Decentralized MRAC for Large Scale Systems with Input and State Delays

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Abstract
In this paper, the problem of decentralized model reference adaptive control (MRAC) for a class of large scale systems with time varying delay in interconnected term and input and state delays is studied. To compensate the effect of input delay indirectly, a Smith predictor built on. To handle the effects of the time delays in input, the adaptive controller part includes two auxiliary dynamic filters with time varying gains. Under a usual assumption that the interconnections are assumed to be Lipschitz in its variables and uniformly in time with unknown Lipschitz gains, the difficulties from unknown interconnections are dealt. A generalized error is defined and by a suitable Lyapunov function, an adaptive controller is designed to stabilize it. Decentralized adaptive feedback controller can render the generalized error system uniformly ultimately bounded stable is designed. Finally, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed design techniques.

Keywords: Input delays, time varying delay, decentralized MRAC, interconnected system.

1. Introduction

Large-scale interconnected systems can be found in many diverse fields such as electrical power systems, manufacturing processes, transportation and communication. Decentralized control for the large-scale systems has been a research focus in the past three decades and a lot of achievements have been made, see [1-3], and the references therein. In such systems, often delay exists and due to the effect of it, these systems may possess instability and control performance of these systems is hardly assured. So far, control of delayed interconnected systems attracted a number of researchers over the past years, see for example [2-8] and references therein. According to be large scaling of these systems, priori knowledge of the nonlinearities like interconnection functions and parameters like delays is impossible. Hence adaptive control is a popular method for controlling of such systems employed in many papers.

In [1] robust adaptive control of large-scale systems without delay was considered by applying dynamic programming. [3] developed a memoryless adaptive output feedback controller for stabilizing a class of large-scale nonlinear systems. An adaptive decentralized neural control laws addressed for a class of pure-feedback interconnected system with unknown time-varying delays in outputs interconnections in [2].

Between adaptive control method, model reference adaptive control (MRAC) has been attracted lot of attention to control of interconnected systems because of its desired closed loop response. Model reference adaptive control for interconnected systems with time delays are considered in [8], but the obtained controllers are dependent on the delays and the interconnections need to be known precisely. Memoryless controller was proposed for large-scale systems with matched nonlinear interconnections in [7] by using MRAC. In [5] by using the adaptive method a memoryless controller was designed to follow a model for nonlinearly interconnected systems with time delays, dead-zone actuators and mismatched time-varying disturbances. In [4] delay-dependent linear state feedback controller was constructed base on the MRAC for a class of large scale systems with time-varying delays and matched interconnections. The final designed controller is very complicated for practical utilizing.

In the all above papers, the actuators delays are not investigated. For systems with input delays, the control problem is very complicated due to the need to predict the input delayed values, especially for large-scale systems. However, to the best of the authors' knowledge, no attempt has been reported to tackle input delayed simultaneously with state and interconnections delays using the model reference method. In [9], globally stable MRAC Smith-predictor like solution for SISO input delayed plants were developed. It was assumed that the process is minimum phase with arbitrary relative degree, though not necessarily stable. The controller structure was similar to the one proposed by [10]. More recently, in [11-13] MRAC controllers were proposed, also with a Smith predictor-like structure. In [12] adaptive control laws were generated by using a high-order tuner and Lyapunov-Krasovski functional was used to adaptive stabilization. The adaptive controller in [13] is based on the Smith predictor and finite spectrum assignment. The problem of input delay with smith predictor was studied in [14], but in the mentioned paper, the

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interconnected term was not considered. In [15] the interconnected term was considered but the input and state delays were not investigated.

In this paper, a new MRAC scheme is developed for a class of interconnected delayed systems with known input and state delays. To handle the effects of the time delays, the adaptive controller part includes two auxiliary dynamic filters with time varying gains. For implicit (indirect) input-delay compensation, a Smith predictor-like filter is used. Decentralized adaptive feedback controller can render the generalized error system uniformly ultimately bounded stable is designed. The paper consists of the following parts. In section2, problem formulation and assumptions are introduced. The controller design is described in section 3. The stability of system is proof by the introducing a suitable Lyapunov function. In section 4 a numerical example is illustrated and the mentioned theoretical process is implemented. Final section is included the conclusion.

