Leader-Following Consensus of Nonlinear Multi-Agent Systems Based on Parameterized Lyapunov Function

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***Abstract*—This paper studies the consensus problem of nonlinear leader-following multi-agent systems (MAS). To do this, the error dynamics between the leader agent and follower ones are described via a Takagi-Sugeno (TS) fuzzy model. If the obtained TS fuzzy model is stable, then all of the nonlinear agents reach consensus. The consensus problem is investigated based on the parameterized or fuzzy Lyapunov function combined with a technique of introducing slack matrices. The slack matrices cause to decouple the Lyapunov matrices from systems ones and therefore, sufficient consensus conditions are obtained in terms of linear matrix inequalities (LMIs). The proposed slack matrices add an extra degree-of-freedom to the LMI conditions and also decrease the conservativeness of the LMI-based conditions. Finally, in order to illustrate the effectiveness and merits of the proposed method, a numerical example for the consensus problem of nonlinear leader-follower MAS with thirteen followers is solved.**

***Index Terms*—Nonlinear multi-agent systems, Consensus, Takagi-Sugeno (T-S) fuzzy model, Parameterized Lyapunov function, Linear Matrix Inequality (LMI).**

I. Introduction

Fuzzy model based (FMB) control provides a framework to design a nonlinear control strategy for a general class of nonlinear systems [1]. The physical phenomena are inherently nonlinear. In order to overcome the complexity of the system nonlinearity, Takagi and Sugeno (in 1985) presented a systematic multi-modeling approach called Takagi-Sugeno (TS) fuzzy model. The TS fuzzy model plays a critical role in FMB control [1]. In TS fuzzy model which introduced via fuzzy IF-THEN rules, the smooth nonlinear system is represented via some local subsystems. Then, by fuzzy blending of the local subsystems, the overall fuzzy model will be achieved in a convex structure. The TS fuzzy model can be constructed via the identification methods based on the input-output data or it can be derived from the existence smooth nonlinear system equations [2]. In order to control the nonlinear system based on the TS fuzzy model, several fuzzy control methods have been presented such as parallel distributed compensation (PDC) or non-parallel distributed compensation (non-PDC) [3]. Furthermore, sufficient stability conditions of the closed-loop system will be obtained based on the Lyapunov direct method in terms of LMIs [3].

One of the common approaches to formulate stability analysis of the TS fuzzy systems is Lyapunov direct method. The sufficient stability conditions are converted to LMIs and solved via convex optimization techniques [2]. In some situations, may be the TS fuzzy system is stable but the common quadratic Lyapunov function (QLF) does not exist [4]. Thus, several kinds of LFs are presented in literature such as parameterized or fuzzy LF (FLF) [5], Max-Min LF [6] and piecewise LF [7]. The piecewise LF is achieved based on a combination of some separate quadratic LFs where each of these quadratic LFs is valid in a particular region. Thus, the piecewise LF suffers from the problem of discontinuity in boundary points of each region. The FLF is also known as basis-dependent LF, and non-quadratic LF. The FLF is achieved based on fuzzy aggregation of some quadratic LFs [4].

Multi-agent systems (MAS) are constructed by multiple interconnecting of intelligent subsystems called agents. The agents joint together to study the problems that are usually very difficult or even impossible for each agent to solve [8]. Consensus of MASs has attracted lots of attentions as a new field of research. The consensus problem is investigated in various background of research such as: control, robotic, biology and computer science backgrounds. Roughly speaking, if the agents of MAS reach an agreement on a specific criterion, then the consensus problem is feasible [9]. According to the control engineering point of view, the consensus problem is controlling of the agents such that the consensus conditions are satisfied.

