Relaxed distributed fuzzy controller design with input constraints for a class of nonlinear first-order hyperbolic PDE systems

Mohammad Mehdi Mardani, Mokhtar Shasadeghi*, Behrouz Safarinejadian

***Abstract*—In this paper, the stability problem of nonlinear first order hyperbolic partial differential equations (PDE) systems is investigated. Based on Lyapunov stability theorem, the sufficient conditions to guarantee the stability of Takagi-Sugeno (TS) fuzzy hyperbolic PDE model are achieved in terms of spatially varying linear matrix inequalities (SVLMI). To investigate the exponentially stabilization of nonlinear first order hyperbolic PDE systems, a fuzzy Lyapunov function is considered. Then, some new space varying slack matrices are introduced to conduct the stability analysis. The proposed stability conditions are more relaxed than the newly published one. Furthermore, the problem of applying some constraints on control input is studied through this paper. Hence, the performance of the controller is improved in the proposed approach. Finally, in order to evaluate the validity of the proposed approach, a practical application of nonisothermal plug flow reactor (PFR) is considered.**

***Index Terms*—First order hyperbolic PDE systems, TS fuzzy PDE model, Fuzzy Lyapunov function, Slack matrices, Space varying LMI.**

I. INTRODUCTION

MOST of the real world processes are described by nonlinear systems [1-2]. Takagi-Sugeno (TS) fuzzy model constructs a framework to introduce a nonlinear system in a convex combination of some local linear subsystems [3-4]. Synthesizing a controller for TS fuzzy model has been attracted lots of researchers' attentions. However, most of the research topics have devoted to ordinary differential equations (ODE) [5-7]. Meanwhile, large amount of industrial processes are distributed in space as well as time such as chemical reactors, heat conduction, and fluid flow. Hence, their behaviors depend on more than one independent variable [8-10]. Generally, the mathematical models of these processes are described by partial differential equations (PDE) [8, 11]. Whereas PDE applications have infinite-dimensional characteristics, applying ODE methods to design a controller for PDE systems is more difficult. On the other hand, designing a distributed controller needs more efforts and energy [12-14].

Due to the behavior of the spatially differential operator (SDO), the PDE systems are split into three well-known categories: hyperbolic, parabolic and elliptic [15]. In parabolic PDE systems, the Eigen-spectrums are usually partitioned into two groups: a finite-dimensional slow component and an infinite-dimensional fast one [16]. One of the main features of hyperbolic PDE systems is their same (or nearly same) amount of energy [13]. Hence, their dynamical behavior can be exactly represented by utilizing an infinite number of modes.

During recent years, significant number of research topics have devoted to analyzing the stability of nonlinear first order hyperbolic PDE systems. Ref. [13] investigates the problem of designing a distributed fuzzy controller. The TS fuzzy PDE model of nonlinear first order hyperbolic PDE systems is presented in [13] for the first time. Then, sufficient stability conditions are achieved in terms of spatially differential LMI conditions [13]. Ref. [14] designs a constraint distributed H_∞ fuzzy controller. However, obtained space varying LMI-based conditions depends on the delta-Dirac function [14]. Ref. [12] designs a relaxed distributed fuzzy controller which can guarantee the exponentially stability of the closed-loop system. In [10], the problem of designing a reliable static output feedback controller is investigated. Ref. [10] guarantees the performance of the system in the presence of exogenous disturbances. More recently, we design a distributed saturated polynomial fuzzy controller for a class of semi-linear hyperbolic PDE systems [8, 11]. According to the best of our knowledge, this paper tries the first attempt to design relaxed stability conditions with input constraints by introducing slack matrices.

This paper investigates the problem of designing relaxed distributed fuzzy controller with input constraints for a class of nonlinear first order hyperbolic PDE systems. Initially, the nonlinear first order hyperbolic PDE system is accurately modeled in a TS fuzzy PDE structure by sector nonlinearity approach. Then, the fuzzy Lyapunov function is proposed to investigate the stability of the closed-loop system. Through the stability analysis, some new slack matrices are introduced and employed to increase the degrees of freedom in space varying LMI conditions. Furthermore, some input constraints on control input are proposed to improve the performance of the controller. The main key ideas of this paper can be enumerated as follows:

1. Utilizing fuzzy Lyapunov function to obtain more relaxed stability conditions.
2. Employing slack matrices results in increasing the degrees of freedom in space varying LMI conditions.
3. Utilizing input energy constraints causes improving the performance of the controller.

Manuscript received December 15, 2016; accepted May 22, 2017
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4. Omitting some strict constraints on Lyapunov and convection matrices.

Finally, in order to illustrate the merits and effectiveness of the proposed approach, a chemical tubular reactor called “nonisothermal PFR” is used.