2. Problem formulation and assumptions

The interconnected system considered in this paper composed by N subsystems with the ith subsystem

$$S_i : \dot{x}_i(t) = A_i\, x_i(t) + A_{di}\, x_i(t-t_i) + B_i\, u_i(t-h_i) + B_i \sum_{j=1}^{N} x_{ij}\left(x_{ij}(t), x_{ij}(t-d_{ij}(t))\right) \tag{1}$$

where $x_i \in \Re^{n_i}$ and $u_i \in \Re^{m_i}$ represent the state and control vectors of the i-th subsystem respectively; A_i, $A_{di} \in \Re^{n_i \times n_i}$ and $B_i \in \Re^{n_i \times m_i}$ are known constant matrices, t_i and h_i are known constant delay, $x_{ij}\left(x_{ij}(t), x_{ij}(t-d_{ij}(t))\right)$ are uncertain interconnections, which indicate the interconnections among the current states and the delayed states of systems S_i and S_j , and $d_{ij}(t)$ are bounded time-varying delays, which are differentiable and satisfy:

$$0 \le d_{ij}(t) \le \bar{d}_{ij} < \infty, \quad 0 \le \dot{d}_{ij}(t) \le \overset{*}{d}_{ij} < 1, \tag{2}$$

where $\bar{d}_{ij}$ and $\overset{*}{d}_{ij}$ are positive scalars. The initial conditions are given by:

$$x_i(t) = \Lambda_i(t), \quad t \in [t_0 - i_i, t_0], \quad i = 1, 2, ..., N$$

where $i_i = \max\{t_i, \bar{d}_{ij}\}$ and $\Lambda_i(t)$ are continuous functions.

The objective in this paper is to design a decentralized controller $u_i(t)$ adaptively to ensure that the system states $x_i(t)$ follow the state of $x_{mi}(t)$ given by the local reference model following:

$$\dot{x}_{mi}(t) = A_{mi} x_{mi}(t) + B_{mi} R_i(t) \tag{3}$$

with the known piecewise continuous and bounded reference $R_i(t)$.

For system (1) and model reference system (3), the following assumptions hold:

Assumption1. There exist matrices K_i and positive matrices P_i satisfying the following inequality:

$$(A_{mi} + B_i K_i)^T P_i + P_i (A_{mi} + B_i K_i) = -Q_i \tag{4}$$

where $i = 1, 2 \ldots, N$ and Q_i are positive definite matrices.

Assumption2. There exit constant vectors $z_i \in \Re^{n_i}$, $m_i \in \Re^{n_i}$ and a nonzero constant scalar q_{ri} such that the following equations are satisfied:

$$A_i - A_{mi} = B_i z_i\ , \quad A_{id} + B_i m_i = 0, \quad B_{mi} = B_i q_{ri}$$

Assumption3. There exist unknown positive constants a_{ij} and b_{ij} such that:

$$\sum_{j=1}^{N} \left\| x_{ij}\left(x_j(t), x_j(t-d_{ij}(t))\right)\right\| \le \sum_{j=1}^{N} a_{ij} \left\|x_j(t)\right\| + \sum_{j=1}^{N} b_{ij} \left\|x_j(t-d_{ij}(t))\right\| \tag{5}$$

Assumption4. The states of system (3) are bounded.
Assumption5. All subsystems are stable.

Remark1. It should be noted that Assumption 1 is to guarantee that the pair (A_{mi}, B_i) can be stabilized. If (A_{mi}, B_i) is completely controllable, Assumption 1 will always hold. Assumptions 1 and 2 are used to determine some important design parameters. Assumption 4 is to assure that the underlying model reference system is bounded stable.

3. Model reference adaptive controller design

The problem is to design an adaptive feedback control in order to achieve desired closed loop specifications. The desired specification in this paper is that all signals of the closed loop system remain bounded and that approximate tracking be achieved with small enough asymptotic error. The tracking error and its derivatives is defined as:

$$e_i(t) = x_i(t) - x_{mi}(t) \tag{6}$$

$$\dot{e}_i(t) = A_i\, x_i(t) + A_{di}\, x_i(t-t_i) + B_i\, u_i(t-h_i) + B_i \sum_{j=1}^{N} x_{ij}\left(x_{ij}(t), x_{ij}(t-d_{ij}(t))\right) - A_{mi} x_{mi}(t) - B_{mi} R_i(t) \tag{7}$$

