller controllers. In section 2, we describe basis of He-Ne lasers operation. In section 3, analysis and simulation results of parallel fuzzy P+ fuzzy I controller performance for stabilizing He-Ne lasers compared to other controllers is expressed. Finally, the conclusion is discussed in section 4.

## 2. He-Ne Laser Principles

The internal-mirror gas laser is a tube that filled with proper atoms and enclosed by two highly reflective mirrors and produces coherent light (e.g. wavelength  $\lambda=633\text{nm}$  for He-Ne laser) [8-10]. The mirrors form a resonant cavity whose length  $L$  determines the exact frequencies of emitted light through the condition that only exact integer multiples of a half-wavelength can resonate [8]. The relationships between mode wavelength  $\lambda$  and cavity length of laser  $L$ , mode frequency  $\nu$  and cavity length of laser  $L$  are described as follow:

$$L = \frac{n\lambda}{2} \quad (1)$$

$$\nu = \frac{nc}{2L} \quad (2)$$

The difference in frequency  $\Delta\nu$ , between successive resonant modes is given by:

$$\Delta\nu = \nu_{n+1} - \nu_n = \frac{c}{2L} \quad (3)$$

According to Eq. (1), by increasing cavity length of laser, the difference in frequency decreases. In the



output signal that is sent back into the process to match its output to the desired value [15]. A widely implemented feedback system is proportional, integral control (PI Controller), which is used extensively in industry [16]. The two elements of PI control respond differently to the error signal [16]. The proportional component produces a response proportional to the magnitude of the error signal and is responsible for reducing much of the error signal. The final output of a proportional-only system results in a steady-state error, a constant difference between the desired value and the actual output, because the proportional component needs some error signal to function [15-16]. The integral component accumulates the error signal over time and removes the steady-state error [15-16]. The output of PI controller is proportional to coefficient of input error plus integral of input error. If  $e(t)$  is input error and  $C(t)$  is output value, PI controller can be modeled by:

$$C(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_0^t e(t) dt \right] \tag{6}$$

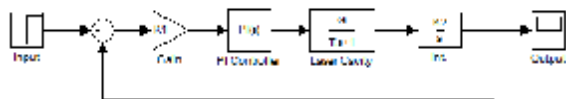
where,  $K_p$  and  $T_i$  are the proportional gain, and integral time constant, respectively. The block diagram of cavity laser and its stabilizer by using PI controller is shown in Fig. 3. The result of stabilization by PI controller to the step input is shown in Fig. 4.  $K_1$ , and  $K_2$  are phase comparator gain and VCO gain, respectively. We assume that  $K_1$ , and  $K_2$  are equal to 1.6 and 1M, respectively. The transfer function of PI controller is as follow:

$$\frac{C(s)}{E(s)} = K_p \left( 1 + \frac{1}{T_i s} \right) \tag{7}$$

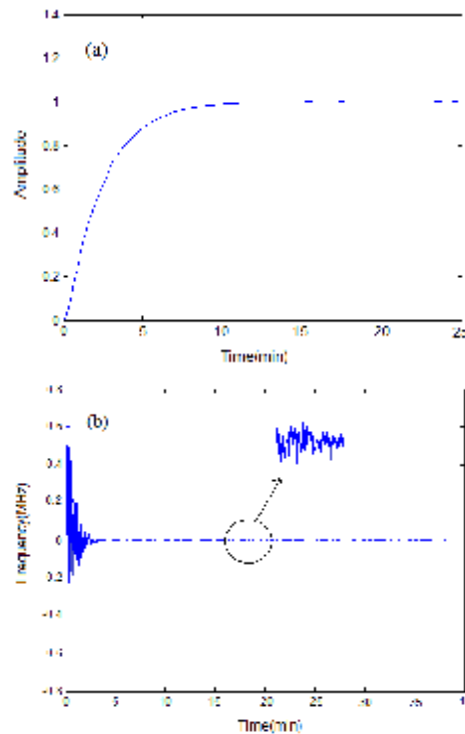
The frequency instability can be estimated from the error signal fluctuations which it is confined to about 150kHz, theoretically. It means that the frequency instability is about  $3 \times 10^{-11}$ , approximately.