The rest of this paper is organized as follows: Section 2 studies the preliminaries; the main results and contributions of this paper are presented in Section 3. Section 4 deals with simulation results and discussions; and finally, the paper will be closed by conclusions in Section 5.

II. Preliminaries

Consider the following nonlinear first order hyperbolic PDE system:

$$\frac{\partial y(x,t)}{\partial t} = \theta(y(x,t),x)\frac{\partial y(x,t)}{\partial x} + f(y(x,t),x) + g(y(x,t),x)u(x,t) \quad (1)$$

where $y(x,t)$ and $u(x,t)$ are state variables and control input, respectively. $\theta(y(x,t),x) \in R^{2\times 2}$, $f(y(x,t),x) \in R^2$ and $g(y(x,t),x) \in R^2$ are known smooth nonlinear functions.

The nonlinear system (1) can be described with the following fuzzy IF-THEN rules:

Plant Rule *i*:

IF $\xi_1(x,t)$ is M_{i1} and … and $\xi_l(x,t)$ is M_{il}, THEN:

$$\frac{\partial y(x,t)}{\partial t} = \theta_i(x)\frac{\partial y(x,t)}{\partial x} + A_i(x)y(x,t) + B_i(x)u(x,t) \quad (2)$$

where $i \in \{1,\cdots,r\}$ and r denotes the number of fuzzy rules. $\theta_i(x)$, $A_i(x)$, and $B_i(x)$ are known space varying matrices with appropriate dimensions. The overall TS fuzzy PDE model is achieved as follows:

$$\frac{\partial y(x,t)}{\partial t} = \sum_{i=1}^{r} h_i(\xi)\left\{\theta_i(x)\frac{\partial y(x,t)}{\partial x} + A_i(x)y(x,t) + B_i(x)u(x,t)\right\} \quad (3)$$

where

$$h_i(\xi) = \frac{\prod_{j=1}^{r} M_{ij}(\xi_j(x,t))}{\sum_{i=1}^{r}\prod_{j=1}^{r} M_{ij}(\xi_j(x,t))} \quad (4)$$

Based on the PDC concept, the fuzzy controller is defined as follows:

Control Rule *i*:

IF $\xi_1(x,t)$ is M_{i1} and … and $\xi_l(x,t)$ is M_{il}, THEN:

$$u(x,t) = K_i(x)y(x,t) \quad (5)$$

where $K_i(x)$ are control gain matrices which will be calculated during the design procedure. The overall fuzzy controller is given by

$$u(x,t) = \sum_{i=1}^{r} h_i(\xi)K_i(x)y(x,t) \quad (6)$$

Substituting (6) in (3), yields

$$\frac{\partial y(x,t)}{\partial t} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)\, h_j(\xi)\left\{\theta_i(x)\frac{\partial y(x,t)}{\partial x} + \Big(A_i(x)y(x,t) + B_i(x)K_j(x)\Big)y(x,t)\right\} \quad (7)$$

In order to increase the degrees of freedom, we propose the following null terms:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)\left\{2\left(M_1^T(x)y(x,t) + M_2^T(x)\frac{\partial y(x,t)}{\partial t} + M_3^T(x)\frac{\partial y(x,t)}{\partial x}\right)^T\left(\frac{\partial y(x,t)}{\partial t} - \theta_i(x)\frac{\partial y(x,t)}{\partial x} - \left[A_i(x) + B_i(x)K_j(x)\right]y(x,t)\right)\right\} = 0 \quad (8)$$

where matrices $M_1(x)$, $M_2(x)$, and $M_3(x)$ are space varying slack matrices which will be calculated during the design procedure. In addition, based on the property of membership functions, the following null term will be introduced:

$$\sum_{i=1}^{r}\frac{\partial h_i(\xi)}{\partial t} y^T(x,t)M_4(x)y(x,t) = 0 \quad (9)$$

where $M_4(x)$ is symmetric slack matrix.

In order to investigate the stability of the closed-loop system, the following Lyapunov function is considered:

$$V(t) = \int_{l_1}^{l_2}\sum_{i=1}^{r} h_i(\xi)\{y^T(x,t)P_i(x)y(x,t)\}\,dx \quad (10)$$

where $P_i(x)$ are Lyapunov matrices.

From now on, for brevity, we use $y, u, \theta_i, A_i, B_i, K_i, M_1, M_2, M_3, M_4$ and P_i instead of $y(x,t), u(x,t), \theta_i(x), A_i(x), B_i(x), K_i(x), M_1(x)$, $M_2(x), M_3(x), M_4(x)$, and $P_i(x)$, respectively.

Lemma 1 [13]. If the Lyapunov function (10) satisfy

$$\frac{dV(t)}{dt} + 2\rho V(t) \le 0 \quad (11)$$

Then, the nonlinear system is exponentially stable with decay rate ρ.