From assumption 3, we further obtain the following inequalities with the help of assumption 4:

$$\begin{aligned} &\sum_{j=1}^{N} \left\| x_{ij}\left(x_j(t), x_j(t-d_{ij}(t))\right)\right\| \\ &\le \sum_{j=1}^{N} a_{ij} \left\|x_j(t)\right\| + \sum_{j=1}^{N} b_{ij} \left\|x_j(t-d_{ij}(t))\right\| \\ &= \sum_{j=1}^{N} a_{ij} \left\|x_{mj}(t) + e_j(t)\right\| \\ &+ \sum_{j=1}^{N} b_{ij} \left\|x_{mj}(t-d_{ij}(t)) + e_j(t-d_{ij}(t))\right\| \\ &\le a_{1ij} \left\|e_j(t)\right\| + b_{1ij} \left\|e_j(t-d_{ij}(t))\right\| + d_{1i} \end{aligned} \tag{8}$$

Since

$$\|e_{gi}(t)\| \le \|e_i(t)\| + \|v_{i1}(t)\| + \|v_{i2}(t)\| \tag{9}$$

Then relationship (8) can be written as:

$$\begin{aligned}&\sum_{j=1}^{N}\left\|x_{ij}\left(x_j(t), x_j(t-d_{ij}(t))\right)\right\| \\ &\le a_i\,\|e_{gi}(t)\| + b_i\,\|e_{gi}(t-d_{ij}(t))\| + d_i\end{aligned} \tag{10}$$

where a_i, b_i and d_i are unknown positive scalars, and since the states x_{mi} of reference model system are bounded, there always exit positive this scalars such that inequality (10) holds.

Now, we are ready to present our main result in this paper.

Theorem. For system (1), the following adaptive feedback controller:

$$u_i = u_{i1} + u_{i2} + u_{i3} \tag{11}$$

where

$$\begin{aligned}u_{i1} &= -z_i x_{mi}(t) + k_i e_{gi}(t) \\ &\quad + q_{ri}\, R_i(t) + m_i\, x_{mi}(t-t_i) \\ u_{i2} &= -q_i\, B_i^T\,\frac{\partial V_i^T(e_{gi})}{\partial e_{gi}}, \\ u_{i3} &= \frac{-h_i(t) B_i^T\,\dfrac{\partial V_i^T(e_{gi})}{\partial e_{gi}}}{\left\|B_i^T\,\dfrac{\partial V_i^T(e_{gi})}{\partial e_{gi}}\right\|}.\end{aligned} \tag{12}$$

In which $q_i(t)$ and $h_i(t)$ are adaptive parameters with adaptive laws:

$$\begin{aligned}\dot{q}_i &= \frac{1}{2}\Gamma_i\left\|B_i^T\,\frac{\partial V_i^T(e_{gi})}{\partial e_{gi}}\right\|^2 - \frac{1}{2}\Gamma_i s_{i1} q_i, \\ \dot{h}_i &= \frac{1}{2}f_i\left\|B_i^T\,\frac{\partial V_i^T(e_{gi})}{\partial e_{gi}}\right\| - \frac{1}{2}f_i s_{i2} h_i.\end{aligned} \tag{13}$$

where Γ_i, f_i, s_{i1} and s_{i2} are positive scalars.

Also, with definition generalized as follows:

$$e_{gi}(t) = e_i(t) + v_{i1}(t) + v_{i2}(t) \tag{14}$$

where is the sum of the tracking error $e_i(t)$ and the internal states $v_{i1}\in R^{n_i}$ and $v_{i2}\in R^{n_i}$ of the adaptive controller. The controller states v_{i1}, v_{i2} are generated by the stable dynamic filters F_1 and F_2 with the gains $\alpha_{i1}(t)\in R^{n_i}$ and $\alpha_{i2}(t)\in R^{2n_i}$:

$$\begin{aligned}F_1:\quad &\dot{v}_{i1}(t) = A_{mi}v_{i1}(t) \\ &+ B_{mi}a_{i1}\left(u_i(t) - u_i(t-h_i)\right)\end{aligned} \tag{15}$$

$$F_2:\quad \dot{v}_{i2}(t) = A_{mi}v_{i2}(t) + B_{mi}\, a_{i2}^T\, w_i(t) \tag{16}$$

where the input of F_2 is the signal $w_i(t) = \left[e_i(t) \quad e_i(t-t_i)\right]^T \in \Re^{2n_i}$.