Consensus of MASs is a promising research topic during recent years. Control theory plays an important role for solving a consensus problem. Several kinds of conventional control protocol are applied on MAS to solve the problem of consensus such as: H_∞ control protocol [10], Pinning control [11] and sampled-data control [12]. Designing control protocol for linear discrete-time agents to solve the consensus problem is considered in [13]. Moreover, the consensus problem is studied for MASs that the dynamic behavior of the agents are linear [14, 15] and nonlinear [16]. Ref. [16] studies an H_∞ leader-following consensus problem of nonlinear MASs'. The consensus problem is o approachs the nonlinear follower agents to the unforced leader agent. Thus, the nonlinear error dynamics between the follower agents and unforced leader

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agent are achieved. Subsequently, the TS fuzzy model of nonlinear error dynamics are calculated and the sufficient stability conditions are derived in terms of LMIs. As aforementioned, Ref. [4] investigates the problem of stability analysis of TS fuzzy systems based on FLF and introducing some slack matrices. Slack matrices generate a degree of freedom in LMI conditions and have a direct effect on converting the stability conditions to the LMI ones [4]. Whereas, by increasing the number of agents, the dimension of LMI conditions will be increased and subsequently the feasibility of LMI conditions will be converted to a challenging problem. According to the authors' best knowledge, this paper tries the first attempt to analyze the consensus problem of nonlinear MASs with FLF and slack matrices.

In this paper, we seek to solve the consensus control problem in more relax scheme by using the FLF and introducing slack matrices. First, define the leader-following as the consensus problem. Second, the nonlinear error dynamics between the agents of nonlinear unforced leaders and the followers will be achieved. Thus, the consensus problem will be converted to the stability analysis one. Third, based on the nonlinear error dynamics, the exact TS fuzzy model will be calculated via sector nonlinear approach. Fourth, in order to analyze the stability of the TS fuzzy model based closed-loop system, the FLF and some new null terms will be defined and sufficient stability conditions will be derived in terms of LMIs. The main contributions of this paper can be classified as follows:

1. The FLF will be used to solve the consensus problem.
2. Some new null terms will be defined. Slack matrices in null terms increase the degree of freedom and also converted the stability conditions to the LMI ones.
3. The control protocol will be designed.
4. Compare to the recent published works [16, 18-20], the LMI-based stability conditions are more relaxed.

Finally, the proposed approach is applied to the consensus problem of nonlinear leader-follower MAS with thirteen followers.

The remainder of this paper is organized as follows. Section II is divided into two parts. First part presents some basic concepts of graph theory, and second part discusses about problem formulation. The main results are given in Section III. In section IV, simulations are carried out to illustrate the effectiveness of the main results. Finally, conclusions are drawn in section V.

In the current paper, the superscript T stands for matrix transpose, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $diag\{\cdots\}$ stands for a block-diagonal matrix. In symmetric block matrices, $*$ is used to represent a term that is induced by symmetry, I_n is an identity matrix of dimension $n\times n$ and $\otimes$ denotes the Kronecker product.

2. PRELIMINARIES AND PROBLEM DESCRIPTION

A. Basic Concepts on Graph theory

A MAS consisting N agents represented by an undirected graph G consists of a vertex set $V(G)=\{v_1,v_2,\dots,\mathrm{v}_N\}$, an edge set $E(G)\subseteq\{(v_i,v_j):v_i,v_j\in V(G)\}$. If $(v_i,v_j)\in E(G)$, it means that, agent i can send its information to the agent j and vice versa. In other words, there is a directed connection from node i to node j. Also, adjacency matrix $A=[a_{ij}]\in\mathbb{R}^{N\times N}$ is defined such that for a directed connection from node i to node j, $a_{ij}\succ 0$, if $(v_i,v_j)\in E(G)$. Furthermore, it is supposed that $a_{ii}=0\ (i=1,2,...,N)$. The neighbor set of node v_i is denoted by $N_i=\{v_j\in V(G):(v_i,v_j)\in E(G)\}$. The Laplacian matrix associated with G is defined as follows:

$$l_{i,j}=\begin{cases}-\sum_{k=1,k\neq j}^{N}a_{ik} & j=i\\ a_{ij} & j\neq i\end{cases} \tag{1}$$

A graph containing the leader and all followers is represented by $\bar{G}$.

B. Problem Formulation

Consider a group of N follower agents and one leader. The dynamic of each follower agent is given by

$$\dot{x}_i(t)=f(x_i(t))+u_i \quad i=1,...,\mathrm{N} \tag{2}$$

where $x_i(t)\in\mathbb{R}^n$ is the state of agent (node) i, $f(x_i(t))$ is a nonlinear continuously differentiable vector function representing the intrinsic nonlinear dynamics of the i-th agent, and $u_i(t)$ is the control protocol to be designed.