III. Main Results

In this section, the sufficient conditions to guarantee the exponentially stability of the TS fuzzy PDE model (7) will be presented in theorem 1. Furthermore, the sufficient conditions to guarantee the energy constraint on control input will be obtained in theorem 2.

Theorem 1. Assume the time derivatives of membership functions are bounded ($\frac{\partial h_i}{\partial t} \le \phi_i$), and α is a pre-given scalar value. The TS fuzzy PDE model of nonlinear hyperbolic PDE system (1) is exponentially stable, if there exist space varying matrices S_j, N_1 and N_2 and symmetric space varying matrices T_i and N_3 such that

$$\begin{cases} T_i > 0 \quad : \ i = \{1,\cdots,r\} \\ T_i + N_3 > 0 \quad : \ i = \{1,\cdots,r\} \\ \Xi_{ii} < 0 \quad : \ i = \{1,\cdots,r\} \\ \frac{1}{r-1}\Xi_{ii} + \frac{1}{2}\left(\Xi_{ij} + \Xi_{ji}\right) < 0 \\ \quad : \ 1 \le i \ne j \le r \end{cases} \quad (12)$$

where

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11,ij} & \Xi_{12,ij} & -\theta_i N_2^T - N_1 A_i^T - S_j^T B_i^T \\ * & \alpha(N_1 + *) & -\theta_i N_2^T + \alpha N_1 \\ * & * & -(\theta_i N_2^T + *) \end{bmatrix}$$

$\Xi_{11,ij} = T_\emptyset + 2\rho T_i - (B_i S_j + A_i N_1^T + *)$
$T_\emptyset = \sum_{k=1}^{r} \emptyset_k (T_k + N_3)$
$\Xi_{12,ij} = \alpha T_i + \alpha N_1^T - S_j^T B_i^T - N_1 A_i^T$
are satisfied. Then the controller gain matrices are achieved as follows:

$$K_j = S_j N_1^{-T}, \quad j = 1, \dots r \tag{13}$$

Proof. From (11), we have

$$\begin{aligned}\frac{dV(t)}{dt} &+ 2\rho V(t) \\ &= \int_{l_1}^{l_2} \sum_{i=1}^{r} \frac{\partial h_i}{\partial t} \{y^T P_i y\}\, dx \\ &+ \int_{l_1}^{l_2} \sum_{i=1}^{r} h_i \left\{ \left(\frac{\partial y}{\partial t}\right)^T P_i y + y^T P_i \left(\frac{\partial y}{\partial t}\right) \right\} dx \\ &+ \int_{l_1}^{l_2} \sum_{i=1}^{r} h_i \{y^T 2\rho P_i y\}\, dx \\ &= \int_{l_1}^{l_2} \sum_{i=1}^{r} h_i \left\{ y^T \left\{ \sum_{k=1}^{r} \frac{\partial h_k}{\partial t} P_k \right\} y + \left(\frac{\partial y}{\partial t}\right)^T P_i y \right. \\ &\left. + y^T P_i \left(\frac{\partial y}{\partial t}\right) + y^T 2\rho P_i y \right\} dx < 0\end{aligned} \tag{14}$$

By utilizing the null terms (8) and (9), inequality (14) can be rewritten as follows:

$$\begin{aligned}\frac{dV(t)}{dt} + 2\rho V(t) \le \int_{l_1}^{l_2} \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j & \left\{ 2 \left(M_1^T y + M_2^T \frac{\partial y}{\partial t} + M_3^T \frac{\partial y}{\partial x} \right)^T \left(\frac{\partial y}{\partial t} - \theta_i \frac{\partial y}{\partial x} - [A_i + B_i K_j] y \right) \right. \\ & + y^T \left\{ \sum_{k=1}^{r} \frac{\partial h_k}{\partial t} (P_k + M_4) \right\} y \\ & + \left(\frac{\partial y}{\partial t}\right)^T P_i y + y^T P_i \left(\frac{\partial y}{\partial t}\right) \\ & \left. + y^T 2\rho P_i y \right\} dx < 0\end{aligned} \tag{15}$$

Assume that the time derivatives of membership functions are bounded as $\frac{\partial h_i}{\partial t} \le \phi_i$. If $P_k(x) + M_4 > 0$, then, we have

$$\begin{aligned}\frac{dV(t)}{dt} + 2\rho V(t) \le \int_{l_1}^{l_2} \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j & \left\{ 2 \left(M_1^T y + M_2^T \frac{\partial y}{\partial t} + M_3^T \frac{\partial y}{\partial x} \right)^T \left(\frac{\partial y}{\partial t} - \theta_i \frac{\partial y}{\partial x} - [A_i + B_i K_j] y \right) \right. \\ & + y^T \left\{ \sum_{k=1}^{r} \phi_k (P_k + M_4) \right\} y \\ & + \left(\frac{\partial y}{\partial t}\right)^T P_i y + y^T P_i \left(\frac{\partial y}{\partial t}\right) \\ & \left. + y^T 2\rho P_i y \right\} dx\end{aligned} \tag{16}$$