The time varying gains $a_{i1}(t)$ and $a_{i2}(t)$ are as:

$$a_{i1} = -q_{ri}^{-1} \tag{17}$$

$$a_{i2} = [-q_{ri}^{-1}n_i \quad q_{ri}^{-1}m_i]^T \tag{18}$$

Definition. The solution generalized error $e_{gi}(t,t_0,y_i)$ of interconnected system (14) is said to be uniformly ultimately bounded with respect to the bound e_i if for each $\delta_i > 0$ there exists $T_i = T_i(e_i, d_i) > 0$ independent of t_0 such that $\|e_{gi}(t,t_0,y_i)\| \le e_i$ for all $t \ge t_0 + T$ when $\|x_{it0}\| \le d_i$.

Proof. Define the following Lyapunov function candidate:

$$\begin{aligned}&\tilde{V}(e_{gi},q_i,h_i,t) = \sum_{i=1}^{N}\bar{V}_i(e_{gi},q_i,h_i,t) = \\ &\sum_{i=1}^{N}\left\{\begin{aligned}&V_i(e_{gi}) + \Gamma_i^{-1}\left(q_i - \tilde{q}_i\right)^2 + f_i^{-1}\left(d_i^* - h_i\right)^2 \\ &+ q_i\int_{t-d_{ij}}^{t}\|e_{gi}(x)\|^2\,dx\end{aligned}\right\}\end{aligned} \tag{19}$$

where

$$V_i(e_{gi}) = e_{gi}^T P_i\, e_{gi}\,. \tag{20}$$

q_i are sufficiently small positive scalars, $\tilde{q}_i$ are also positive scalars defined in (25) (below).

Generalized error $e_{gi}(t)$ is equal to:

$$\begin{aligned}\dot{e}_{gi}(t) &= \dot{e}_i(t) + \dot{v}_{i1}(t) + \dot{v}_{i2}(t) \\ &= A_{mi}e_{gi}(t) + B_{mi}a_{i1}\left[u_i(t) - u_i(t-h_i)\right] \\ &\quad + B_{mi}a_{i2}^T w_i(t) + B_i z_i x_i - B_i m_i x_i(t-t_i)\ . \\ &\quad - B_{mi}R_i(t) + B_i\sum_{j=1}^{N}x_{ij}(x_j(t), x_j(t-d_{ij}(t))) \\ &\quad + B_i u_i(t-h_i)\end{aligned} \tag{21}$$

By taking the time derivative of $\tilde{V}(.)$ along the trajectories of the closed loop system, we have

$$\begin{aligned}&\frac{\partial \bar{V}_i(e_{gi},q_i,h_i,t)}{\partial t} = -e_{gi}^T(t)Q_i\, e_{gi}(t) \\ &+ \frac{\partial V_i(e_{gi})}{\partial e_{gi}}B_i\sum_{j=1}^{N}x_{ij}(x_j(t), x_j(t-d_{ij}(t))) \\ &+ \frac{\partial V_i(e_{gi})}{\partial e_{gi}}B_i\left(u_{i2}(t) + u_{i3}(t)\right) \\ &+ 2\Gamma_i^{-1}\left(q_i - \tilde{q}_i\right)\dot{q}_i - 2f_i^{-1}\left(d_i^* - h_i\right)\dot{h}_i \\ &+ q_i\,\|e_{gi}(t)\|^2 - q_i(1 - d_{ij}^*(t))\|e_{gi}(t-d_{ij}(t))\|^2 \\ &= -e_{gi}^T(t)Q_i\, e_{gi}(t) \\ &+ \frac{\partial V_i(e_{gi})}{\partial e_{gi}}B_i\sum_{j=1}^{N}x_{ij}(x_j(t), x_j(t-d_{ij}(t))) \\ &+ \frac{\partial V_i(e_{gi})}{\partial e_{gi}}B_i\left(u_{i2}(t) + u_{i3}(t)\right) \\ &\left(q_i - \tilde{q}_i\right)\left\|B_i^T\,\frac{\partial V_i^T(e_{gi})}{\partial e_{gi}}\right\|^2 - s_{i1}q_i\left(q_i - \tilde{q}_i\right) \\ &-\left(d_i - h_i\right)\left\|B_i^T\,\frac{\partial V_i^T(e_{gi})}{\partial e_{gi}}\right\| + s_{i2}h_i\left(d_i - h_i\right) \\ &+ q_i\,\|e_{gi}(t)\|^2 - q_i(1 - d_{ij}^*(t))\|e_{gi}(t-d_{ij}(t))\|^2\end{aligned} \tag{22}$$