It is assumed that the leader has the following nonlinear time-varying dynamic

$$\dot{s}(t)=f(s(t)) \tag{3}$$

where $s(t)$ is the states vector of the leader and should be tracked by all the followers. It can be an equilibrium point, a chaotic orbit or a periodic orbit [16].

Assuming that $e_i(t)$ denotes the error between the states of the followers and the states of the leader, i.e., $e_i(t)=x_i(t)-s(t)$, the error dynamics can be represented as

$$\dot{e}_i(t)=f(x_i(t))-f(s(t))+u_i,\quad i=1,...,N \tag{4}$$

Consider a distributed consensus protocol as follows:

$$u_i(t) = c\Gamma \sum_{j \in N_i} a_{ij}\left(x_j(t) - x_i(t)\right) + cd_i \Gamma\left(s(t) - x_i(t)\right), \quad i = 1,...,N \tag{5}$$

where N_i denotes the neighboring set of node i, c is the coupling strength, $\Gamma \in R^{n\times n}$ is the feedback gain matrix to be designed, it also should be positive definite [16]. a_{ij} represents the coupling strength of the information flowing from node j to node i, and $a_{ij} = a_{ji} > 0$ when $j \in N_i$. If agent i has access to the leader's state information, $d_i > 0$ and otherwise, $d_i = 0$.

Based on the TS fuzzy model introduced in [16], we rewrite $f\left(x_i(t)\right) - f\left(s(t)\right)$ as $g\left(x_i(t), e_i(t)\right) + h\left(x_i(t), e_i(t)\right)$, where $g\left(x_i(t), e_i(t)\right)$ can be approximated by a TS fuzzy model with the chosen premise variables, meanwhile $h\left(x_i(t), e_i(t)\right)$ cannot be approximated but can be restated as a product of bounded time-varying matrix $\bar{A}\left(x_i(t)\right)$ and $e_i(t)$. The fuzzy model of (4) is represented as the following rules:

Rule p: If $\theta_{i,1}(t)$ is M_{p1} and $\theta_{i,2}(t)$ is M_{p2} and …and $\theta_{i,k}(t)$ is M_{pk}, then

$$\dot{e}_i(t) = A_p e_i(t) + \bar{A}\left(x_i(t)\right)e_i(t) + u_i, \quad p = 1,...,r \tag{6}$$

where $\theta_i(t) = \left[\theta_{i,1}^T(t), \theta_{i,2}^T(t), ...\theta_{i,k}^T(t)\right]^T$ is the premise variable vector, M_{pl} for $p = 1,2,...,r$ and $l = 1,2,...,k$ represents the fuzzy sets, r is the number of if-then rules and A_p is the constant matrix. The compact form of the fuzzy model is as follows:

$$\dot{e}_i(t) = \sum_{p=1}^{r} h_p\left(\theta_i(t)\right)A_p e_i(t) + \bar{A}\left(x_i(t)\right)e_i(t) + u_i \quad i = 1,...,N \tag{7}$$

where

$$h_p\left(\theta_i(t)\right) = \omega_p\left(\theta_i(t)\right) \Big/ \sum_{p=1}^{r} \omega_p\left(\theta_i(t)\right)$$

$$\omega_p\left(\theta_i(t)\right) = \prod_{l=1}^{k} M_{pl}\left(\theta_{i,l}(t)\right)$$

where $M_{pl}\left(\theta_{i,l}(t)\right)$ is the grade of the membership of $\theta_{i,l}(t)$ in M_{pl}. Then

$$h_p\left(\theta_i(t)\right) \geq 0, \quad p = 1,2,...,r. \qquad \sum_{p=1}^{r} h_p\left(\theta_i(t)\right) = 1 \tag{8}$$

Assumption 1: $\bar{A}^T\left(x_i(t)\right)\bar{A}\left(x_i(t)\right) \leq Q$, where Q is a positive definite matrix and can be written as $Q = R^T R$.

3. THE MAIN RESULTS

In this section, based on the TS fuzzy modeling, the consensus problem of nonlinear MAS is investigated.