Based on congruence lemma, one has

$$\int_{l_1}^{l_2} \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \begin{bmatrix} y \\ \frac{\partial y}{\partial t} \\ \frac{\partial y}{\partial x} \end{bmatrix}^T \hat{\Xi}_{ij} \begin{bmatrix} y \\ \frac{\partial y}{\partial t} \\ \frac{\partial y}{\partial x} \end{bmatrix} dx < 0 \tag{17}$$

where

$$\hat{\Xi}_{ij} = \begin{bmatrix} \hat{\Xi}_{11,ij} & \hat{\Xi}_{12,ij} & -M_1\theta_i - A_i^T M_3^T - K_j^T B_i^T M_3^T \\ * & M_2 + * & -M_2\theta_i + M_3^T \\ * & * & -(M_3\theta_i + *) \end{bmatrix}$$

$\hat{\Xi}_{11,ij} = P_\emptyset + 2\rho P_i - (M_1 B_i K_j + M_1 A_i + *)$
$P_\emptyset = \sum_{k=1}^{r} \emptyset_k (P_k + M_4)$
$\hat{\Xi}_{12,ij} = P_i + M_1 - K_j^T B_i^T M_2^T - A_i^T M_2^T$
Inequality (17) is equivalent to $\hat{\Xi}_{ij} < 0$. Pre- and post-multiplying both sides of $\hat{\Xi}_{ij}$ by
$\begin{bmatrix} M_1^{-1} & 0 & 0 \\ * & M_2^{-1} & 0 \\ * & * & M_3^{-1} \end{bmatrix}$, one has

$$\begin{bmatrix} \Xi_{11,ij} & \Xi_{12,ij} & \Xi_{13,ij} \\ * & M_2^{-T} + * & -\theta_i M_3^{-T} + M_2^{-1} \\ * & * & -(\theta_i M_3^{-T} + *) \end{bmatrix} < 0 \tag{18}$$

where
$\Xi_{11,ij} = M_1^{-1} P_\emptyset M_1^{-T} + 2\rho M_1^{-1} P_i M_1^{-T} - (B_i K_j M_1^{-T} + A_i M_1^{-T} + *)$
$\Xi_{12,ij} = M_1^{-1} P_i M_2^{-T} + M_2^{-T} - M_1^{-1} K_j^T B_i^T - M_1^{-1} A_i^T$
$\Xi_{13,ij} = -\theta_i M_3^{-T} - M_1^{-1} A_i^T - M_1^{-1} K_j^T B_i^T$
Defining the following change of variables:

$$\begin{aligned} N_1 &= M_1^{-1} \\ \alpha N_1 &= M_2^{-1} \\ N_2 &= M_3^{-1} \\ N_3 &= M_1^{-1} M_4 M_1^{-T} \\ T_\emptyset &= M_1^{-1} P_\emptyset M_1^{-T} \\ S_j &= K_j M_1^{-T} \\ T_i &= M_1^{-1} P_i M_1^{-T} \end{aligned} \tag{19}$$

The proof will be completed.

Theorem 2. Assume the Lyapunov function (10) is bounded ($V(t) = \int_{z_1}^{z_2} y^T (\sum_{i=1}^{r} h_i P_i) y dx < \eta$). The constraint $\|u\|_2^2 < u_{max}$ is guaranteed if there exist positive definite symmetric matrices T_i and any matrices S_i such that

$$\begin{bmatrix} T_i & S_i^T \\ S_i & u_{max}\eta^{-1} \end{bmatrix} > 0 \tag{20}$$

$$\begin{bmatrix} -(l_2 - l_1)^{-1}\eta & y_0^T \\ y_0 & -P_i \end{bmatrix} < 0 \tag{21}$$

for all $i = 1, 2, \cdots, r$.

Proof. Based on the definition of the norm 2 [13], we have

$$\|u\|_2^2 = \int_{l_1}^{l_2} y^T \left(\sum_{i=1}^{r} h_i K_i \right)^T \left(\sum_{i=1}^{r} h_i K_i \right) y\, dx < u_{max} \tag{22}$$