Considering the below relationship

$$\frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \left(u_{i2}(t)+u_{i3}(t)\right)= \tag{23}$$

$$-q_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 - h_i(t) \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

.

and

$$\frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \sum_{j=1}^{N} x_{ij} \left(x_j(t), x_j(t-d_{ij}(t))\right) \tag{24}$$

$$\leq \bar{a}_i \left\| e_{gi}(t) \right\| \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\| +$$

$$\bar{b}_i \left\| e_{gi}(t-d_{ij}(t)) \right\| \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$+d_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$\leq \frac{\left\|\bar{a}_i\right\|^2}{4e_i} \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 + e_i \left\| e_{gi}(t) \right\|^2$$

$$+\frac{\left\|\bar{b}_i\right\|^2}{4q_i(1-d_{ij}^*)} \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2$$

$$+d_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$+q_i(1-d_{ij}^*) \left\| e_{gi}(t-d_{ij}(t)) \right\|^2$$

Let

$$\hat{q}_i = \frac{\left\|\bar{a}_i\right\|^2}{4e_i} + \frac{\left\|\bar{b}_i\right\|^2}{4q_i(1-d_{ij}^*)} \tag{25}$$

Then, we have

$$\frac{\partial V_i(e_{gi})}{\partial e_{gi}} B_i \sum_{j=1}^{N} x_{ij} \left(x_j(t), x_j(t-d_{ij}(t))\right) \tag{26}$$

$$\leq \hat{q}_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|^2 + d_i \left\| B_i^T \frac{\partial V_i^T(e_{gi})}{\partial e_{gi}} \right\|$$

$$+q_i(1-d_{ij}^*) \left\| e_{gi}(t-d_{ij}(t)) \right\|^2 + e_i \left\| e_{gi}(t) \right\|^2$$

Considering the following equality:

$$(q_i-\hat{q}_i)(-s_{i1}q_i)+(d_i-h_i)(s_{i2}h_i) \tag{27}$$

$$=-s_{i1}\left(q_i-\frac{1}{2}\hat{q}_i\right)^2+\frac{1}{4}s_{i1}\hat{q}_i^2$$

$$-s_{i2}\left(d_i-\frac{1}{2}h_i\right)^2+\frac{1}{4}s_{i2}h_i^2$$

Substituting (23) to (27) into (22), we further have

$$\dot{V}(e,q,t)=\sum_{i=1}^{N}\dot{V}_i(e,q,t) \tag{28}$$

$$=\sum_{i=1}^{N}\left(-e_{gi}^T(t)Q_i\,e_{gi}(t)+(q_i+e_i)\left\|e_{gi}(t)\right\|^2\right)$$

$$+\sum_{i=1}^{N}\left(\begin{array}{l} -s_{i1}\left(q_i-\frac{1}{2}\hat{q}_i\right)^2+\frac{1}{4}s_{i1}\hat{q}_i^2 \\ -s_{i2}\left(d_i-\frac{1}{2}h_i\right)^2+\frac{1}{4}s_{i2}h_i^2 \end{array}\right)$$

Because Q_i is positive definite matrices, parameters e_i and q_i can be selected to be small enough to render that the following inequality holds:

$$-l_{\min}(Q_i)+q_i+e_i=-\Pi_i<0 \tag{29}$$

where Π_i are positive scalars. Furthermore, one has

$$\dot{V}(e,q,t)\leq\sum_{i=1}^{N}\left(\begin{array}{l} -\Pi_i\left\|e_{gi}\right\|^2 \\ +\frac{1}{4}s_{i2}d_i^2+\frac{1}{4}s_{i1}\hat{q}_i^2 \end{array}\right) \tag{30}$$

Based on Lyapunov stability theory, the proposed decentralized state feedback controller (11)- (18) will guarantee the closed loop error system is uniformly ultimately bounded stable.