The error dynamics of (6) can be written as follows:

$$\dot{e}(t) = \left(\sum_{p=1}^{r} h_p \mathfrak{A}_p\right) e(t) + \bar{\mathfrak{A}}\left(x_i(t)\right)e(t) + c\bar{L} \otimes \Gamma e(t) \tag{9}$$

where

$$\mathfrak{A}_p = I_N \otimes A_p$$

$$\bar{\mathfrak{A}}\left(x_i(t)\right) = I_N \otimes \bar{A}\left(x_i(t)\right)$$

$$e(t) = \left[e_1^T, e_2^T, ..., e_N^T\right]^T$$

$$\bar{L} = L - diag\left\{d_1, d_2, ..., d_N\right\}$$

To obtain new stability conditions for MAS, consider the following null matrix identities that are obtained from (7) and (8):

$$\sum_{\rho=1}^{r} \dot{h}_\rho = 0 \Rightarrow \sum_{p=1}^{r}\sum_{\rho=1}^{r} h_p \dot{h}_\rho \left(e^T M_3 e\right) = 0 \tag{10}$$

$$\left[M_1 \dot{e} + M_2 e\right]^T \times \left[\dot{e} - \left(\sum_{p=1}^{r} h_p \mathfrak{A}_p + \bar{\mathfrak{A}} + c\bar{L} \otimes \Gamma\right)e\right] = 0 \tag{11}$$

It is assumed that M_1 has a linear relation with M_2 $\left(M_1 = \alpha M_2\right)$ where α is an arbitrary known scalar number, and M_3 is a symmetric matrix with appropriate dimension.

Theorem 1: The follower agents as described in (2) can reach consensus with the leader if $\bar{G}$ is connected or equivalently, at least one agent in each connected component of G has access to the state information of the leader and further, there exist symmetric matrices P_ρ and M_3, any matrix N, positive definite matrix Γ and scalar c such that the following LMIs hold for $p = 1,...,r$

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$$P_p \succ 0 \tag{12}$$

$$M_3 + I_N \otimes P_\rho \succ 0 \tag{13}$$

$$\begin{bmatrix} M_1+M_1^T & H_1 \\ * & H_2 \end{bmatrix} \prec 0$$

$$H_1 = I_N \otimes P_p - M_1^T\left(\mathfrak{A}_p + .5 I_N \otimes (\mathrm{I}_n + \mathrm{R}^T\mathrm{R})\right) + \mathrm{c}\bar{L}\otimes S + \alpha M_1 \tag{14}$$

$$H_2 = E_\phi + \alpha M_1^T(\mathrm{I}_N \otimes(-\mathrm{I}_n - \mathrm{R}^T\mathrm{R})) + \left[-\alpha\,\mathrm{c}\bar{L}\otimes S - \alpha M_1^T\mathfrak{A}_p + *\right]$$

where

$$E_\phi = \sum_{\rho=1}^{r}\phi_\rho\left(M_3 + I_N \otimes P_\rho\right)$$

$$M_1 = I_N \otimes N$$

$$S \triangleq N^T\Gamma$$

Proof: The following parameterized Lyapunov function candidate is chosen

$$V(t) = \sum_{p=1}^{r}\sum_{i=1}^{N} h_p e_i^T P_p e_i \tag{15}$$

By differentiating $V(t)$, one has

$$\dot{V}(t) = \sum_{q=1}^{r}\sum_{i=1}^{N}\frac{\partial h_q}{\partial t}e_i^T P_q e_i + \sum_{p=1}^{r}\sum_{i=1}^{N} h_p\left(\dot{e}_i^T P_p e_i + *\right)$$

$$= \sum_{q=1}^{r}\frac{\partial h_q}{\partial t}e^T\left(I_N \otimes P_q\right)e + \sum_{p=1}^{r}\left[h_p\dot{e}^T\left(I_N\otimes P_p\right)e + *\right] \tag{16}$$