Using $0 < \|u\|_2^2 < \eta^{-1} V(t) u_{max} < u_{max}$ and (22) result in

$$\int_{l_1}^{l_2} y^T \left[\left(\sum_{i=1}^{r} h_i K_i \right)^T \left(\sum_{i=1}^{r} h_i K_i \right) - u_{max}\eta^{-1} \sum_{i=1}^{r} h_i P_i \right] y\, dx < 0 \tag{23}$$

whereas $u_{max}\eta^{-1}$ is positive definite scalar, pre- and post-multiplying both sides of equation (23) by $M_1^{-1}\sqrt{u_{max}^{-1}\eta}$, we have

$$\sum_{i=1}^{r} h_i T_i + \left(\sum_{i=1}^{r} h_i S_i\right)^T u_{max}^{-1}\eta \left(\sum_{i=1}^{r} h_i S_i\right) > 0 \quad (24)$$

Using Schur complement, one has

$$\sum_{i=1}^{r} h_i \begin{bmatrix} T_i & S_i^T \\ S_i & u_{max}\eta^{-1} \end{bmatrix} > 0 \quad (25)$$

Hence, if the LMI conditions (20) are satisfied, then (25) will be guaranteed. Additionally, in order to guarantee $V(t) = \int_{z_1}^{z_2} y^T(\sum_{i=1}^{r} h_i P_i) y dx < \eta$ which is used in inequality (23), we have

$$\begin{aligned} V(t) &= \int_{z_1}^{z_2} y^T \left(\sum_{i=1}^{r} h_i P_i\right) y dx \le V(0) \\ &= \int_{z_1}^{z_2} y_0^T \left(\sum_{i=1}^{r} h_i P_i\right) y_0 dx \\ &< \int_{l_1}^{l_2} (l_2 - l_1)^{-1} \eta dx \end{aligned} \quad (26)$$

(26) is equivalent to

$$y_0^T \left(\sum_{i=1}^{r} h_i P_i\right) y_0 - (l_2 - l_1)^{-1}\eta < 0 \quad (27)$$

Using Schur complement, we have

$$\sum_{i=1}^{r} h_i \begin{bmatrix} -(l_2 - l_1)^{-1}\eta & y_0^T \\ y_0 & -P_i \end{bmatrix} < 0 \quad (28)$$

The equation (21) is directly concluded from (28). The proof is completed. ■

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, in order to illustrate the effectiveness and merits of the proposed approach, the nonisothermal PFR system is considered. In PFR tubular reactor, the reaction $A \to \tilde{b}B$ takes places. The PFR application can be modeled in the nonlinear first order hyperbolic PDE structure (1) with the following elements [8, 11]:

$$\begin{aligned} \theta(y(x,t),x) &= -\frac{v}{L}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ f(y(x,t),x) &= \begin{bmatrix} \Theta_1 f_0 - b y_1 \\ \Theta_2 f_0 \end{bmatrix} \\ g(y(x,t),x) &= \begin{bmatrix} b \\ 0 \end{bmatrix} \\ f_0 &= (1-\Xi_{2e})Z_2 - y_2 Z_1 \\ Z_1 &= \exp\left(\frac{\beta y_1}{1+y_1}\right) \\ Z_2 &= Z_1 - 1 \end{aligned} \quad (29)$$

where

$$\Xi_{2e}(x) = 1 - \exp\left(-\frac{\Theta_2 L}{v} x\right)$$

where $\beta = \frac{E}{RT_{in}}, \Theta_1 = \delta\Theta_2$, and $\Theta_2 = k_0 \exp(-\beta)$. The numerical values of nonisothermal PFR parameters are presented in Table. I.

The overal TS fuzzy PDE model of PFR system is obtained based on sector nonlinearity approach [13, 17]. The PFR system is exactly represented by the TS fuzzy PDE model (3) by defining the following matrices [13]:

TABLE I
THE NUMERICAL VALUES OF PFR SYSTEM

Parameters	Numerical values
v	0.025 m/s
L	1 m
E	11250 cal/mol
k_0	06 s-1
$b = 4h/\rho_p C_p d$	0.2 s-1
$C_{A,in}$	0.02 mol/L
R	1.986 cal/(mol.K)
T_{in}	340 K
δ	0.25

$$\begin{aligned} \theta_i &= \theta = diag\left\{-\frac{v}{L}, -\frac{v}{L}\right\}, \ \forall i \in \{1,2,\cdots,r\} \\ B_i &= B = \begin{bmatrix} b \\ 0 \end{bmatrix}, \quad \forall i \in \{1,2,\cdots,r\} \\ A_1 &= \begin{bmatrix} \Theta_1(1-\Xi_{2e}(x))c_1 - b & -\Theta_1 a_1 \\ \Theta_2(1-\Xi_{2e}(x))c_1 & -\Theta_2 a_1 \end{bmatrix} \\ A_2 &= \begin{bmatrix} \Theta_1(1-\Xi_{2e}(x))c_2 - b & -\Theta_1 a_1 \\ \Theta_2(1-\Xi_{2e}(x))c_2 & -\Theta_2 a_1 \end{bmatrix} \\ A_3 &= \begin{bmatrix} \Theta_1(1-\Xi_{2e}(x))c_1 - b & -\Theta_1 a_2 \\ \Theta_2(1-\Xi_{2e}(x))c_1 & -\Theta_2 a_2 \end{bmatrix} \\ A_4 &= \begin{bmatrix} \Theta_1(1-\Xi_{2e}(x))c_2 - b & -\Theta_1 a_2 \\ \Theta_2(1-\Xi_{2e}(x))c_2 & -\Theta_2 a_2 \end{bmatrix} \end{aligned} \quad (30)$$