Remark2. In this section, robust adaptive tracking control is proposed for a class of systems with delayed input and state and without so-called distributed delay blocks. To handle the time delays effects, the auxiliary dynamic filters ***F₁*** and ***F₂*** with variable gains $a_{i1}(t)$ and $a_{i2}(t)$ are introduced. The Smith predictor-like filter F_1 is used for indirect elimination of the effect of the dead time h_i. The filter F_2 with the nonlinear input $a_{i2}^T(t)w_i(t)$ is introduced to compensate plant parameter uncertainties, perturbations. Note that in our case the Smith predictor has a "classical" structure, with the difference that it is based on the reference model transfer function rather than the plant transfer function, and that it has the input gain $a_{i1}(t)$.

Remark3. For inequality (30), it is easy to obtain that the stable bounds of error e_{gi} can be rendered sufficiently small by reducing the values of parameters s_{i1} and s_{i2}.

4. Numerical example

To illustrate the utilization of our approach, in this section, we consider the following numerical example. Here, a large scale time-delay system with interconnected terms is composed of two subsystems described by

$$\dot{x}_1(t)=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}x_1(t)+\begin{bmatrix}0\\1\end{bmatrix}u_1(t-h_1) \quad (31)$$
$$+\begin{bmatrix}0 & 0\\ -0.6 & -0.8\end{bmatrix}x_1(t-t_1)$$
$$+\begin{bmatrix}0\\1\end{bmatrix}\left(d_{11}x_{11}(t)+d_{12}x_{22}(t-d_1(t))\right)$$
$$\dot{x}_2(t)=\begin{bmatrix}0 & 1\\ -2.5 & -3.5\end{bmatrix}x_2(t)++\begin{bmatrix}0\\1\end{bmatrix}u_2(t-h_2)$$
$$+\begin{bmatrix}0 & 0\\ -0.6 & -0.8\end{bmatrix}x_2(t-t_2)$$
$$+\begin{bmatrix}0\\1\end{bmatrix}\left(d_{21}x_{21}(t)+d_{22}x_{12}(t-d_2(t))\right)$$

To build the adaptive controller we choose the reference model:

$$\dot{x}_{m1}(t)=\begin{bmatrix}0 & 1\\ -4 & -5\end{bmatrix}x_{m1}(t)+\begin{bmatrix}0\\1\end{bmatrix}R_1(t) \quad (32)$$
$$\dot{x}_{m2}(t)=\begin{bmatrix}0 & 1\\ -5 & -6\end{bmatrix}x_{m2}(t)+\begin{bmatrix}0\\1\end{bmatrix}R_2(t)$$

with $R_1(t)=100\sin\frac{p}{8}t,\ R_2(t)=100\cos\frac{p}{8}t.$

Therefore, we obtain the following controller from theorem 1:

$$u_1=-(2x_{m11}+2x_{m12})+(0.6x_{m11}(t-t_1) \quad (33)$$
$$+0.8x_{m12}(t-t_1))-\left(e_{g11}+10e_{g12}\right)+R_1(t)$$
$$-2q_1B_1^T P_1e_{g1}-2h_1\frac{B_1^T P_1e_{g1}}{\left\|B_1^T P_1e_{g1}\right\|}$$

$$u_2=-(2.5x_{m21}+2.5x_{m22})+(0.6x_{m21}(t-t_2) \quad (34)$$
$$+0.8x_{m22}(t-t_2))-\left(e_{g21}+10e_{g22}\right)+R_2(t)$$
$$-2q_2B_2^T P_2e_{g2}-2h_2\frac{B_2^T P_2e_{g2}}{\left\|B_2^T P_2e_{g2}\right\|}$$

and the adaptive laws

$$\dot{q}_i=\left\|2B_i^T P_i e_{gi}\right\|^2-0.01q_i\ , \quad (35)$$
$$\dot{h}_i=\left\|2B_i^T P_i e_{gi}\right\|-0.01h_i,\ \ i=1,2.$$

We chose

$$t_1=t_2=1,\ h_1=h_2=1.5,\ d_{11}=d_{21}=1,$$
$$d_{22}=d_{12}=0.5, d_1(t)=d_2(t)=0.5\left(1+\cos t\right)$$
$$,Q_i=100I.$$

and the initial conditions are

$$x_1(0)=\begin{bmatrix}8\\4\end{bmatrix},\ x_2(0)=\begin{bmatrix}-4\\-8\end{bmatrix},$$
$$x_{m1}(0)=\begin{bmatrix}1\\3\end{bmatrix},\ x_{m2}(0)=\begin{bmatrix}-1\\-2\end{bmatrix}.$$

The state responses $x_i(t)$ and $x_{mi}(t)$ are shown in Figures 1-4. From figures it can be seen that the closed-loop system is uniformly ultimately bounded stable. Also the control signals and the generalized errors $e_{gi}(t)$ are shown in Figures 5 and 6 respectively.