By using the identities (10)-(11), one has

$$\dot{V}(t) = \sum_{q=1}^{r}\frac{\partial h_q}{\partial t}e^T\left(I_N\otimes P_q\right)e + \sum_{p=1}^{r}h_p\left[\dot{e}^T\left(I_N\otimes P_p\right)e + *\right] + \sum_{q=1}^{r}\frac{\partial h_q}{\partial t}e^T M_3 e + \{\left[M_1\dot{e} + M_2 e\right]^T\left[\dot{e} - \left(\sum_{p=1}^{r}h_p\mathfrak{A}_p + \bar{\mathfrak{A}} + c\bar{L}\otimes\Gamma\right)e\right] + *\} \tag{17}$$

$$= \sum_{p=1}^{r}h_p\Big(\sum_{q=1}^{r}\Big(\frac{\partial h_q}{\partial t}e^T\left(I_N\otimes P_q\right)e\Big) + \left[\dot{e}^T\left(I_N\otimes P_p\right)e + *\right] + \sum_{q=1}^{r}\frac{\partial h_q}{\partial t}e^T M_3 e + \left\{\left[M_1\dot{e} + M_2 e\right]^T\left[\dot{e} - \left(\mathfrak{A}_p + \bar{\mathfrak{A}} + c\bar{L}\otimes\Gamma\right)e\right] + *\right\} \tag{18}$$

Define

$$E_\phi = \sum_{\rho=1}^{r}\phi_\rho\left(M_3 + I_N\otimes P_\rho\right) \tag{19}$$

If $\left|\frac{\partial h_\rho}{\partial t}\right| \le \phi_\rho, \quad \rho = 1,\dots,r$ and $M_3 + I_N\otimes P_\rho \succ 0$ then

$$\dot{\mathrm{V}}(\mathrm{t}) \le \sum_{p=1}^{r}h_p\xi^T Z_p\xi \tag{20}$$

where

$$\xi = \left[\dot{e}^T \quad e^T\right]^T$$

$$Z_p = \begin{bmatrix} M_1+M_1^T & H_3 \\ * & H_4\end{bmatrix}$$

$$H_3 = I_N\otimes P_p - M_1^T\left(\mathfrak{A}_p + \bar{\mathfrak{A}} + c\bar{L}\otimes\Gamma\right) + M_2$$

$$H_4 = E_\phi + \left[M_2^T\left(-\mathfrak{A}_p - \bar{\mathfrak{A}} - c\bar{L}\otimes\Gamma\right) + *\right]$$

If $Z_p < 0$, then for any $\xi \ne 0$, $\dot{V}(t) < 0$

By using assumption 1, defining $M_1 \triangleq I_N\otimes N$ and remembering the constraint $M_2 = \alpha M_1$, one has

$$\begin{bmatrix} M_1+M_1^T & H_5 \\ * & H_6\end{bmatrix} \prec 0$$

$$H_5 = I_N\otimes P_p - M_1^T(\mathfrak{A}_p + .5 I_N \otimes(\mathrm{I}_n + \mathrm{R}^T\mathrm{R}) + \mathrm{c}\bar{\mathrm{L}}\otimes\Gamma) + \alpha M_1 \tag{21}$$

$$H_6 = E_\phi + \alpha M_1^T(\mathrm{I}_N\otimes(-\mathrm{I}_n - \mathrm{R}^T\mathrm{R})) + \left[\alpha M_1^T(-\mathfrak{A}_p - c\bar{L}\otimes\Gamma) + *\right]$$

To convert (21) into LMI, further manipulations are required to do on the bilinear term $M_1^T\cdot(c\bar{L}\otimes\Gamma)$ of (21) as follows

$$M_1^T\cdot(c\bar{L}\otimes\Gamma) = (I_N\otimes N^T)\cdot(c\bar{L}\otimes\Gamma) = c\bar{L}\otimes N^T\Gamma$$

Define the new decision variable $S \triangleq N^T\Gamma$, one has

$$M_1^T\cdot(c\bar{L}\otimes\Gamma) = c\bar{L}\otimes N^T\Gamma = c\bar{L}\otimes S$$

Consequently, (21) is obtained as

$$\begin{bmatrix} M_1+M_1^T & H_1 \\ * & H_2\end{bmatrix} \prec 0$$

$$H_1 = I_N\otimes P_p - M_1^T\left(\mathfrak{A}_p + .5 I_N\otimes(\mathrm{I}_n + \mathrm{R}^T\mathrm{R})\right) + \mathrm{c}\bar{L}\otimes S + \alpha M_1 \tag{22}$$

$$H_2 = E_\phi + \alpha M_1^T(\mathrm{I}_N\otimes(-\mathrm{I}_n - \mathrm{R}^T\mathrm{R})) + \left[-\alpha\,\mathrm{c}\bar{L}\otimes S - \alpha M_1^T\mathfrak{A}_p + *\right]$$

Therefore, the errors converge to zero and the consensus is achieved. The proof is completed. ■

4. ILLUSTRATIVE EXAMPLE

In this section, a numerical example is given to show the effectiveness of the theoretical results.