Also, the membership functions are defined as follows:

$$\begin{aligned} h_1 &= M_{11}M_{12} \\ h_2 &= M_{11}M_{22} \\ h_3 &= M_{21}M_{12} \\ h_4 &= M_{21}M_{22} \end{aligned} \quad (31)$$

where

$$M_{11} = \frac{Z_1 - \alpha_2}{\alpha_1 - \alpha_2},$$

$$M_{21} = 1 - M_{11},$$

$$M_{12} = \begin{cases} \frac{Z_2 - c_2 y_1}{y_1(c_1 - c_2)}, y_1 \ne 0 \\ \frac{\beta - c_2}{c_1 - c_2}, y_1 = 0 \end{cases},$$

$$M_{22} = 1 - M_{12}.$$

A. Feasibility region

The obtained stability conditions in Theorem 1 are exploited to investigate the stabilization region of the TS fuzzy hyperbolic PDE model. To compare the results of the proposed approach and [13], we assume $\rho = 2000$. Furthermore, consider $0.018 \le \beta_2 \le 0.034$ and $0 \le v \le 0.16$ at the interval of 0.01 and 0.02, respectively. The feasibility region is presented in Fig. 1. The results illustrate that, because of utilizing the slack matrices with more degree of freedom in LMI-based conditions, the proposed approach is less conservative than [13].

B. Design controller with input constraint

In this section, we assume ρ=0.005, v=0.025, and β_2=0.0581. Then, the proposed conditions in theorems 1 and 2 are numerically solved by utilizing Yalmip toolbox in MATLAB. Hence, the feasible solutions are obtained and

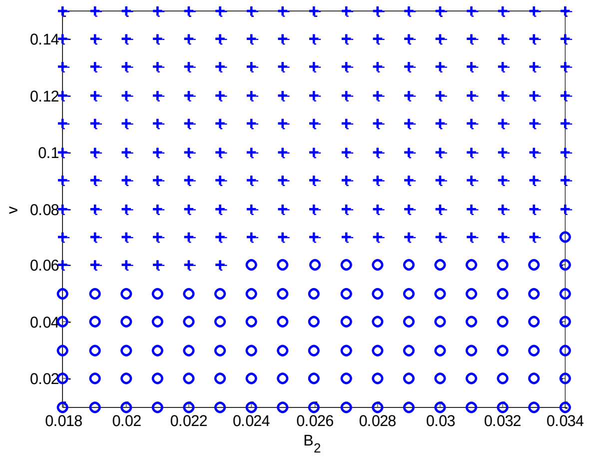


Fig.1. Comparing the feasibility region between the proposed approach (+,o) and [13] (+).