5. Conclusion

In this paper, model reference adaptive control problem for a class of large-scale time-delay systems with both input delay and state delay and interconnection is investigated. The decentralized feedback controllers and corresponding adaptive laws are designed. Our approach relies on a decomposition of the adaptive control design by introducing the "generalized error", and two auxiliary linear filters with time varying gains. The first filter is the "classical" Smith predictor, but based on the reference model transfer function rather than on the plant transfer function. The second filter has the same dynamics as the reference model but driven by the signal $a_{i2}^T w_i(t)$. The effect of such decomposition is to pull the input delay out of the design procedure. That leads to control laws with the following distinctive features: (i) they do not incorporate distributed-delay elements and (ii) they provide robustness with respect to plant parameter uncertainties. The "price" paid for this controller configuration is its restrictive applicability to stable plants, to plants stabilizable with memory less state feedback or to plants stabilizable with so called "delayed state feedback". Based on Lyapunov stability theory, we prove the resulting closed-loop error system is uniformly ultimately bounded stable. A numerical example is given to verify the feasibility and validity of the main results. Therefore, the results obtained are expected to apply to a large class of

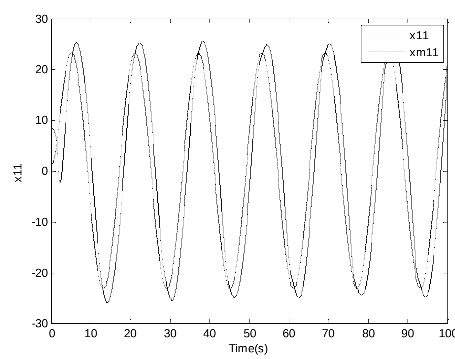

Fig. 1. The response of the state variable x_{11} and x_{m11}.

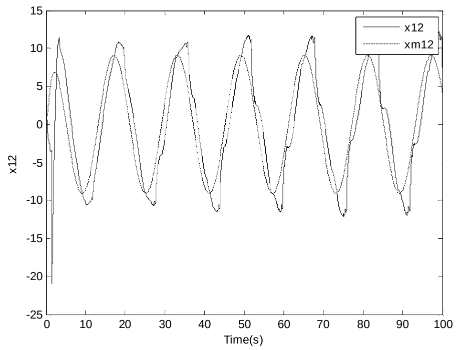

Fig. 2. The response of the state variable x_{12} and x_{m12}.

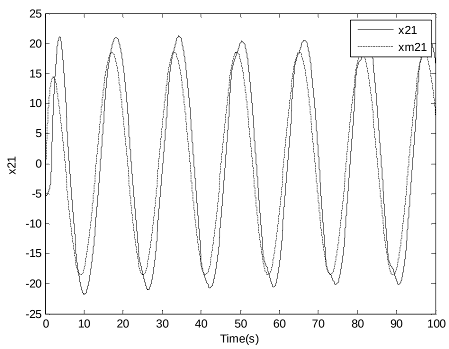

Fig. 3. The response of the state variable x_{21} and x_{m21}.

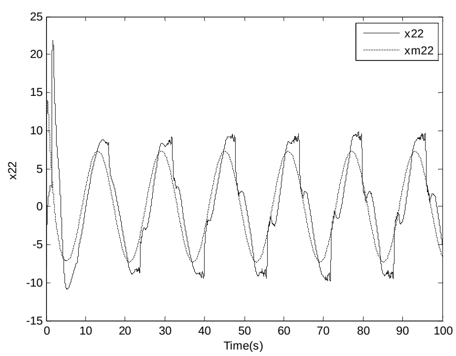

Fig. 4. The response of the state variable x_{22} and x_{m22}.