Example: Consider the multi-agent network in Fig. 1, consisting of N identical systems [16]. The dynamics of each node is described by the following chaotic equation

$$\dot{x}_i(t) = f(x_i(t)) + u_i \qquad i = 1,\dots,N \tag{23}$$

where

$$f(x(t)) = \begin{bmatrix} -\sigma x_{i1}(t) + \sigma x_{i2}(t) \\ \eta_1 x_{i1}(t) - \eta_2 x_{i2}(t) - x_{i1}(t)x_{i3}(t) \\ x_{i1}(t)x_{i2}(t) - bx_{i3}(t) \end{bmatrix} \tag{24}$$

By choosing the values of $(\sigma,\eta_1,\eta_2,b) = (10,28,-1,8/3)$ for chaos to emerge, the system in (23) becomes the Lorenz system. The error dynamics system, the difference between (23) and the leader $\dot{s}(t) = f(s(t))$, can be derived as [17]:

$$\dot{e}_i(t) = \begin{bmatrix} -\sigma e_{i1}(t) + \sigma e_{i2}(t) \\ \eta_1 e_{i1}(t) - \eta_2 e_{i2}(t) - x_{i1}(t)e_{i3}(t) + e_{i1}(t)e_{i3}(t) \\ -bx_{i3}(t) + x_{i1}(t)e_{i2}(t) - e_{i1}(t)e_{i2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -x_{i3}(t) & 0 & 0 \\ x_{i2}(t) & 0 & 0 \end{bmatrix} e_i(t) + u_i \tag{25}$$

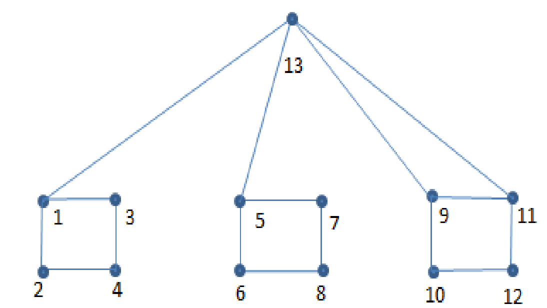


Fig. 1. Connected network topology.

Here, we choose $N = 13$. The TS fuzzy model (6) is used for modeling the nonlinear MAS as following:

Rule jk : if $x_{i1}(t)$ is M_{1j} and $e_{i1}(t)$ is M_{ek}, then

$$\dot{e}_i(t) = \breve{A}_{jk} e_i(t) + \bar{A}(x_i(t)) e_i(t) + u_i, \quad j,k = 1,2 \tag{26}$$

where $x_{i1}(t) \in [M_{11} \quad M_{12}]$, $e_{i1}(t) \in [M_{e1} \quad M_{e2}]$, $M_{11} = -20$, $M_{12} = 30$, $M_{e1} = -50$, and $M_{e2} = 50$

The augmented error system is as follows

$$\dot{e}_i(t) = \sum_{p=1}^{4} \lambda_p(x_i(t), e_i(t))(A_p e_i(t)) + \bar{A}(x_i(t)) e_i(t) + u_i \tag{27}$$

where

$A_1 = \breve{A}_{11}$, $A_2 = \breve{A}_{22}$, $A_3 = \breve{A}_{12}$, $A_4 = \breve{A}_{21}$

$$\breve{A}_{jk} = \begin{bmatrix} -\sigma & \sigma & 0 \\ \eta_1 & \eta_2 & M_{ek} - M_{1j} \\ 0 & M_{1j} - M_{ek} & -b \end{bmatrix}$$

$$\bar{A}(x_i(t)) = \begin{bmatrix} 0 & 0 & 0 \\ -x_{i3}(t) & 0 & 0 \\ x_{i2}(t) & 0 & 0 \end{bmatrix}$$

Remembering assumption 1, $\bar{A}(x_i(t))^T \bar{A}(x_i(t))$ is bounded by $Q = R^T R$ which

$$R = \begin{bmatrix} 0 & -60 & 40 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\lambda_p(x_i(t), e_i(t))$ can be calculated according to (6), and the membership functions are considered as in [2].