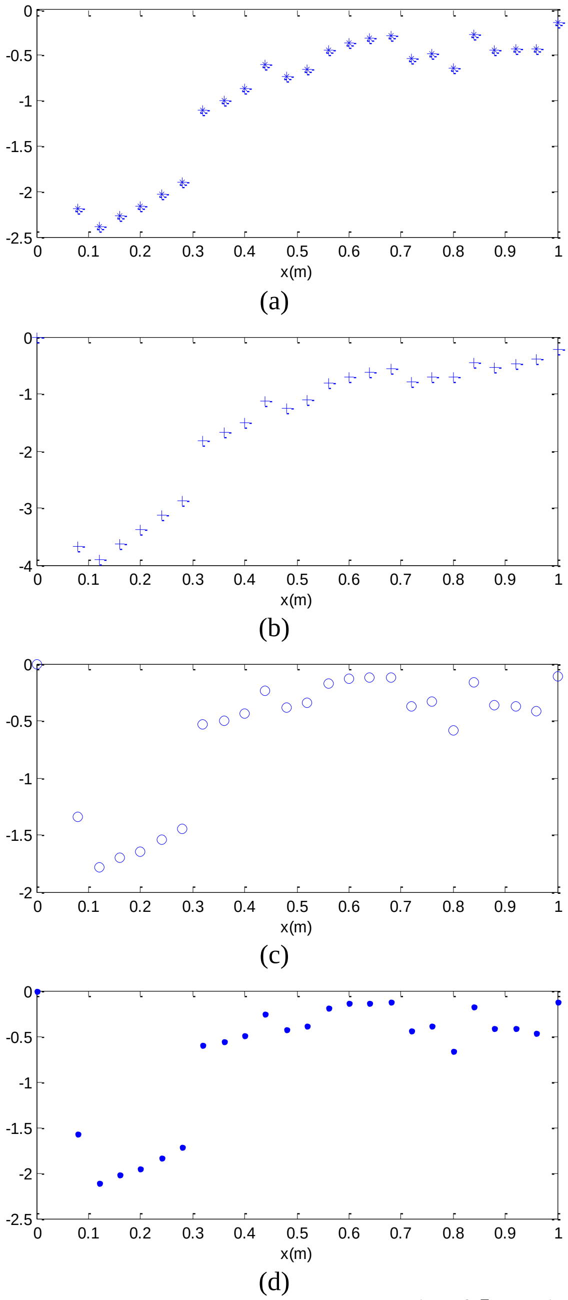


Fig. 2. The first array of control gain matrices $K_i = [K_i^1 \quad K_i^2]^T$. (a) K_1^1 (b) K_2^1 (c) K_3^1 (d) K_4^1.

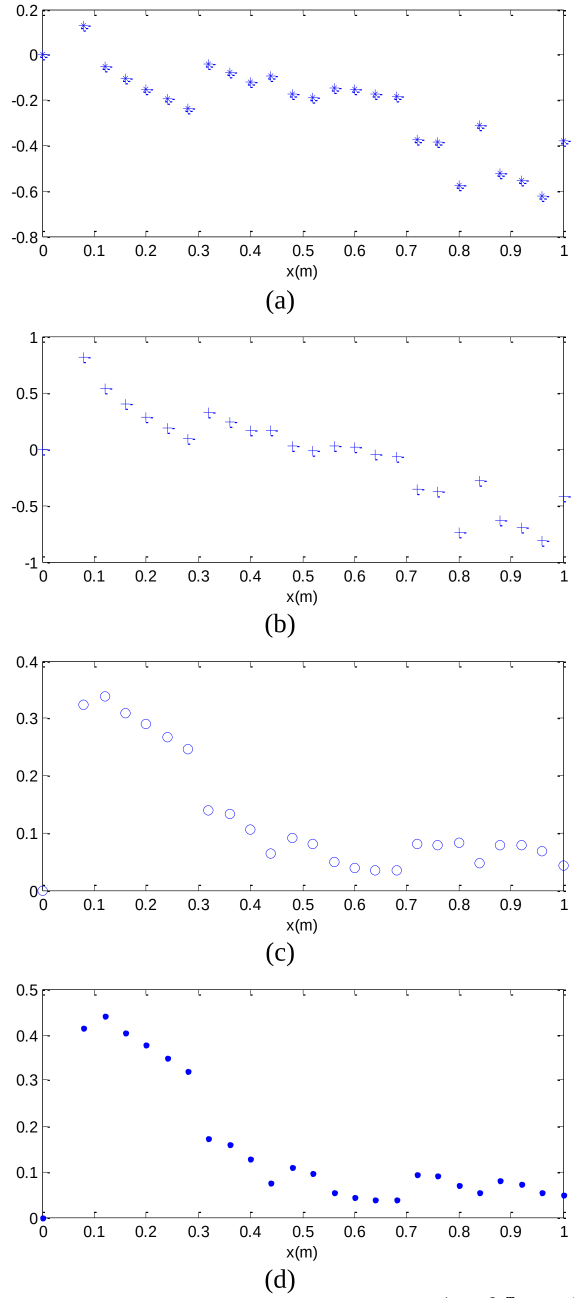
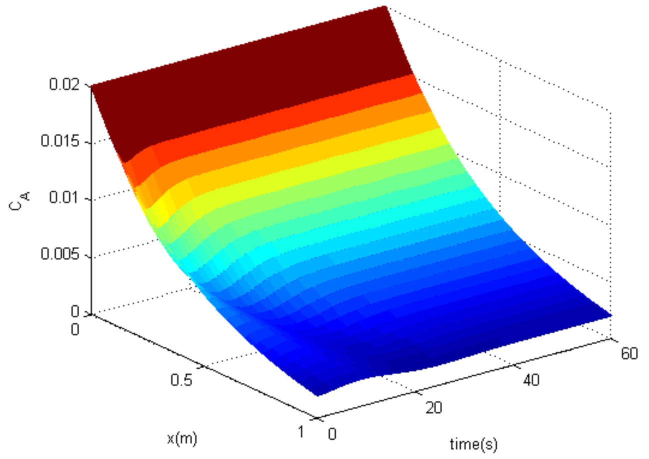