**3.2. PI-like Fuzzy Controller**

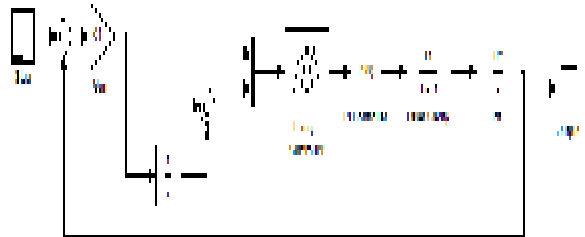
The PI-like fuzzy controller is the combination of PI and fuzzy controllers. The error and error changing rate are used as the input variables in control system. The block diagram of cavity laser and its stabilizer by using PI-like fuzzy controller is shown in Fig. 5. The input and output membership functions of fuzzy controller that defined in Simulink model as shown in Fig. 6 (N: negative; Z: zero and P: positive). Also, the result of stabilization by PI-like fuzzy controller to the step input is presented in Fig. 7.



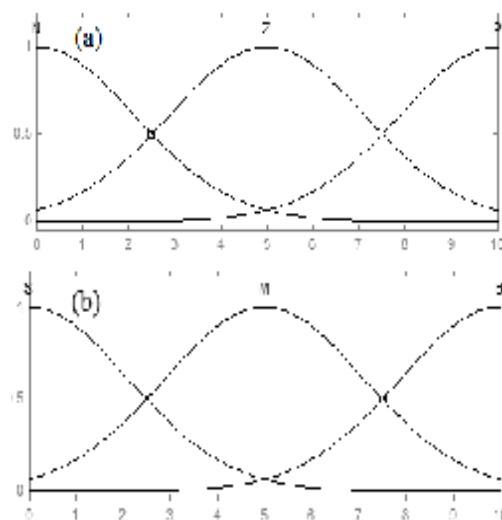
**Figure 3.** The block diagram of laser cavity and its stabilizer by PI controller



**Figure 4.** The result of stabilization by PI controller, (a) response to the step and (b) laser frequency fluctuations



**Figure 5.** The block diagram of laser cavity and its stabilizer by PI-like fuzzy controller



**Figure 6.** The membership functions, (a) input membership function and (b) output membership function

The frequency is confined to about 130kHz, theoretically. It means that the frequency instability is

about  $2.3 \times 10^{-11}$ , approximately.

### 3.3. Parallel Fuzzy P+ Fuzzy I Controller Structure

The fuzzy P+ fuzzy I control action is obtained by summing fuzzy P control action, and fuzzy I control action. Now, we review the fuzzy P control action and fuzzy I control action, respectively. The output of the conventional P controller is as follow:

$$u_p(t) = K_p e(t) \quad (8)$$

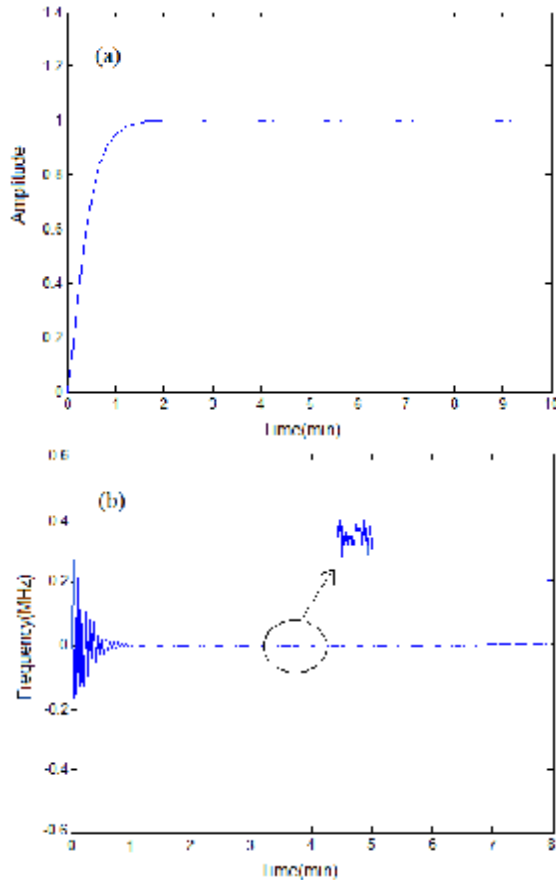
where,  $u_p(t)$ ,  $K_p$  and  $e(t)$  are output of conventional P controller, proportional constant and error signal, respectively. The discrete version of Eq. (8) is:

$$u_p(z) = K_p e(z) \quad (9)$$

By taking inverse z-transform of Eq. (9), we have:

$$u_p(nT) = K_p e(nT) \quad (10)$$

The  $K\Delta e(t)$  should be added to above equation. Then the Eq. (10) is converted to:



**Figure 7.** The result of stabilization by PI-like fuzzy controller, (a) response to the step and (b) laser frequency fluctuations

$$u_p(nT) = K_p e(nT) + K \Delta(nT) \quad (11)$$

where,  $\Delta e(nT) = e(nT) - e(nT-T)$  and  $K$  is a constant. Therefore, we change  $K_p$  and  $K$  coefficient to  $K_{p1}$  and  $K_{p2}$ , respectively and rewrite above equation as:

$$u_p(nT) = K_{p1} e(nT) + K_{p2} \Delta(nT) \quad (12)$$

In order to increase the degree of freedom, an additional gain  $G_p$  multiply in  $u_p(nT)$ . So, the fuzzy P

control action is as follow:

$$u_p(nT) = G_p u_p(nT) \quad (13)$$

The output of the conventional I controller is as follow:

$$u_i(t) = K_i \int e(t) dt \quad (14)$$

where,  $u_i(t)$ ,  $K_i$  and  $e(t)$  are output of conventional I controller, integral constant and error signal, respectively. In frequency s-domain, the above equation is converted to:

$$u_i(s) = \frac{K_i}{s} e(s) \quad (15)$$

The discrete version of equation 15 by applying transformation  $s = (2/T)(z-1)/(z+1)$  is as follow:

$$u_i(z) = \left( -\frac{K_i T}{2} + \frac{K_i T}{1-z^{-1}} \right) e(z) \quad (16)$$

where,  $T > 0$  is sampling time. By taking inverse z-transform of equation 16, we have:

$$u_i(nT) - u_i(nT-T) = K_i T e(nT) - \frac{K_i T}{2} (e(nT) - e(nT-T)) \quad (17)$$

Therefore, we change  $K_i T$  and  $K_i T/2$  coefficient to  $K_{i1}$  and  $K_{i2}$ , respectively and rewrite above equation as:

$$u_i(nT) - u_i(nT-T) = K_{i1} e(nT) - K_{i2} (e(nT) - e(nT-T)) \quad (18)$$

By dividing the above equation to  $T$ , we have:

$$\frac{u_i(nT) - u_i(nT-T)}{T} = \frac{K_{i1}}{T} e(nT) - K_{i2} \left( \frac{e(nT) - e(nT-T)}{T} \right) \quad (19)$$

we rewrite the Eq. (19) as:

$$\Delta u_i(nT) = K_{i1} e(nT) - K_{i2} r(nT) \quad (20)$$

where,  $\Delta u_i(nT)$ ,  $e(nT)$  and  $r(nT)$  are incremental control output of conventional integral controller, error signal and rate of change of the error signal.  $K_{i1}$  is equal to  $K_i/T$ . According to Eqs. (19) and (20),  $\Delta u_i(nT)$  is equal to  $(u_i(nT) - u_i(nT-T))/T$ . Then we have:

$$u_i(nT) = u_i(nT-T) + T \Delta u_i(nT) \quad (21)$$

The term  $T \Delta u_i(nT)$  is replaced by the fuzzy control action as follow:

$$u_i(nT) = u_i(nT-T) + G_i \Delta u_i(nT) \quad (22)$$

where,  $G_i$  is a fuzzy I controller gain. Finally, the fuzzy P+ fuzzy I control action is:

$$u_{PI}(nT) = u_p(nT) + u_i(nT) \quad (23)$$

$$u_{PI}(nT) = G_p u_p(nT) + u_i(nT-T) + G_i \Delta u_i(nT)$$

In this controller,  $K_{p1}e(nT)$  and  $K_{p2}\Delta e(nT)$  are the fuzzy p controller inputs,  $K_{i1}e(nT)$  and  $K_{i2}r(nT)$  are the fuzzy I controller inputs.  $u_p(nT)$ , and  $\Delta u_i(nT)$  are outputs. The block diagram of cavity laser and its stabilizer by using parallel fuzzy P+ fuzzy I controller

is shown in Fig. 8. The input and output membership functions of parallel fuzzy P+ fuzzy I controller that defined in Simulink model display in Fig. 9(n: negative; p: positive; on: output negative; oz: output zero; op: output positive). The fuzzy control rules for fuzzy P controller and fuzzy I controller are selected as shown in Table 1 and 2, respectively. Also, the result of stabilization by parallel fuzzy P+ fuzzy I controller to the step input is presented in Fig. 10.



Figure 8. The block diagram of laser cavity and its stabilizer by parallel fuzzy P+ fuzzy I controller

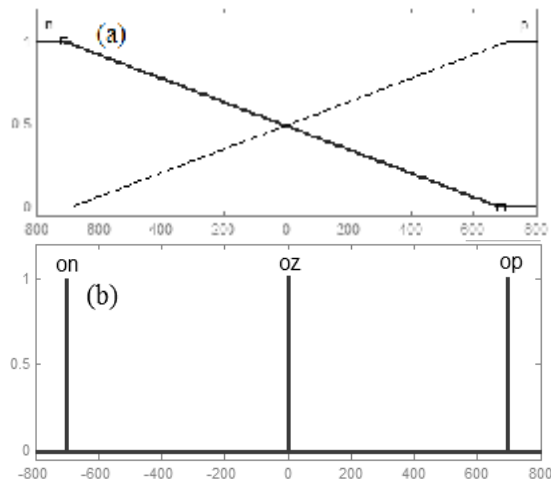


Figure 9. Membership functions, (a) input membership functions and (b) output membership functions

TABLE 1

Fuzzy control rules for the fuzzy P controller		
$K_{P1}e(nT)$	$K_{P2}\Delta e(nT)$	$u_P(nT)$
n	n	z
n	p	p
p	n	n
p	p	z

TABLE 2

Fuzzy control rules for the fuzzy I controller		
$K_{I1}e(nT)$	$K_{I2}e(nT)$	$\Delta u_I(nT)$
n	n	n
n	p	z
p	n	z
p	p	p

The frequency is confined to about 100kHz, theoretically. It means that the frequency instability is about  $2 \times 10^{-11}$ , approximately. Also, the value of IAE by different controllers is shown in Fig. 11. The

results show that the system modeled by parallel fuzzy P+ fuzzy I controller has better performance and IAE. The low IAE is the main factor in sensitive applications. The bode diagram of open-loop transfer function of designed system is shown in Fig. 12. As seen clearly in Fig. 12, the phase margin and gain margin are  $55.2^\circ$  and  $+\infty$ dB, respectively.

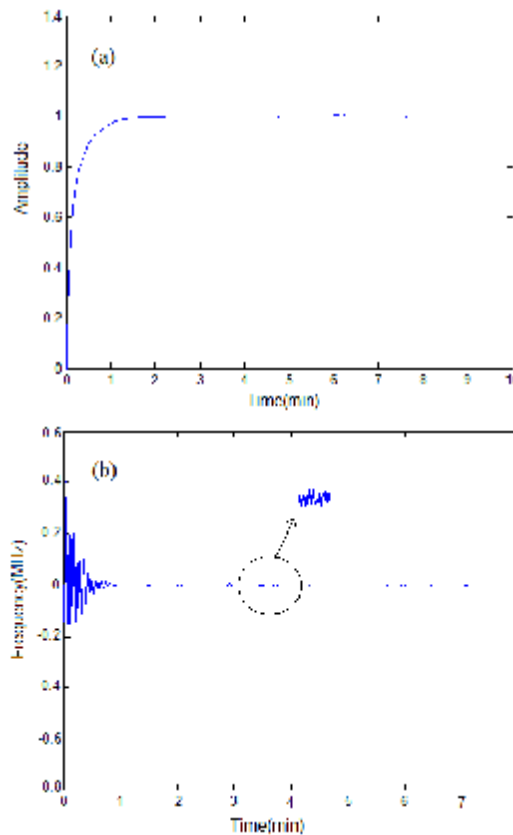


Figure 10. The result of stabilization by parallel fuzzy P+ fuzzy I controller, (a) response to the step and (b) laser frequency fluctuations