Here, it is supposed that $d_1 = 1$ and others $d_k = 0$ for $k = 2,\dots,13$ that means, only agent one has access to the states information of the leader.

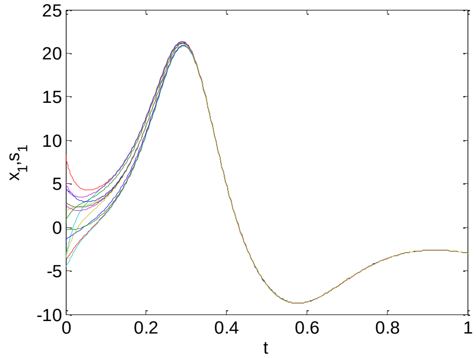
Solving the LMIs (11-13), the gain matrix Γ is obtained as follows:

$$\Gamma = \begin{bmatrix} 8.3923 & 6.6976 & 9.8409 \\ -0.0745 & 158.5477 & -4.4160 \\ -0.2948 & -1.8867 & 155.6960 \end{bmatrix}$$

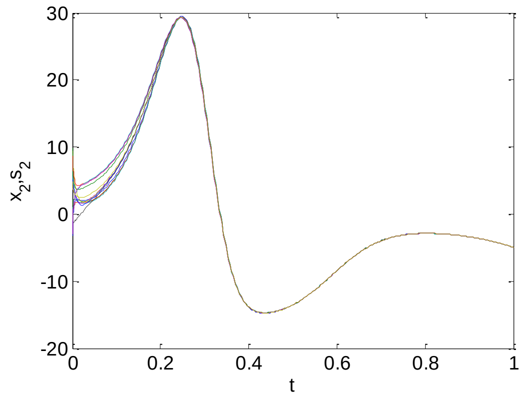
Fig. 2 shows the states of the agents using Γ and randomly chosen initial conditions. It is concluded from Fig. 2, that the consensus take place quickly and the follower agents track the leader's states for the future.

5. CONCLUSIONS

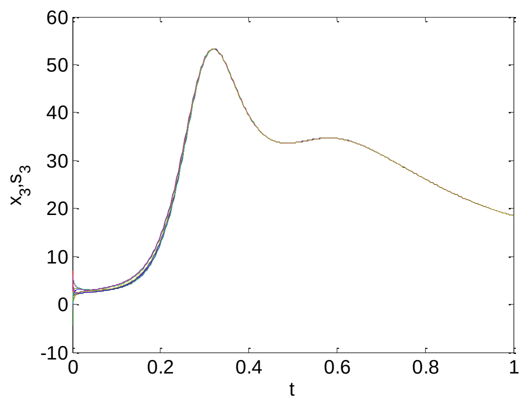
This paper has considered a consensus control problem of leader-following nonlinear MASs. Initially, the nonlinear error dynamics between the leader agent and nonlinear follower agents were calculated. Thus, the consensus problem of leader-following system changed into the stability analysis of the error dynamic system. Then, the exact TS fuzzy model of error dynamic achieved via the concept of sector nonlinearity. Sufficient asymptotically stability conditions obtained based on the FLF in terms of LMIs. Moreover, based on the behavior of the closed-loop system, some new null terms proposed. The slack matrices, defined in null terms, had some advantages such as: increasing the degree of freedom, converting the sufficient stability conditions to the LMI ones, decoupling the system matrices from the Lyapunov ones, and also, generate more relax conditions. LMI conditions were achieved by utilizing the Kronecker product. The simulation results were shown the effectiveness of the proposed approach.



(a)



(b)



(c)

Fig. 2. Consensus of multi-agent network and states evolutions of the followers ($x_i, i = 1,2,3$) and the leader ($s_i, i = 1,2,3$).

6. References

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