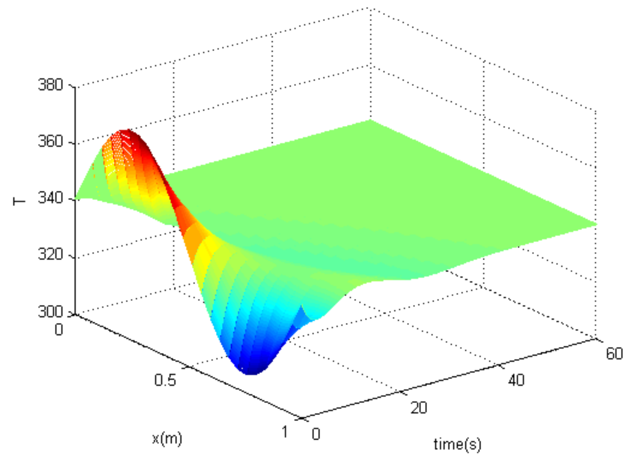
Fig.3. The second array of control gain matrices $K_i = [K_i^1 \quad K_i^2]^T$. (a) K_1^2 (b) K_2^2 (c) K_3^2 (d) K_4^2.

the control gain matrices are achieved. Figures 2 and 3 illustrate the control gain matrices. The first and second arrays of control gain matrices are illustrated in Figs. 2 and 3, respectively. Fig. 4 illustrates the behavior of the controlled state variables. Furthermore, the evolution of signal control is depicted in Fig. 5 (a). Fig. 5 (b) indicates the evolution of controller signal based on the method presented in [13].

From practical points of view, one of the important issues to deal with designing a controller for nonisothermal PFR application is the amplitude of the control signal. The amplitude of the control signal must be applicable. As it is observed in Fig. 5, the amplitude of the control signal for the proposed approach (Fig. 5 (a)) is more suitable than [13]

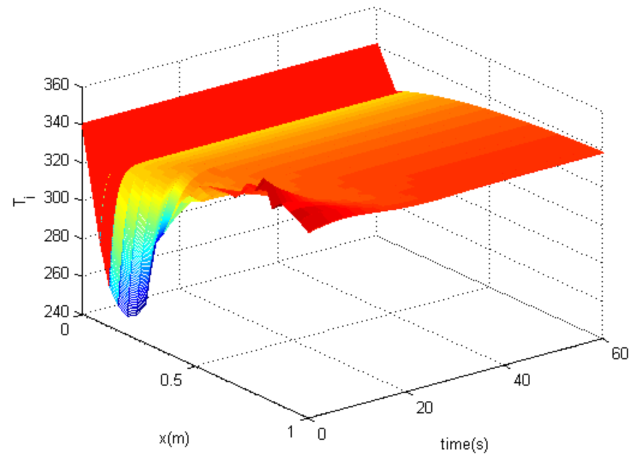


(a)

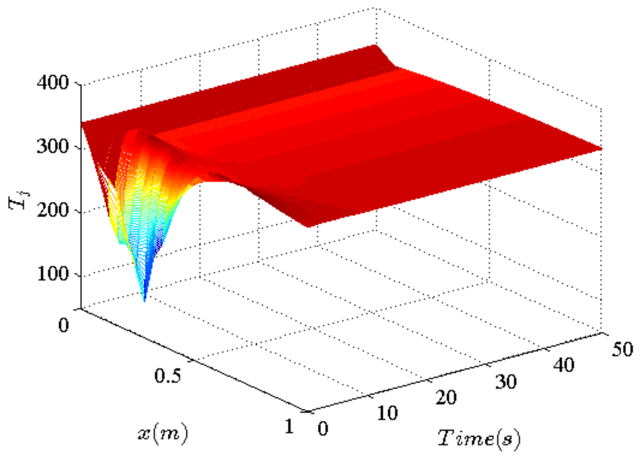


(b)

Fig.4. Evolutions of state variables: (a) Reactant concentration, (b) Reactant temperature.



(a)



(b)

Fig.5. Evolutions of jacket temperature. (a) proposed approach, (b) Ref. [13].

which is illustrated in Fig. 5 (b). Furthermore, it is more applicable than [10]. Hence, utilizing the control constraint on the control input improve the performance of the control signal than [10, 13].

V. CONCLUSIONS

This paper investigated the stabilization problem of nonlinear first order hyperbolic PDE systems. From this paper, we can conclude that, utilizing the fuzzy Lyapunov function and introducing new slack matrices resulted in increasing the degrees of freedom in space varying LMI conditions. Hence, the proposed approach decreases the conservativeness in control synthesis and stability analysis of TS fuzzy PDE models. Furthermore, employing constraints on the control input improve the performance of the controller. In addition, some strict conditions on Lyapunov and convection matrices are omitted by the proposed approach. Finally, the PFR application is used to investigate the applicability of the proposed approach. The simulation results clearly indicate the performance and applicability of the controller than timely references. Additionally, the results show that, the stabilization conditions are more relaxed than newly published works.

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