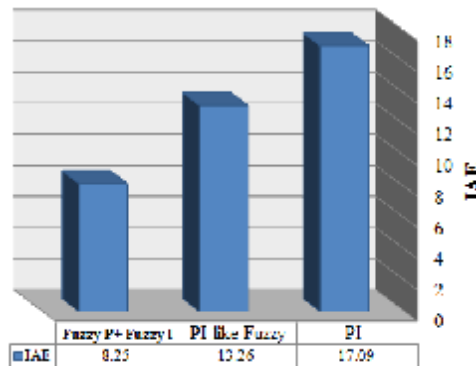


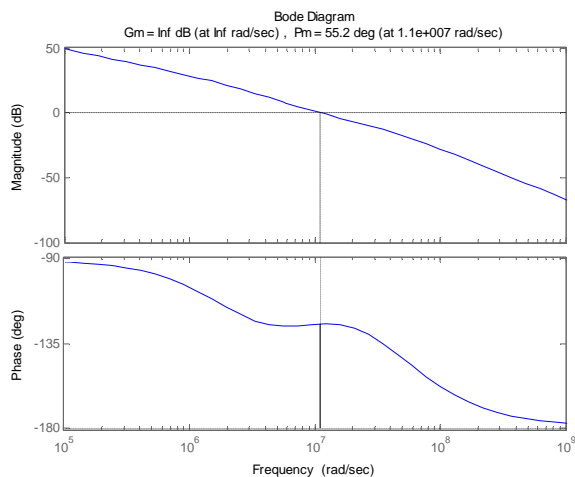
Figure 11. Simulation result of IAE

We have designed the GUI of parallel fuzzy P+ fuzzy I controller for better interaction with the user and the preparation of technical reports. The designed GUI is shown in Fig. 13. As shown in Fig. 13, by changing the parallel fuzzy P+ fuzzy I controller

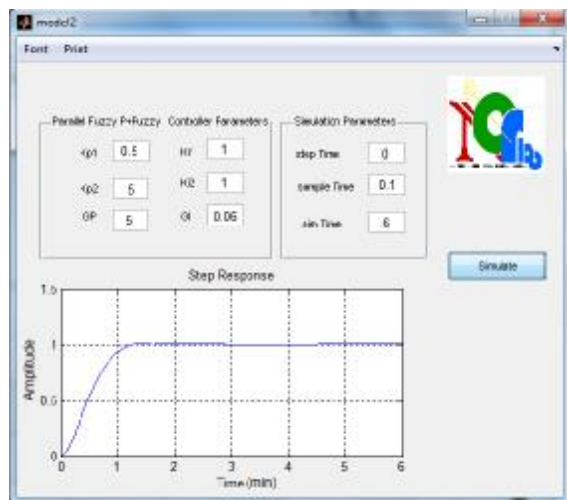
parameters and simulation parameters and pushing simulate bottom, the users can analysis the step response of controlled system and tune it for achieving best IAE.

**4. Conclusion**

In this paper, the parallel fuzzy P+ fuzzy I controller for stabilizing He-Ne lasers has been proposed. Moreover, we compared this controller to other controllers. The results of simulation in SIMULINK showed that performance of system was better and IAE was less than other states. Also, the frequency fluctuations by parallel fuzzy P+ fuzzy I controller has been obtained as  $2 \times 10^{-11}$ .



**Figure 12.** The bode diagram of open-loop transfer function of designed system



**Figure 13.** The designed GUI of parallel fuzzy P+ fuzzy I controller

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