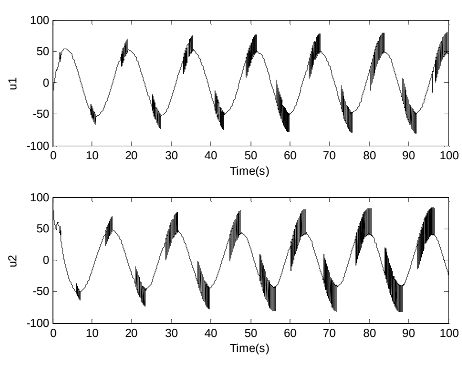

Fig. 5. Adaptive control signals u_1 and u_2.

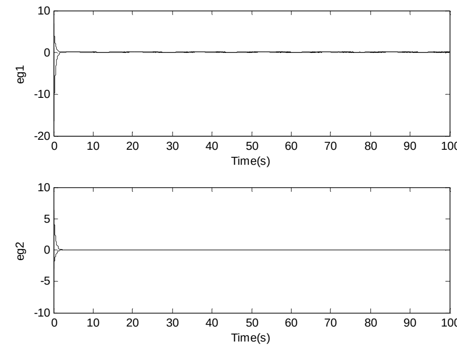

Fig. 6. The generalized errors e_{g1} and e_{g2} converge to zero.

Reference

[1] T. Bian, Y. Jiang, Z.-P. Jiang, Decentralized adaptive optimal control of large-scale systems with application to power systems. Industrial Electronics, IEEE Transactions on 62 (2015) 2439-2447.

[2] C. He, J. Li, L. Zhang, Decentralized adaptive control of nonlinear large scale pure feedback interconnected systems with time varying delays. International Journal of Adaptive Control and Signal Processing 29 (2015) 24-40.

[3] X. Zhang, Y. Lin, Adaptive output feedback control for a class of large-scale nonlinear time-delay systems. Automatica 52 (2015) 87-94.

[4] C.-C. Hua, J. Leng, X.-P. Guan, Decentralized MRAC for large-scale interconnected systems with time-varying delays and applications to chemical reactor systems. Journal of Process Control 22 (2012) 1985-1996.

[5] C.-C. Hua, S.X. Ding, Model following controller design for large-scale systems with time-delay interconnections and multiple dead-zone inputs. Automatic Control, IEEE Transactions on 56 (2011) 962-968.

[6] M.S. Mahmoud, Decentralized stabilization of interconnected systems with time-varying delays. Automatic Control, IEEE Transactions on 54 (2009) 2663-2668.

[7] C. Hua, X. Guan, P. Shi, Decentralized robust model reference adaptive control for interconnected time-delay systems. Journal of Computational and Applied Mathematics 193 (2006) 383-396.

[8] C.-H. Chou, C.-C. Cheng, A decentralized model reference adaptive variable structure controller for large-scale time-varying delay systems. Automatic Control, IEEE Transactions on 48 (2003) 1213-1217.

[9] R. Ortega, R. Lozano, Globally stable adaptive controller for systems with delay. International Journal of Control 47 (1988) 17-23.

[10] K. Ichikawa, Adaptive control of delay system. International Journal of Control 43 (1986) 1653-1659.

[11] S.-I. Niculescu, A.M. Annaswamy, An adaptive Smith-controller for time-delay systems with relative degree n* 2. Systems & control letters 49 (2003) 347-358.

[12] S. Evesque, A. Annaswamy, S. Niculescu, A. Dowling, Adaptive control of a class of time-delay systems. Journal of dynamic systems, measurement, and control 125 (2003) 186-193.

[13] Y. Yildiz, A. Annaswamy, I.V. Kolmanovsky, D. Yanakiev, Adaptive posicast controller for time-delay systems with relative degree n* 2. Automatica 46 (2010) 279-289.

[14] B. Mirkin, P.-O. Gutman, Adaptive following of perturbed plants with input and state delays, Control and Automation (ICCA), 2011 9th IEEE International Conference on, IEEE, 2011, pp. 865-870.

[15] H. Wu, A simple robust adaptive control scheme for model reference adaptive control systems with nonlinear delayed state perturbations, Industrial Electronics Society, IECON 2013-39th Annual Conference of the IEEE, IEEE, 2013, pp. 3287-3292.